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Normality in Idempotent Commutative Γ-Semi group

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Abstract:- **In this paper, a normality in an idempotent commutative Γ-semigroup is defined. A notion of left (right) normal, left (right) quasi-normal, regular, normal, left (right) semi-normal, left (right) semi-regular, in a normal idempotent commutative Γ-semigroup S are defined. Any left (right) normal is left (right) quasi-normal in an idempotent commutative Γ-semigroup and vice versa. Also, it is regular if and only if it is normal and the same statement is proved with respect to semi-regular and semi-normal substructure. Any quasi-normal is also semi-regular as well as semi-normal and also the converse in an idempotent commutative Γ-semigroup. In a commutative idempotent Γ-semigroup, left regularity implies both left and right normality.**

Keywords:- Γ-semigroup, idempotent, commutative, left and right regular, left and right normal, left and right quasi-normal, left and right semi-normal, left and right semi-regular.

I. INTRODUCTION

The formal study of semigroups began in the early 20th century. Early results include a Cayley theorem for semigroups realizing any semigroup as transformation semigroup, in which arbitrary functions replace the role of bijections from group theory. Other fundamental techniques of studying semigroups like Green's relations do not imitate anything in group theory though. In other areas of applied mathematics, semigroups are fundamental models for linear time-invariant systems. As a generalization of a semigroup SEN [17] introduced the notion of Γ-semigroup in 1981 and developed some theory on Γ-semigroup. JIROJKL, SRIPAKORN, CHINRAM extended many classical notions of semigroups to Γ-semigroup. DUTTA, T.K. CHATTERJE [5] genelized the green's relations in semigroups to Γ-semigroups.

In mathematics a Γ-semigroup is an algebraic structure consisting of a set together with an associative gamma operation. Associativity is formally expressed as that $(x\gamma_1 y)\gamma_2 z = x\gamma_1(y\gamma_2 z)$ for all x, y, z in semigroup and γ_1, γ_2 in gamma semigroup and in this full paper Normal Idempotent Commutative Gamma Semigroup satisfying some properties.

2. PRELIMINARIES

In this section we present some basic concepts of Γ-semigroup and definitions needed for the study of this chapter.

2.1 Definition:-
Let S = {a, b, c, …} and $\Gamma = \{\gamma_1, \gamma_2, \dots\dots\}$ be two non-empty sets. S is called *Γ-semigroup*.
(a) $a\gamma b \in S$;
(b) $(a\gamma_1 b)\gamma_2 c = a\gamma_1(b\gamma_2 c)$ for all a, b, c in S and γ_1, γ_2 in Γ.

2.2 Definition:-
An element a of Γ -semigroup S is said to be an *idempotent*. If $a\gamma a = a$ for all γ in Γ.

2.3 Definition:-
A Γ-semigroup S is said to be *commutative* provided $a\gamma b = b\gamma a$ for all a, b in S and γ in Γ.

2.4 Definition:-
A Γ-semigroup S is called *left (right) regular*, if it satisfies the identity $a\gamma_1 b\gamma_2 a = a\gamma_1 b$ $(a\gamma_1 b\gamma_2 a = b\gamma_2 a)$ for all a, b in S, $\gamma_1, \gamma_2 \in \Gamma$.

2.5 Definition:-
A Γ-Semigroup S is called *regular* if it satisfies the identity $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 a\gamma_3 c\gamma_4 a$ for all a, b, c in S and $\gamma_1, \gamma_2, \gamma_3, \gamma_4 \in \Gamma$.

2.6 Definition:-
A Γ-semigroup S is said to be *left (right) normal* if $a\gamma_1 b\gamma_2 c = a\gamma_1 c\gamma_2 b$ $(a\gamma_1 b\gamma_2 c = b\gamma_1 a\gamma_2 c)$ for all a, b, c in S and $\gamma_1, \gamma_2 \in \Gamma$.

