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On Strong (G, D)-Number of Inflated Graphs

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***Abstract:*-Strong (G, D)-number of Graphs was introduced by Palani K and SanthaanaGomathi C. Let G be a (V, E) graph. A dominating set is said to be a strong dominating set of G if it strongly dominates all the vertices of its complement. A (G, D)-set D of G is said to be a strong (G, D)-set of G if it strongly dominates all the vertices of V-D. Strong (G, D)-number of Product graphs was discussed by S.Velammal,S.Rajalakshmi and K Palani. Strong (G, D)-number of Middle graphs was discussed by M.Mahalakshmi,A.Sony& K Palani. In this paper, we find the strong (G, D)-number of Inflated Graphs of some standard graphs.**

I. INTRODUCTION

“Graph Theory” is an important branch of Mathematics. It has grown rapidly in recent times with a lot of research activities. In 1958, domination was formulized as a theoretical area in graph theory by C. Berge. He referred to the domination number as the coefficient of external stability and denoted as β(G). In 1962, Ore [6] was the first to use the term ‘Domination’ number by δ(G) and also he introduced the concept of minimal and minimum dominating set of vertices in graph. In 1977, Hedetniemiet.al[5] introduced the accepted notation γ(G) to denote the domination number. Let G = (V,E) be any graph. A dominating set of a graph G is a set D of vertices of G such that every vertex in V-D is adjacent to atleast one vertex in D. The minimum cardinality among all dominating sets of G is called the domination number of G. It is denoted by γ(G).The concept of geodominating (or geodetic) set was introduced by Buckley and Harary in [1] and Chartrand, Zhang and Harary in [2, 3, 4]. Let u, v ∈V(G). A u-v geodesic is a u-v path of length d(u, v). A vertex x is said to lie on a u-v geodesic p if x is any vertex on p. A set S of vertices of G is a geodominating (or geodetic) set if every vertex of G lies on an x-y geodesic for some x,y in S. The minimum cardinality of geodominating set is the geodomination (or geodetic) number of G. It is denoted by g(G). K. Palaniet.al[7,8,9] introduced the concept (G,D)- set of graphs. A (G,D)- set of graph G is a subset S of vertices of G which is both dominating and geodominating (or geodetic) set of G. A (G,D)- set of G is said to bea minimal (G,D) set of G if no proper subset of S is a (G,D)- set of G. The minimum cardinality of all minimal (G,D)-set of G is called the (G,D)- number of G. It is denoted byγ_G(G). In [10] C. SanthaanaGomathi K. Palani and S.Kalavathi initiated the study of strong (G,D)-number of a graph. A strong (G,D)-set is a (G,D)-set D which strongly dominates all the vertices of V-D. K. Palani et.al [11,12] investigate the (G,D)- number of Middle and Product Graphs. Given a graph G with $\delta(G) \geq 1$, a graph denoted by G_I is obtained as follows:To each u∈V(G), a clique A_u of order $\deg_G u$is obtained and a bijection$\phi_u : N(u) \to A_u$ is constructed. $\phi_u(v)$ is denoted by v′ for all v ∈ N(u), $V(G_I) = \cup_{u\in V(G)} A_u$ and $E(G_I) = \cup E(A_u) \cup \{u'v' : uv \in E(G)\}$. $v' \in A_u$, $u' \in A_v$. The graph G_I is known as th*e inflated graph of G*. In this paper, we investigate the strong (G,D)-number of Inflated graphs of some standard graphs. The following theorems are from [10].

1.1 Theorem: $s\gamma_G(P_n) = 2 + \left\lceil \frac{n-2}{3} \right\rceil$

1.2 Theorem: $s\gamma_G(C_n) = \left\lceil \frac{n}{3} \right\rceil$

1.3 Theorem: Any strong (G,D)-set contains all the extreme vertices of G. In particular, all the end vertices of G.

2. Strong (G,D)-Number of Inflated graphs.

Here we investigate the strong (G,D)-Number of Inflated graphs of P_n, C_n, K_n,$K_{1,n}$, $D_{m,n}$etc,.

2.1 Proposition:

$s\gamma_G(I(P_n)) = 2 + \left\lceil \frac{2n-4}{3} \right\rceil$.

