Abstract: -- The data rates and the supported range in communication systems, can be increased using MIMO (Multiple input and multiple output) technique. MIMO technique uses multiple antennas at both transmitter and receiver. MIMO systems use Orthogonal frequency division multiplexing (OFDM) technique for multicarrier modulation. QR decomposition (QRD) is the first step in the decoding of the MIMO receiver. Gram Schmidt, Householder and Givens described QR decomposition method which are computationally intensive as these involve division operation for normalization. The computation complexity of these methods for MIMO-OFDM systems is difficult to handle because QR decomposition is performed for each subcarrier. Sphere decoder is an efficient decoder for MIMO systems. In this paper we use Modified Householder’s method for reducing the computation complexity without affecting the system packet error rate (PER) performance. The simulation process is carried out in all different models of 802.11 TGAC channels.

Keywords: -- MIMO decoder, QR decomposition, Sphere decoder.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) is one of the main techniques for achieving high throughput in wireless communication. Currently many upcoming cutting edge wireless communication technologies like, IEEE 802.11ac, IEEE 802.16e and LTE adapted MIMO technology. With MIMO technology, the complexity of the signal processing to detect the transmitted signal with low probability of error increases. This process is usually termed as MIMO detector or decoder. The Singular Value Decomposition (SVD) [1] technique is used in closed loop MIMO systems. In the open loop MIMO systems, QR decomposition (QRD) of the channel matrix is generally used in the MIMO detection module due to its simplicity. The QR decomposition can be used in numerous MIMO decoders like Zero Forcing (ZF), minimum mean squared error (MMSE), and maximum likelihood performance achieving sphere decoder [2]. To support all the MIMO decoders mentioned above, a high efficient reduced complex QR decomposition module is needed. The QRD factorizes the channel matrix H in to the product of a unitary matrix Q and triangular matrix R. The authors in [3] have proposed a QR decomposition method that requires fewer multiplications compared to conventional methods. But, there is no mention about the number of divisions required for QR decomposition, which consume significant resources when implemented. The Sorted QR decomposition as discussed in [4] reduces the complexity in terms of multipliers, but it requires several square root operations and divisions. There have been numerous papers in the literature, focusing on reducing the complexity of MIMO decoders in terms of multipliers, adders and not on divisions and square root operations required. Modified Householder method reduces complexity for MIMO decoder using QRD without any degradation in system performance. We provide packet error rate (PER) simulation results with proposed method and the conventional method for wireless LAN systems based on IEEE 802.11ac.

II. MIMO SYSTEM MODEL AND DETECTION

A. System Model

MIMO system model with $n_T$ transmit and $n_R$ receive antennas as show in Fig. 1. The matrix $H_c$ describes the $n_T \times n_T$ complex MIMO channel, $x_c$ is the $n_T \times 1$ modulated complex transmitted vector and is given by

$$x_c = [x_1 \ldots x_{n_T}] \quad (1)$$

where the channel $H_c$ can be described as

$$H_c = [h_{11} \ h_{12} \ \ldots \ h_{1n_T}]$$

$$h_{21} \ h_{22} \ \ldots \ h_{2n_T}$$

$$\vdots$$

$$h_{n_R1} \ \ldots \ h_{n_Rn_T} \quad (2)$$
and \( n_c \) is a \( n_g \times 1 \) complex additive white Gaussian noise vector and subscript 'c' indicates that the matrices and the vectors have complex elements.

**B. MIMO Detection based on QR decomposition**

In MIMO systems with spatial mapping scheme, multiple signals are transmitted independently over the channel to increase the data transmission rate. At the receiver, separating these signals combined in the channel poses a big challenge. Here, \( x_c \) has to be estimated from \( y_c \) assuming channel information is known. Using QR decomposition, the channel matrix \( H_c \) can be decomposed into an upper triangular matrix \( R_c \) and a unitary matrix \( Q_c \). The complex channel can be represented as \( H_c = Q_c R_c \). Pre-multiplying Eq. (1) with \( Q_c^* \) on both sides,

\[
Q_c^* y_c = Q_c^* Q_c R_c x_c + Q_c^* n_c
\]

where \( y_c = Q_c^* y_c \) and * in superscript indicates Hermitian transpose operation. Using Eq. (3), \( x_c \) can be obtained with linear techniques.

**III. QR-DECOMPOSITION METHODS**

Gram-Schmidt \[5\], Householder (HH) transformation \[6\], \[7\] and Givens rotation (GR) \[7\] are three well known algorithms used to perform QR decomposition. We briefly describe each of these techniques.

![Fig. 1MIMO system modal](image)

**A. Gram-Schmidt Orthogonalization**

In numerical analysis, we come across the Gram Schmidt process \[5\] for orthonormalization of a set of vectors by subtracting the projection of one on the others. The Gram Schmidt procedure is numerically unstable for the system with finite precision and usually not preferred in system implementation.

**B. Householder Transformation**

Householder transformation is a QRD technique to decompose any matrix into an upper triangular matrix \( R_c \) and a unitary matrix \( Q_c \). The main idea of this Householder reflections technique is to find reflection matrix \( P \) \[6\] and also known as Householder matrix. The Householder matrix annihilates all elements in a vector except the first element, which will be replaced with the norm of the corresponding vector.

**C. Givens Rotation**

Givens Rotation is also used to decompose the channel matrix \( H_c \) using a plane rotation \[8\]. Using a sequence of such Givens rotations in the form of a matrix, we can selectively manipulate the entries of a matrix \( H_c \) to reduce into upper triangular form \( (R_c) \).
IV. MODIFIED HOUSEHOLDER TRANSFORMATION

The system model in Eq. (3) has complex terms, the direct method of implementing QR decomposition requires multiplications, square root and division operations. The Eq. (1) can be reframed into real system Eq. (4) with transmitted real vector X, real channel matrix H and received real vector y of order $(2n^R X 1), (2n^R X 2n^T)$ and $(2n^T X 1)$ respectively.