2.7 Definition:-
A Γ-semigroup S is said to be *normal* if it satisfies the identity $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 c\gamma_2 b\gamma_3 a$ for all a, b, c in S and $\gamma_1, \gamma_2, \gamma_3 \in \Gamma$.

2.8 Definition:-
A Γ-semigoroup S is said to be *left (right) quasi-normal*, if it satisfies the identity $a\gamma_1 b\gamma_2 c = a\gamma_1 c\gamma_2 b\gamma_3 c$ ($a\gamma_1 b\gamma_2 c = a\gamma_1 b\gamma_2 a\gamma_3 c$) for all a, b, c in S and $\gamma_1, \gamma_2, \gamma_3$ in Γ.

2.9 Definition:-
A Γ-semigroup S is said to be *left (right) semi-normal* if it satisfies the identity $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 c\gamma_2 b\gamma_3 c\gamma_4 a$ ($a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 b\gamma_4 a$) for all a, b, c in S and $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ in Γ.

2.10 Definition:-
A Γ-semigroup S is said to be *left (right) semi-regular* if it satisfies the identity
$a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 a\gamma_3 c\gamma_4 a\gamma_5 b\gamma_6 c\gamma_7 a$ ($a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 a\gamma_4 b\gamma_5 a\gamma_6 c\gamma_7 a$) for all a, b,c in S and $\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7$ in Γ.

3. Normality in Idempotent Commutative Γ-Semigroup:-
In this section, we will see various theorem on normal idempotent commutative gamma semigroup satisfying some properties.

3.1 Theorem:-
An idempotent commutative Γ-semigroup S is left (right) normal if and only if it is left (right) quasi-normal.

Proof:-
Let S be an idempotent commutative Γ-semigroup. Now, let S be left normal, then $a\gamma_1 b\gamma_2 c = a\gamma_1 c\gamma_2 b$ =>$a\gamma_1 b\gamma_2 c\gamma_3 c = a\gamma_1 c\gamma_2 b\gamma_3 c$ =>$a\gamma_1 b\gamma_2 c = a\gamma_1 c\gamma_2 b\gamma_3 c$ [$c\gamma_3 c = c$]. Therefore S is left quasi-normal. Conversely, let S be a left quasi-nromal, then $a\gamma_1 b\gamma_2 c\ a\gamma_1 c\gamma_2 b\gamma_3 c => a\gamma_1 b\gamma_2 c = a\gamma_1 c\gamma_2 c\gamma_3 b$[$b\gamma_3 c = c\gamma_3 b$] $=> a\gamma_1 b\gamma_2 c = a\gamma_1 c\gamma_3 b[c\gamma_2 c = c]$. Therefore S is left normal. Now, let S be right normal, then $a\gamma_1 b\gamma_2 c = b\gamma_1 a\gamma_2 c$ => $a\gamma_1 a\gamma_2 b\gamma_3 c = a\gamma_1 b\gamma_2 a\gamma_3 c$ $[a\gamma_2 b = b\gamma_2 a]$ => $a\gamma_2 b\gamma_3 c = a\gamma_1 b\gamma_2 a\gamma_3 c$ [$a\gamma_1 a = a$]. Therefore S is right quasi-normal. Conversely, let S be right quasi-normal, then $a\gamma_1 b\gamma_2 c = a\gamma_1 b\gamma_2 a\gamma_3 c => a\gamma_1 b\gamma_2 c = b\gamma_1 a\gamma_2 a\gamma_3 c$ $[a\gamma_1 b = b\gamma_1 a]$ => $a\gamma_1 b\gamma_2 c = b\gamma_1 a\gamma_3 c$ $[a\gamma_2 a = a]$. Hence S is right normal.