Proof:

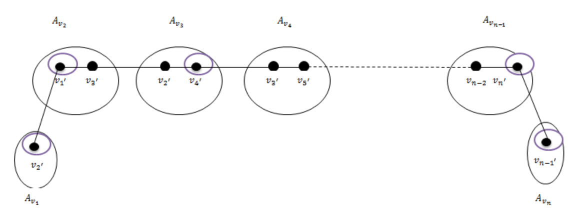

Figure 2.1

In a path, there are 2 end vertices and n-2 vertices of degree 2. Therefore, Inflated graph of P_n is again a path on $2(n-2)+2 = 2n-2$vertices. Hence, by theorem1.1,

$$s\gamma_G(I(P_n)) = s\gamma_G(P_{2n-2})$$
$$= 2 + \left\lceil \frac{2n-4}{3} \right\rceil$$

2.2. Illustration:

$s\gamma_G(I(P_7)) = 6$

Proof:

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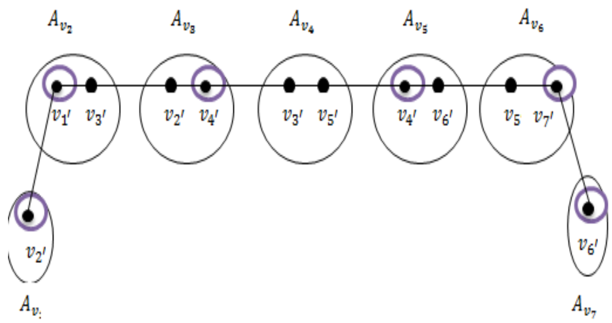

Figure 2.2

$I(P_7) \cong P_{12}$

Therefore, $s\gamma_G(I(P_7)) = s\gamma_G(P_{12})$

$$= 2 + \lceil (\frac{14-4}{3} \rceil$$

$s\gamma_G(I(P_7))$ =2+4 = 6

2.3. Proposition:

$s\gamma_G(I(C_n)) = \left\lceil \frac{2n}{3} \right\rceil$.

Proof:

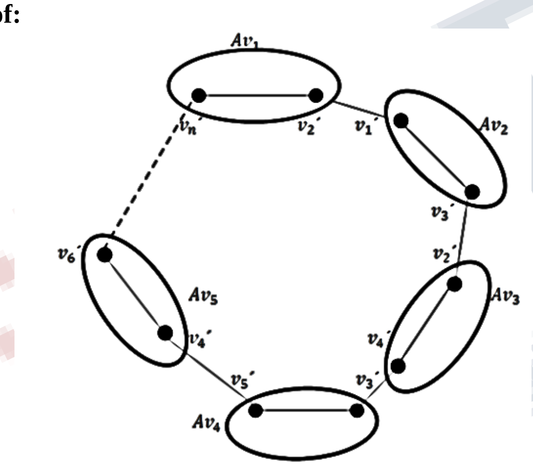

Figure 2.3

In a cycle, every vertex is of degree 2.Therefore, the inflated graph of C $_n$ is again a cycle of 2n vertices.Hence,by theorem 1.2, $s\gamma_G(I(C_n)) = s\gamma_G(C_{2n}) = \left\lceil \frac{2n}{3} \right\rceil$.

2.4. Illustration:

Inflated graph of $C_3$is given in the following figure.

Proof:

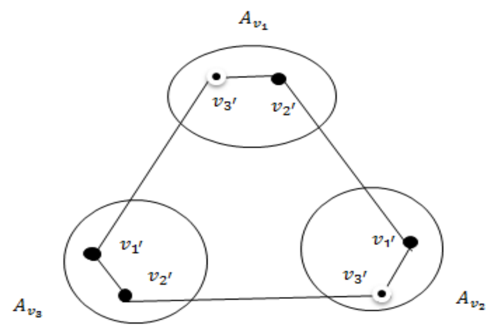

Figure 2.4

Therefore, $s\gamma_G(I(C_3)) = s\gamma_G(C_6)$

$$= 2 + \left\lceil \frac{10}{3} \right\rceil = 6.$$

2.5.Proposition:

$s\gamma_G(I(K_n)) = n - 1$

Proof:

$K_n$is a regular graph of degree $n - 1$.

Hence, $I(K_n)$ contains n cliques with $n - 1$ vertices. Let them be $K_{n-1}^1, K_{n-1}^2, K_{n-1}^3, \dots \dots K_{n-1}^n$.

Consider one of the cliques with $n - 1$ vertices, say K_{n-1}^1. Label the vertices of this clique as $v_1, v_2, v_3, \dots v_{n-1}$. Now, label the vertices adjacent to $v_1, v_2, v_3, \dots v_{n-1}$ in the remaining $n - 1$ cliques as $v_{11}, v_{21}, v_{31}, \dots v_{(n-1)1}$ $-----\rightarrow$ (1) as in figure 2.5.

