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On Strong (G,D) Number of Middle Graphs

[1] A.Sony, [2]M.Mahalakshmi, [3]K.Palani, [4] S.Kalavathi
[1][2] PG Mathematics
[3] Department of mathematics, A.P.C.Mahalaxmi college for women, Thoothukudi.
[4] Research Scholar, Manonmaniam Sundaranar University, Tirunelveli

Abstract:- **(G,D)-number of graphs was introduced by Palani K and Nagarajan A . Let G be a (V,E) graph. A subset D of V(G) is said to be a (G,D)-set of G if it is both a dominating and a geodetic set of G. A dominating set is said to be a strong dominating set of G if it strongly dominates all the vertices of its complement. In this paper, we introduce the concept strong (G,D) number of middle graph and find the same for some standard graphs and its bounds.**

INTRODUCTION

"Graph Theory" is an important branch of Mathematics. It has grown rapidly in recent times with a lot of research activities. In 1958, domination was formulized as a theoretical area in graph theory by C. Berge. He referred to the domination number as the coefficient of external stability and denoted as β(G). In 1962, Ore [6] was the first to use the term 'Domination' number by δ(G) and also he introduced the concept of minimal and minimum dominating set of vertices in graph. In 1977, Hedetniemiet.al[5] introduced the accepted notation γ(G) to denote the domination number. Let G = (V,E) be any graph. A dominating set of a graph G is a set D of vertices of G such that every vertex in V-D is adjacent to atleast one vertex in D. The minimum cardinality among all dominating sets of G is called the domination number of G. It is denoted by γ(G).The concept of geodominating (or geodetic) set was introduced by Buckley and Harary in [1] and Chartrand, Zhang and Harary in [2, 3, 4]. Let u, v ∈V(G). A u-v geodesic is a u-v path of length d(u, v). A vertex x is said to lie on a u-v geodesic p if x is any vertex on p. A set S of vertices of G is a geodominating (or geodetic) set if every vertex of G lies on an x-y geodesic for some x,y in S. The minimum cardinality of geodominating set is the geodomination (or geodetic) number of G. It is denoted by g(G). K. Palani et.al[7,8,9] introduced the concept (G,D)-set of graphs. A (G,D)- set of graph G is a subset S of vertices of G which is both dominating and geodominating (or geodetic) set of G. A (G,D)- set of G is said to bea minimal (G,D) set of G if no proper subset of S is a (G,D)-set of G. The minimum cardinality of all minimal (G,D)-set of G is called the (G,D)- number of G. It is denoted by $\gamma_G(G)$. In [10] C. Santhaana Gomathi K. Palani and S.Kalavathi initiated the study of strong (G,D)-number of a graph. A strong (G,D)-set is a (G,D)-set D which strongly dominates all the vertices of V-D. K. Palani et.al [11,12] investigate the (G,D)- number of Inflated Graphs and Product Graphs.. The middle graph of M(G) of a graph is the graph whose vertex set is V(G)∪ E(G) and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it. In this paper , we investigate the strong (G,D)-number of some Middle Graphs. The following theorems are from [10] .

1.1 Theorem: $s\gamma_G(P_n) = 2+\left\lceil\frac{n-2}{3}\right\rceil$

1.2 Theorem: $s\gamma_G(C_n) = \left\lceil\frac{n}{3}\right\rceil$

1.3 Theorem: Any strong (G,D)-set contains all the extreme vertices of G. In particular, all the end vertices of G.

<u>Theorem 2.1:</u>

$s\gamma_G(M(P_n)) = n+2+\left\lceil\frac{n-7}{3}\right\rceil$

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Proof:

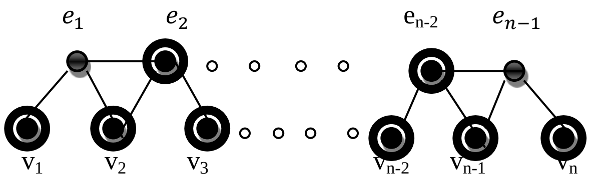

Figure:2.1

Let $V(M(P_n))$= $\{v_, v_2 \dots\dots\dots. v_{n-2}, v_{n-1}, v_n, e_1, e_2, \dots\dots\dots e_{n-1}\}$ where v_1, v_2, v_3,…….. v_{n-2}, v_{n-1}, v_n represent the vertices of P_n and $e_1, e_2, \dots\dots\dots e_{n-1}$ represent the edges of P_n.