3.2 Theorem:-
An idempotent commutative Γ-semigroup S is regular if and only if it is normal.

Proof:-
Let S be an idempotent commutative Γ-semigroup. Assume that S is regular, then $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 a\gamma_3 c\gamma_4 a$ => $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 a\gamma_4 a$ $[a\gamma_3 c = c\gamma_3 a]$ =>$a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 a$ $[a\gamma_4 a = a]$ => $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 c\gamma_2 b\gamma_3 a$ $[b\gamma_2 c = c\gamma_2 b]$. Therefore S is normal.
Conversely, assume that S is normal, then $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 c\gamma_2 b\gamma_3 a => a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 a$ $[c\gamma_2 b = b\gamma_2 c] => a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 a\gamma_2 b\gamma_3 c\gamma_4 a = [a = a\gamma_1 a]$ => $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 a\gamma_3 c\gamma_4 a$ $[a\gamma_2 b = b\gamma_2 a]$. Hence S is regular.

3.3 Theorem:-
A normal idempotent commutative Γ-semigroup S is left semi-normal if and only if it is left and right semi-regular.

Proof:-
Let S be a normal idempotent commutative Γ-semigroup. Assume that S is left semi-normal.Then $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 c\gamma_2 b\gamma_3 c\gamma_4 a => a\gamma_1 b\gamma_2 c\gamma_3 a$ = =$a\gamma_1 a\gamma_2 c\gamma_3 b\gamma_4 b\gamma_5 c\gamma_6 a$ $[a = a\gamma_1 a \& b\gamma_4 b = b]$ =>$a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 a\gamma_2 b\gamma_3 c\gamma_4 b\gamma_5 c\gamma_6 a$ $[c\gamma_3 b = b\gamma_3 c]$=>$a\gamma_1 b\gamma_2 c\gamma_3 a =$ $a\gamma_1 b\gamma_2 a\gamma_3 c\gamma_4 b\gamma_5 c\gamma_6 a[a\gamma_2 b = b\gamma_2 a] => a\gamma_1 b\gamma_2 c\gamma_3 a =$ $a\gamma_1 b\gamma_2 c\gamma_3 a\gamma_4 b\gamma_5 c\gamma_6 a[c\gamma_3 a = a\gamma_3 c] => a\gamma_1 b\gamma_2 c\gamma_3 a =$ $a\gamma_1 a\gamma_2 b\gamma_3 c\gamma_4 a\gamma_5 b\gamma_6 c\gamma_7 a$ [$a\gamma_1 a$=a] =>$a\gamma_1 b\gamma_2 c\gamma_3 a =$ $a\gamma_1 b\gamma_2 a\gamma_3 c\gamma_4 a\gamma_5 b\gamma_6 c\gamma_7 a$ $[a\gamma_2 b = b\gamma_2 a]$. Hence S is left semi-regular. Conversely, assume that S is left semi-regular then, $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 a\gamma_3 c\gamma_4 a\gamma_5 b\gamma_6 c\gamma_7 a$ => $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 a\gamma_3 a\gamma_4 c\gamma_5 b\gamma_6 c\gamma_7 a$ $[c\gamma_4 a = a\gamma_4 c]$ => $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 a\gamma_4 c\gamma_5 b\gamma_6 c\gamma_7 a$ $[a\gamma_3 a = a]$ => $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 a\gamma_2 b\gamma_4 c\gamma_5 b\gamma_6 c\gamma_7 a$ $[b\gamma_2 a = a\gamma_2 b]$ => $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_2 c\gamma_4 b\gamma_5 b\gamma_6 c\gamma_7 a$ $[a\gamma_1 a = a \ \& \ b\gamma_4 c = c\gamma_4 b]$ => $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_2 c\gamma_4 b\gamma_6 c\gamma_7 a$ $[b\gamma_5 b = b]$. Hence S is left semi-normal.