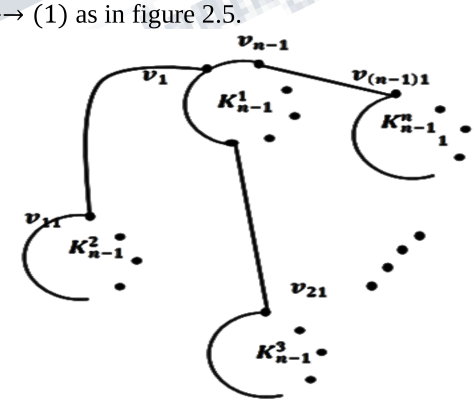

Figure2.5

These $n - 1$ vertices dominate the vertices of $I(K_n)$.

Also, $I(K_n)$is a regular graph. Therefore, any dominating set of $I(K_n)$ is also a strong dominating set of $I(K_n)$.

Further, distance between any two vertices in (1) is 3 and every vertex other than those in (1) lie in a geodesic joining two vertices of (1).

Therefore, $S = \{v_{11} v_{21}, v_{31}, \dots v_{(n-1)1}\}$ is a strong (G, D) set of$I(K_n)$.

Therefore, $s\gamma_G(I(K_n)) \le |S| = n - 1$

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Further, no set with less than $|S|$vertices is a strong $(G, D)-$ set of $I(K_n)$.

$$\therefore sY_G(I(K_n)) \geq |S|$$

$\therefore sY_G(I(K_n)) = |S| = n-1.$

2.6. Illustration:

$sY_G(I(K_4)) = 3.$

Proof:

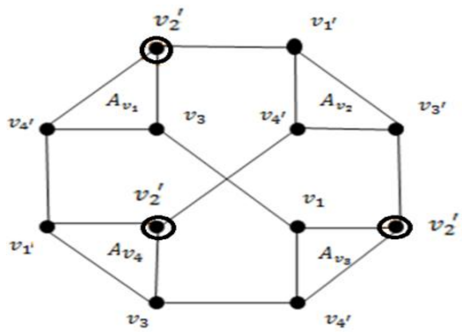

Figure 2.6

From the figure 2.6, we have

$sY_G(I(K_4)) = 3 =$ 4-1.

2.7. Illustration:$sY_G(I(K_5)) = 4.$

Proof:

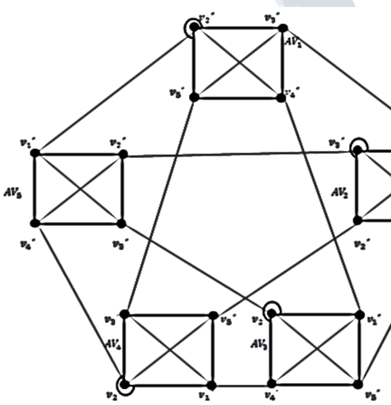

Figure 2.7

From the figure 2.7, $sY_G(I(K_5)) = 4$

$= 5-1.$

2.8. Illustration:

$sY_G(I(K_7)) = 6.$

Proof:

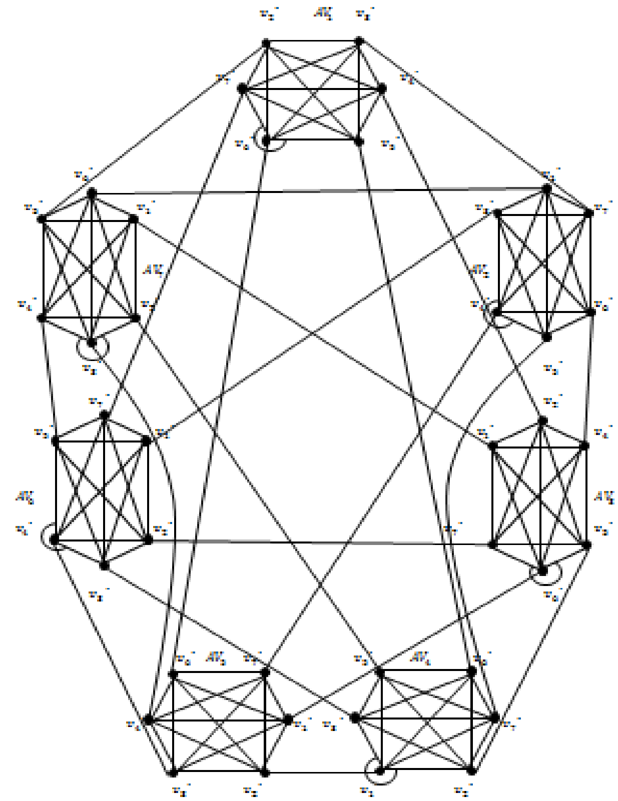

Figure 2.8

In the inflated graph of K_7, each vertex is of degree 6. In figure 2.8, the rounded vertices form a minimum strong (G,D)-set.