Since v_1, v_2, v_3,…….. v_{n-2}, v_{n-1}, v_n are extreme vertices , S=$\{v_, v_2 \dots\dots\dots. v_{n-2}, v_{n-1}, v_n\}$ is a subset of every strong (G,D)-set of $M(P_n)$. Also S is a geodesic set of $M(P_n)$.

Hence, S∪S′ where S′ is any strong dominating set of the path $(e_1, e_2, \dots\dots\dots e_{n-1})$ is obviously a strong (G,D)-set of $M(P_n)$.

Now to find a set S′ with minimun cordinality, we proceed as follows:

Choose e_2,e_{n-2}.They dominate e_1,e_2,e_3,e_{n-3},e_{n-2},e_{n-1}. Now ,any dominating set of the path

P= (e_4,e_5,e_6…..e_{n-4}) together with e_2,e_{n-2} forms a strong dominating set (e_1,e_2,………e_{n-1}).P contains n-1-6 = n-7 vertices. Clearly,

S∪{ e_2,e_{n-2} } ∪ S′′ where S′′ is a minimum dominating set of path containing n-7 vertices forms a minimum strong (G,D) of $M(P_n)$.

$s\gamma_G(M(P_n))$ = n+2+$\gamma(P_n)$

= n+2+ $\lceil \frac{n-7}{3} \rceil$ (By theorem:1.2)

{ $\gamma(P_n) = \lceil \frac{n}{3} \rceil$ }

Illustration:2.2

Consider $M(P_9)$:

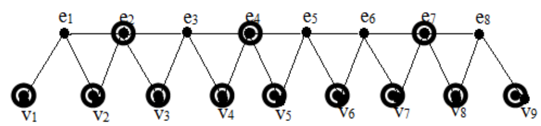

Figure:2.2

S = { v_1,v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,e_2,e_4,e_7 } is a minimum strong (G,D)-set of $M(P_9)$.

Therefore, $s\gamma_G(M(P_9))$ = 9+2+ $\gamma(P_9)$

= 9+2+ $\lceil \frac{9-7}{3} \rceil$

= 11+1

= 12

Illustration:2.3

Consider the graph $M(P_{10})$:

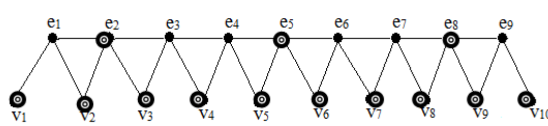

Figure:2.3

Solution:

S={v_1,v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},e_2,e_5,e_8} is a minimum strong (G,D)-set of $M(P_{10})$.