Assume that S is left semi-normal then, $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 c\gamma_2 b\gamma_3 c\gamma_4 a => a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 c\gamma_4 a$ $[c\gamma_2 b = b\gamma_2 c]$ => $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 a\gamma_4 c$ $[c\gamma_4 a = a\gamma_4 c]$ => $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 b\gamma_3 c\gamma_4 a\gamma_5 a\gamma_6 c$ $[b = b\gamma_2 b \ \& \ a = a\gamma_1 a]$ => $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 b\gamma_4 a\gamma_5 c\gamma_6 a$ $[b\gamma_3 c = c\gamma_3 b \ \& \ a\gamma_6 c = c\gamma_6 a]$ => $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 b\gamma_4 a\gamma_5 a\gamma_6 c\gamma_7 a$ $[a\gamma_5 a = a]$ => => $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 a\gamma_4 b\gamma_5 a\gamma_6 c\gamma_7 a$ $[b\gamma_4 a = a\gamma_4 b]$. Therefore S is right semi-regular. Conversely assume that S is right semi-regular then, $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 a\gamma_4 b\gamma_5 a\gamma_6 c\gamma_7 a$ => $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 c\gamma_2 b\gamma_3 b\gamma_4 a\gamma_5 a\gamma_6 c\gamma_7 a[b\gamma_2 c = c\gamma_2 b \ \& \ a\gamma_4 b = b\gamma_4 a]$ => $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 c\gamma_2 b\gamma_4 a\gamma_6 c\gamma_7 a$ [$b\gamma_3 b$ = b & $a\gamma_5 a = a$] => $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 c\gamma_2 b\gamma_4 c\gamma_6 a\gamma_7 a[a\gamma_6 c = c\gamma_6 a]$ =>

$a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 c\gamma_2 b\gamma_4 c\gamma_6 a$ $[a\gamma_7 a = a]$. Therefore S is left semi-normal.

3.4 Theorem:-
Any normal idempotent commutative Γ-semigroup S is left quasi-normal if and only if it is left and right semi-regular.

Proof:-
Let S be a normal idempotent commutative Γ-semigroup. Assume that S is left quasi-normal, then $a\gamma_1 b\gamma_2 c = a\gamma_1 c\gamma_2 b\gamma_3 c => a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 c\gamma_2 b\gamma_3 c\gamma_4 a =>$ $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 a\gamma_2 c\gamma_3 b\gamma_4 b\gamma_5 c\gamma_6 a[a\gamma_1 a = a \;\&\; b\gamma_4 b = b] => a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 a\gamma_2 b\gamma_3 c\gamma_4 b\gamma_5 c\gamma_6 a$ $[c\gamma_3 b = b\gamma_3 c]$ $=> a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 a\gamma_3 c\gamma_4 b\gamma_5 c\gamma_6 a$ $[a\gamma_2 b = b\gamma_2 a] => a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 a\gamma_4 b\gamma_5 c\gamma_6 a[a\gamma_3 c = c\gamma_3 a] => a\gamma_1 b\gamma_2 c\gamma_3 a a\gamma_1 a\gamma_2 b\gamma_3 c\gamma_4 a\gamma_5 b\gamma_6 c\gamma_7 a[a\gamma_1 a = a] => a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 a\gamma_3 c\gamma_4 a\gamma_5 b\gamma_6 c\gamma_7 a[a\gamma_2 b = b\gamma_2 a]$. Hence S is left semi-regular.