Therefore,$sY_G(I(K_7)) = 6 = 7-1.$

2.9. Proposition:

$$sY_G\left(I(K_{1,n})\right) = n+1$$

Proof:

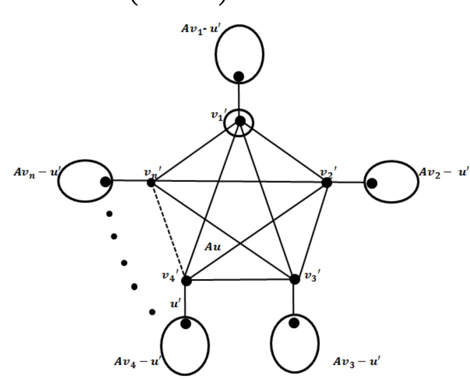

Figure.2.9

Here S= $\{Au_i - u'/$ i=1 to n } is the set of end vertices and hence every strong (G,D)-set contains S. Further, <V(I ($K_{1,n}$))- S> is complete. Hence, S∪ {Au-v_i'} forms a minimum strong (G,D)-set of $I(K_{1,n})\ \forall\ i = 1\ to\ n$.

Therefore, $sY_G\left(I(K_{1,n})\right) = |S| + 1 = n + 1$.

2.10. Illustration:

$$sY_G\left(I(K_{1,5})\right) = 6$$

Proof:

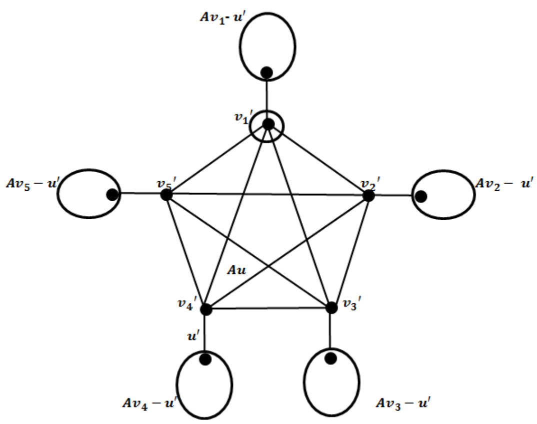

Figure 2.10

From the figure 2.10, it is clear that the set of vertices S= $\{Au_i - u'/$ i=1 to 5 }∪ {Au-v_1'} forms a strong (G,D)-set of I($K_{1,5}$).

Therefore, $sY_G\left(I(K_{1,5})\right) = 6 = 5 + 1$.

2.11. Proposition

Let $D_{m,n}$ denote the Double star. Then,

$$sY_G\left(I(D_{m,n})\right) = m + n + 2.$$

Proof:

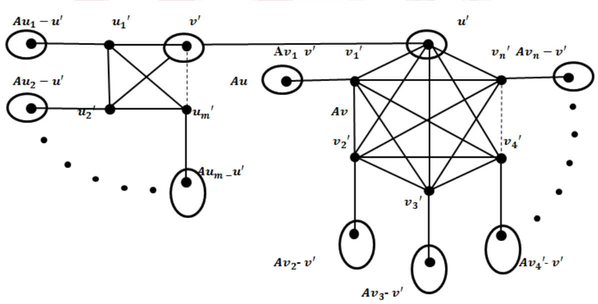

Figure 2.11

Without loss of generality, assume that $m < n$. As in the previous theorem, let the vertices be referred along with the clique name in which they appear. Let S_1 ={Au_1-u', Au_2-u',…..,Au_m-u'}

S_2 ={Av_1-v', Av_2-v',…..,Av_n-v'}

$S_1 \cup S_2$ is the set of extreme vertices of $I(D_{m,n})$.

Therefore, $S = S_1 \cup S_2$ is a subset of every strong (G,D)- set of $I(D_{m,n})$.

Further, S∪{Au-v',Av-u'} is a minimum strong (G,D)- set of $K_{m,n}$.