Therefore, $s\gamma_G(M(P_{10}))$ = 10+2+ $\gamma(P_3)$

$= 10 + 2 ++ \lceil \frac{10-7}{3} \rceil$

$=12 + 1 = 13.$

Theorem:2.4

$$s\gamma_G(\mathrm{M}(C_n)) = \mathrm{n} + \lceil \frac{n}{3} \rceil$$

Proof

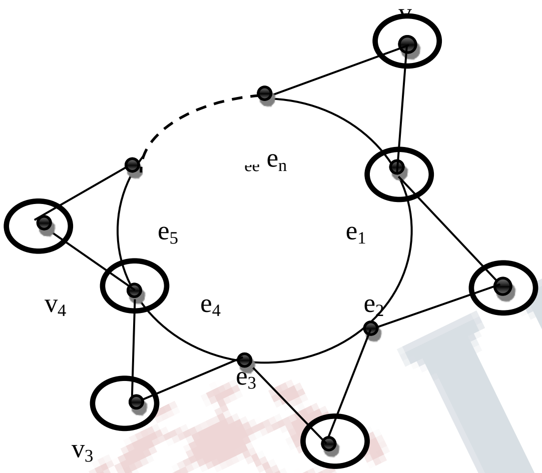

Figure:2.4

Let $V(\mathrm{M}(C_n))$ = $\{v_, v_2 \dots \dots \dots . v_{n-2}, v_{n-1}, v_n, e_{1,} e_{2,} \dots \dots . . e_n\}$

Where v_i's and e_i's represents the vertices and edges of C_n.

S $=\{v_, v_2 \dots \dots \dots . v_{n-2}, v_{n-1}, v_n\}$ is a set of extreme vertices of M(C_n).

Therefore, S is a subset of every strong (G,D)-set of M(C_n).

Also, S is a geodetic set of M(C_n) . < V - S > = a cycle ($e_1,e_2,\dots\dots e_n,e_1$).

In M(C_n), $e_1,e_2,\dots\dots e_n$ are all of same degree.Therefore any dominating set < V - S > is also a strong dominating set of V-S .Hence, S together with any dominating set of ($e_1,e_2,\dots\dots e_n,e_1$) forms a strong (G,D)-Set of M(C_n) and so

$$s\gamma_G(\mathrm{M}(C_n)) \leq |\, S \,| + \gamma(e_1,e_2,\dots\dots e_n,e_1)$$

$$= |\, S \,| + \lceil \frac{n}{3} \rceil$$

It is obviously a minimum strong dominating set of M(C_n) and so

$$s\gamma_G(\mathrm{M}(C_n)) = \mathrm{n} + \lceil \frac{n}{3} \rceil$$

Illutration:2.5

Consider M(C_6):

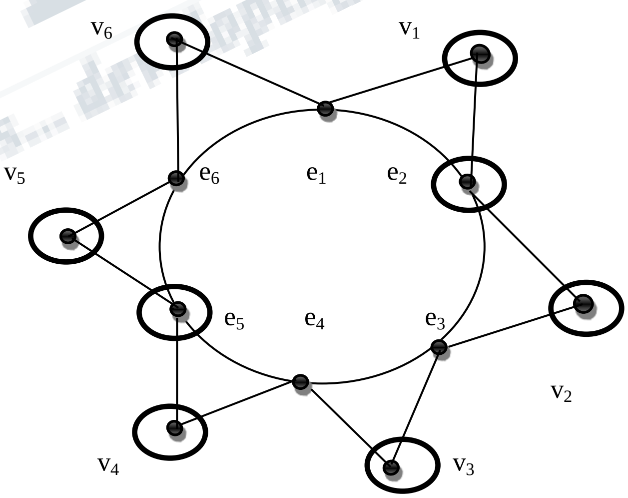

Figure:2.5

Solutuion :

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S = { $v_1,v_2,v_3,v_4,v_5,v_6,e_2,e_5$ } is a minimum strong (G,D)-set of $M(C_6)$ graph.

Therefore, $s\gamma_G(M(C_6)) = |S| + \lceil \frac{n}{3} \rceil$

$$= 6 + \left[\frac{6}{3}\right]$$

$$= 6 + 2 = 8$$

Theorem 2.6

$$s\gamma_G(M(K_{1,n})) = n + 2$$

Proof:

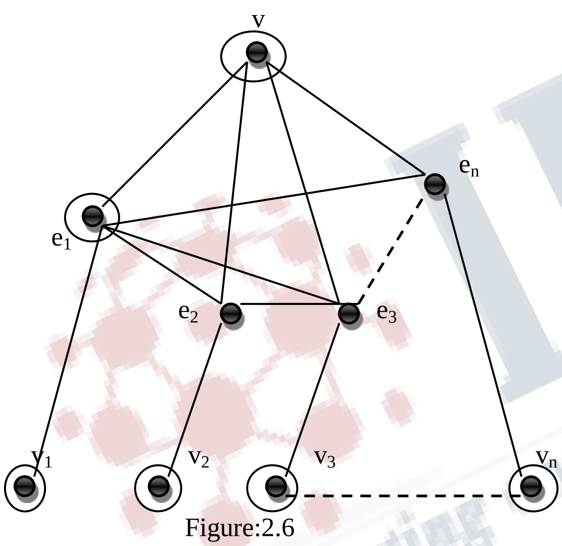

Figure:2.6

Let $V(M(K_{1,n}))$ = { $v,v_1,v_2,\ldots\ldots.v_n,e_1,e_2,\ldots\ldots e_n$ }

where V is the root vertex of $K_{1,n}$; $v_1,v_2,\ldots\ldots.v_n$ are the vertices and $e_1,e_2,\ldots\ldots e_n$ are the edges of $K_{1,n}$ respectively.

In $(M(K_{1,n}))$, $v,v_1,v_2,\ldots\ldots.v_n$ are the extreme vertices. S = { $v,v_1,v_2,\ldots\ldots.v_n$ } is a subset of every strong (G,D)-set of $(M(K_{1,n}))$.

Here, degree v = n

degree $v_i = 1 \ \forall \ i = 1$to n and

degree $e_i = n + 1 \ \forall \ i = 1$to n

Therefore, S do not strong dominate the vertices of $(M(K_{1,n}))$ which represents the edges of $K_{1,n}$. .Also, $< e_1,e_2,\ldots\ldots e_n >$ is complete and every e_i's lie in geodetic path joining a pair of vertices forms $v_1,v_2,\ldots\ldots.v_n$.

Therefore , for, i = 1 to n , S ∪ { e_i } is a minimum strong (G,D)-set of $(M(K_{1,n}))$.

Hence,

$$s\gamma_G(M(K_{1,n})) = |S| + 1$$

$$= n + 1 + 1$$

$$= n + 2$$

Illustration :2.7

Consider $(M(K_{1,4}))$:

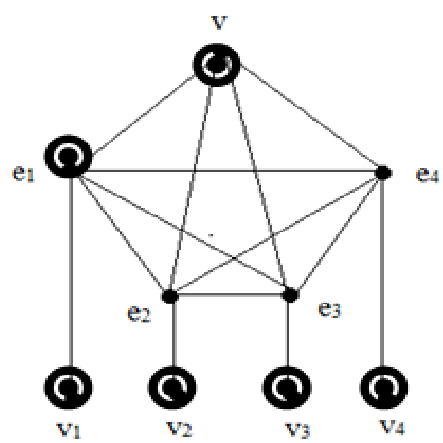

Figure:2.7

Solution:

Here , n = 4 and S = { v, v_1,v_2,v_3,v_4,e_1 } is a minimum strong (G,D)-set of $(M(K_{1,4}))$.

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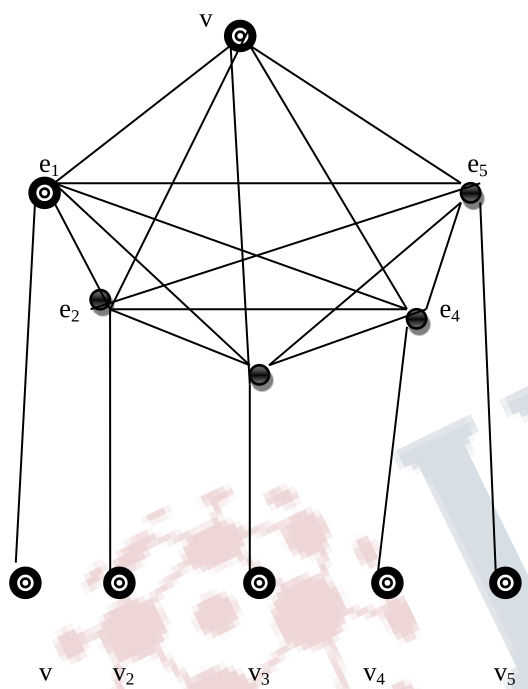
Hence, $s\gamma_G(M(K_{1,4})) = 6$.