Conversely, assume S is left semi-regular, then $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 a\gamma_3 c\gamma_4 a\gamma_5 b\gamma_6 c\gamma_7 a =>$ $a\gamma_1 b\gamma_2 a\gamma_3 c = a\gamma_1 a\gamma_2 b\gamma_3 c\gamma_4 a\gamma_5 b\gamma_6 c\gamma_7 a$ $[c\gamma_3 a = a\gamma_3 c \;\&\; b\gamma_2 a = a\gamma_2 b]$ $=> a\gamma_1 a\gamma_2 b\gamma_3 c = a\gamma_2 c\gamma_3 b\gamma_4 a\gamma_5 b\gamma_6 c\gamma_7 a$ $[b\gamma_2 a = a\gamma_2 b \& a\gamma_1 a = a \& b\gamma_3 c = c\gamma_3 b$ $=> a\gamma_2 b\gamma_3 c = a\gamma_2 c\gamma_3 a\gamma_4 b\gamma_5 b\gamma_6 a\gamma_7 c$ $[b\gamma_4 a = a\gamma_4 b \& c\gamma_7 a = a\gamma_7 c]$ $=> a\gamma_2 b\gamma_3 c = a\gamma_2 c\gamma_3 a\gamma_4 b\gamma_6 a\gamma_7 c[b\gamma_5 b = b] =>$ $a\gamma_2 b\gamma_3 c = a\gamma_2 c\gamma_3 a\gamma_4 a\gamma_6 b\gamma_7 c$ $[b\gamma_6 a = a\gamma_6 b] =>$ $a\gamma_2 b\gamma_3 c = a\gamma_2 a\gamma_3 c\gamma_6 b\gamma_7 c$ $[c\gamma_3 a = a\gamma_3 c]$ $=> a\gamma_2 b\gamma_3 c = a\gamma_3 c\gamma_6 b\gamma_7 c$ $[a\gamma_2 a = a]$. Hence S is left quasi-nromal.

Assume that S is left quasi-normal then, $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 c\gamma_2 b\gamma_3 c => a\gamma_1 b\gamma_2 c\gamma_3 a =$ $a\gamma_1 c\gamma_2 b\gamma_3 c\gamma_4 a$ $=>$ $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 a\gamma_2 c\gamma_3 b\gamma_4 b\gamma_5 c\gamma_6 a\gamma_7 a$ $[a\gamma_1 a = a \;\&\; b\gamma_4 b = b]$ $=>$ $a\gamma_1 b\gamma_2 c\gamma_3 a =$ $a\gamma_1 a\gamma_2 b\gamma_3 c\gamma_4 b\gamma_5 a\gamma_6 c\gamma_7 a$ $[c\gamma_3 b = b\gamma_3 c \;\&\; c\gamma_6 a = a\gamma_6 c] => a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 a\gamma_3 c\gamma_4 b\gamma_5 a\gamma_6 c\gamma_7 a$ $=> [a\gamma_2 b = b\gamma_2 a]$ $=>$ $a\gamma_1 b\gamma_2 c\gamma_3 a =$ $a\gamma_1 b\gamma_2 c\gamma_3 a\gamma_4 b\gamma_5 a\gamma_6 c\gamma_7 a$ $[a\gamma_3 c = c\gamma_3 a]$. Therefore S is right semi-regular.

Conversely assume that S is right semi-regular then, $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 a\gamma_4 b\gamma_5 a\gamma_6 c\gamma_7 a =>$ $a\gamma_1 b\gamma_2 a\gamma_3 c = a\gamma_1 c\gamma_2 b\gamma_3 b\gamma_4 a\gamma_5 a\gamma_6 c\gamma_7 a$ $[b\gamma_2 c = c\gamma_2 b \;\&\; b\gamma_4 a = a\gamma_4 b \;\&\; c\gamma_3 a = a\gamma_3 c] => a\gamma_1 a\gamma_2 b\gamma_3 c = a\gamma_1 c\gamma_2 b\gamma_4 a\gamma_6 c\gamma_7 a$ $[b\gamma_3 b = b \;\&\; a\gamma_5 a = a \;\&\; b\gamma_2 a = a\gamma_2 b] => a\gamma_2 b\gamma_3 c = a\gamma_1 c\gamma_2 b\gamma_4 a\gamma_6 a\gamma_7 c$ $[c\gamma_7 a = a\gamma_7 c \;\&\; a\gamma_1 a = a] => a\gamma_2 b\gamma_3 c = a\gamma_1 c\gamma_2 b\gamma_4 a\gamma_7 c$ $[a\gamma_6 a = a] =>$ $a\gamma_2 b\gamma_3 c = a\gamma_1 c\gamma_2 a\gamma_4 b\gamma_7 c$ $[b\gamma_4 a = a\gamma_4 b] => a\gamma_2 b\gamma_3 c = a\gamma_1 a\gamma_2 c\gamma_4 b\gamma_7 c$ $=>$ $[c\gamma_2 a = a\gamma_2 c] => a\gamma_2 b\gamma_3 c = a\gamma_2 c\gamma_4 b\gamma_7 c$ $[a\gamma_1 a = a]$. Therefore S is left quasi-normal.