Reframed MIMO equation:

$$Y = HX + n.$$  \hspace{1cm} (4)

where $y$, $x$, $H$ and $n$ are computed from the complex quantities $y_c, x_c$ and $H_c$ as follows. Where

$$y = [\text{Re}\{y_1\} \text{Im}\{y_1\} \ldots \ldots \text{Re}\{y_{n_R}\} \text{Im}\{y_{n_R}\}]^T$$

$$X = [\text{Re}\{x_1\} \text{Im}\{x_1\} \ldots \ldots \text{Re}\{x_{n_T}\} \text{Im}\{x_{n_T}\}]$$

and each element of $H$ can be written as

$$H_{ij} = \begin{bmatrix} \text{Re}\{h_{ij}\} & -\text{Im}\{h_{ij}\} \\ \text{Im}\{h_{ij}\} & \text{Re}\{h_{ij}\} \end{bmatrix}$$

where $\text{Re}\{\cdot\}$ and $\text{Im}\{\cdot\}$ denotes the real and imaginary parts of the channel element respectively and $T$ in superscript indicates transpose operation.

Modified Householder (MHH) transform is computationally improved compared to the HH algorithm by changing the Householder matrix. MHH eliminates the division operation in computing QR decomposition [9].

V. MIMO DECODERS USING QR DECOMPOSITION

There are many MIMO decoders, some of them are listed below. The output vector $y$ in Eq. (4) pre-multiplying with $Q^T$ on both sides, we get

$$\hat{y} = RX + \hat{n}.$$  \hspace{1cm} (5)

Where $\hat{y} = Q^Ty$, $\hat{n} = Q^Tn$ and $R$ is an upper triangular matrix formed from QR decomposition of channel matrix.

**A. Zero-Forcing Detector**

ZF solution is obtained from Eq. (5) as

$$W = (R^TR)^{-1}R^T$$  \hspace{1cm} (6)

From Eq. (5) and Eq. (6)

$$W\hat{y} = x + W\hat{n}$$  \hspace{1cm} (7)
The computation saving here increases with increased MIMO size.

**B. MMSE Detector**

MMSE solution is obtained from Eq. (5) as

\[ W = (R^T R + \sigma_n^2 I_n)^{-1} d^2 \]  

(8)

As in ZF we can reduce the complexity in finding W.

**C. Sphere Decoder**

Sphere decoder solution is obtained from Eq. (5) as

\[ ||y - Rx||^2 < d^2 \]  

(9)

The hypothesis value Rx which lies inside the sphere of radius d and the real received vector y can be represented as [2], the cost function of sphere decoder is defined as

\[ \arg_{x \epsilon D} \min_{i} \left( \sum_{i=1}^{n} (y_i - \sum_{j=1}^{n} r_{ij} x_j)^2 \right) \leq d^2 \]  

(10)

we can save few multiplications in the sphere decoder where the entries of \( r_{ij} \) are zero in Eq. (10).

**VI SIMULATION RESULTS**

Simulation is carried, in the multipath fading channels for IEEE 802.11ac (VHT) system. We considered VHT mixed format packet consisting of preamble, signal fields and data as in [5] and [6]. We have modeled Analog Front End (AFE) section with analog filter, DAC/ADC, TGac channel [10] and RF impairments. We selected simulation parameters as given in Table I.

<table>
<thead>
<tr>
<th>MCS Index</th>
<th>Parameters</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Modulation</td>
</tr>
<tr>
<td>3</td>
<td>16 QAM</td>
</tr>
<tr>
<td>5</td>
<td>64 QAM</td>
</tr>
</tbody>
</table>

We considered different SNRs in dB with block fading using TGac channel models A, B, C, D, E, F. For each SNR, we run simulations for 1000 realizations of channel and noise. In each realization, we generate a different transmit frame and pass through Analog Front End (AFE), the channel. We add AWGN noise and introduce other impairments like frequency offset, clock offset and phase noise. At the receiver, we perform timing and frequency corrections and estimate the channel using long training field (LTF). The estimated channel can be used in the detection of QR based MIMO. We performed PER simulations with both modified HH (MHH) and conventional HH using a channel bandwidth of 20MHz.
VII. COMPUTATIONAL COMPLEXITIES

A. Computational Complexities for proposed Method

Computational complexities of the QRD techniques are obtained from the Algorithm 1 and Algorithm 2 and, are tabulated in Table II for $n_R = n_T = n$. In Table II, it is clearly shown that, The HH requires several division operations compared to that of modified HH. For a MIMO channel matrix of size 2x2, modified HH saves around 30 division operations. Similarly, we can show that, for other values of $n_R$ and $n_T$, the modified HR is computationally less intensive.

Table II. Saving in complexities for proposed methods

<table>
<thead>
<tr>
<th>QRD</th>
<th>Basic method</th>
<th>Modified method</th>
<th>Saving in computations</th>
</tr>
</thead>
<tbody>
<tr>
<td>HH</td>
<td>$\frac{2n^2 + 2n^2 - 1}{3}$</td>
<td>$\frac{8n^2 + 2n^2 - 1}{3}$</td>
<td>$n^2$</td>
</tr>
</tbody>
</table>

VIII. CONCLUSION

Complexity of MIMO decoder is reduced based on modified QR decomposition. The method was simulated using VHT wireless LAN system under different channel conditions. MIMO decoder gives same PER performance as that of the conventional QR based MIMO decoder.

REFERENCES