Hence, $sY_G\left(I(D_{m,n})\right) = |S| + 2 = m + n + 2$.

2.12. Illustration:

Inflated graph of $D_{3,5}$

Proof:

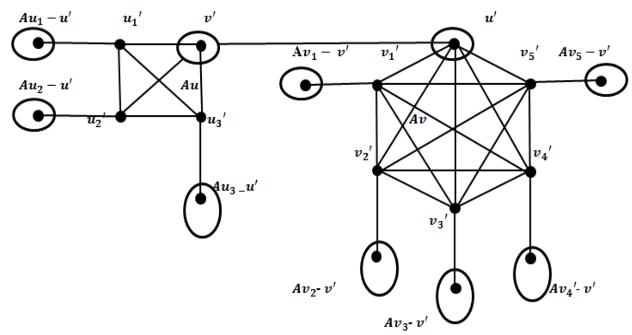

Figure .2.12

From the figure 2.12, it is clear that the rounded vertices form a minimum strong (G,D)- set of $I(D_{3,5})$.

$$\therefore sY_G\left(I(D_{3,5})\right) = 10$$
$$= 3 + 5 + 2.$$

2.13Proposition:

The inflated graph of W_5

Proof:

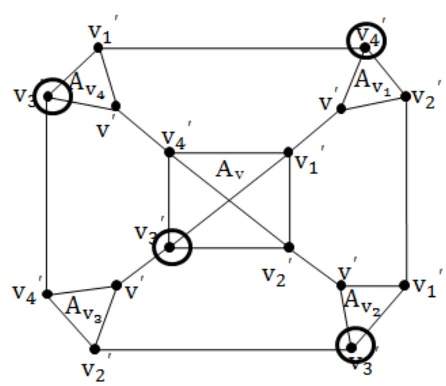

Figure 2.13

From the figure 2.13, it is clear that the rounded vertices form a minimum strong (G,D)-set of $I(W_5)$.

$\therefore sY_G\left(I(W_5)\right)$= 4.

2.14.Proposition:
$sϒ_G(I(W_7))$ = 6.
Proof:

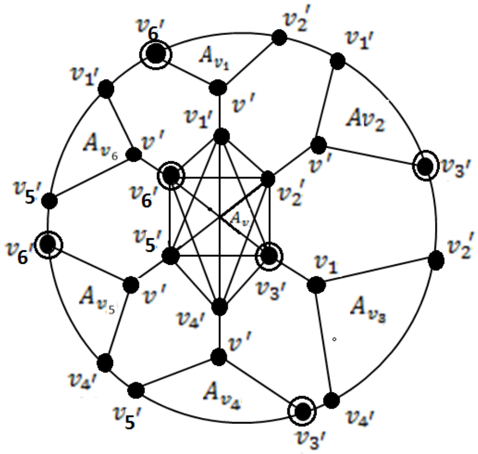

Figure 2.14

Let the vertices be referred along with the clique name in which they appear (i.e)Au-v_1', Av$_1$-v_4' etc. Obviously, S={Av-v_3', Av-v_6', Av$_1$-v_6', Av$_2$-v_3', Av$_4$$v_3'$, Av$_5$-$v_6'$} is one of the minimum strong (G,D)- sets of $I(W_7)$ as in figure 2.14.
Hence, $sϒ_G(I(W_7))$ =|S|= 6.

2.15. Proposition:
Inflated graph of $K_{2,3}$
Proof:

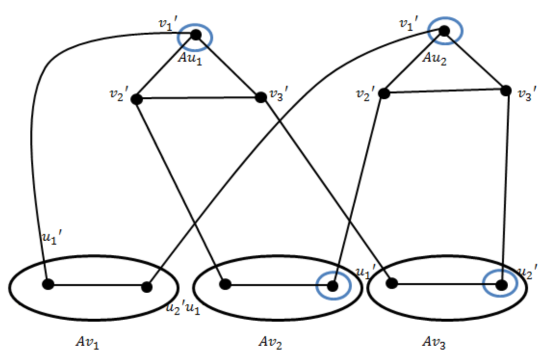

Figure.2.15

I ($K_{2,3}$) as in figure 2.15.It is clear that $S = \{Au_1 - v_1', Au_2 - v_1' Av_2 - u_2', Av_3 - u_2'\}$ isa minimum strong (G, D) set of $I(K_{2,3})$.
Hence, $sϒ_G(I(K_{2,3}))$ =|S|= 4.

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