Illustration:2.8

Consider the graph $M(K_{1,5})$



solution:

Here , n = 5 and

S = $\{v, v_1, v_2, v_3, v_4, v_5, e_1\}$ is a minimum strong dominating set of $M(K_{1,5})$.

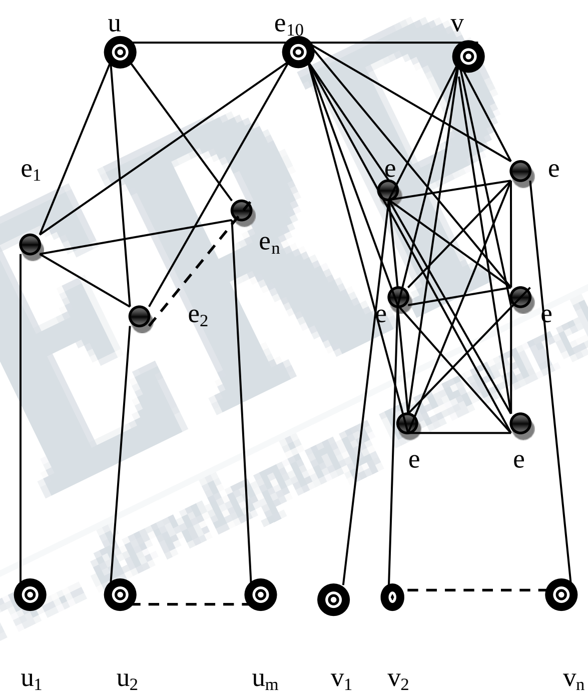
Hence,

$s\gamma_G(M(K_{1,4})) = 6$.

Theorem :2.9

$s\gamma_G(M(D_{m,m})) = m + n + 3$

Proof :



Here, S = $\{u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_n\}$ is the set of extreme vertices of $(M(D_{m,m}))$.

Therefore, S is contained in every strong (G,D)-set of $(M(D_{m,m}))$

Also, every vertex of V – S except u and v lie in a geodesic joining two vertices of S.

Therefore, S∪ { u, v } is a geodetic set of G.

Also, S∪ { u, v } strong dominates all the vertices of $(M(D_{m,m}))$ except e_{m+n+1}.

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Hence, S∪ { u, v } ∪ { e_{m+n+1} }

is a strong (G,D)-set of (M($D_{m,m}$)).

Furthere , it is a minimum strong (G,D) – set , since no set with lesser number of elements is a strong (G,D) – set.

Hence,

$$s\gamma_G(M(D_{m,m})) = |S| + 2 + 1$$

$$= m + n + 3$$

Illustration :2.10

Consider (M($D_{3,6}$)) :

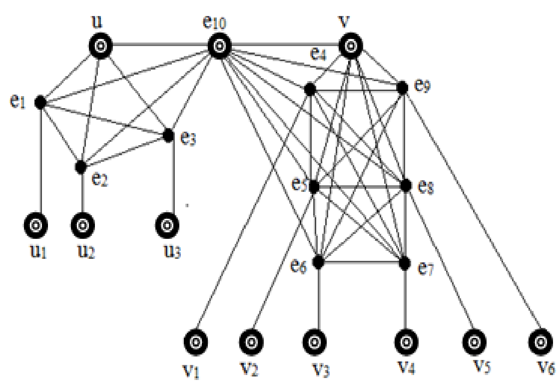

Figure:2.10

Solution:

Here , m = 3, n = 6 and

S = {u,v, u_1,u_2,u_3,v_1,v_2,v_3, v_4,v_5,v_6,e_{10}} is a minimum strong (G,D)-set of (M($D_{3,6}$)).

Hence,

$$s\gamma_G(M(D_{m,m})) = m + n + 3$$

$$= 12$$

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