3.5 Theorem:-
An idempotent commutative Γ-semigroup S when normal is left quasi-normal if and only if it is left and right semi-normal.

Proof:-
Let S be a normal idempotent commutative Γ-semigroup. Assume S is left quasi-normal then, $a\gamma_1 b\gamma_2 c = a\gamma_1 c\gamma_2 b\gamma_3 c => a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 c\gamma_2 b\gamma_3 c\gamma_4 a$. Hence S is left semi-normal.

Conversely assume that S is left semi-normal then $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 c\gamma_2 b\gamma_3 c\gamma_4 a => a\gamma_1 b\gamma_2 a\gamma_3 c =$ $a\gamma_1 c\gamma_2 b\gamma_3 a\gamma_4 c[c\gamma_4 a = a\gamma_4 c \& c\gamma_3 a = a\gamma_3 c] =>$ $a\gamma_1 a\gamma_2 b\gamma_3 c = a\gamma_1 c\gamma_2 a\gamma_3 b\gamma_4 c[b\gamma_2 a = a\gamma_2 b \;\&\; b\gamma_3 a = a\gamma_3 b] => a\gamma_2 b\gamma_3 c = a\gamma_1 a\gamma_2 c\gamma_3 b\gamma_4 c[a\gamma_1 a = a \;\&\; c\gamma_2 a = a\gamma_2 c] => a\gamma_2 b\gamma_3 c = a\gamma_2 c\gamma_3 b\gamma_4 c$ $[a\gamma_1 a = a]$. Hence S is left quasi-normal.

Assume that S is left quasi-normal, then $a\gamma_1 b\gamma_2 c = a\gamma_1 c\gamma_2 b\gamma_3 c => a\gamma_1 b\gamma_2 c\gamma_3 a =$ $a\gamma_1 c\gamma_2 b\gamma_3 c\gamma_4 a => a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 c\gamma_4 a =>$ $[c\gamma_2 b = b\gamma_2 c] => a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_4 a =>$ $[c\gamma_3 c = c] => a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 b\gamma_3 c\gamma_4 a[b = b\gamma_2 b] => a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 b\gamma_4 a$ $[b\gamma_3 c = c\gamma_3 b]$. Hence S is right semi-normal.

Convesely, assume that S is right semi-normal, then $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 b\gamma_4 a => a\gamma_1 b\gamma_2 a\gamma_3 c =$ $a\gamma_1 c\gamma_2 b\gamma_3 b\gamma_4 a[c\gamma_3 a = a\gamma_3 c \;\& b\gamma_2 c = c\gamma_2 b] =>$ $a\gamma_1 a\gamma_2 b\gamma_3 c = a\gamma_1 c\gamma_2 b\gamma_3 a$ $[b\gamma_2 a = a\gamma_2 b \;\&\; b\gamma_3 b = b] => a\gamma_2 b\gamma_3 c = a\gamma_1 c\gamma_2 a\gamma_3 b$ $[b\gamma_3 a = a\gamma_3 b \& a\gamma_1 a = a] =>$ $a\gamma_2 b\gamma_3 c = a\gamma_1 a\gamma_2 c\gamma_3 b[c\gamma_2 a = a\gamma_2 c] => a\gamma_2 b\gamma_3 c =$ $a\gamma_2 c\gamma_3 b => a\gamma_2 b\gamma_3 c\gamma_4 c = a\gamma_2 c\gamma_3 b\gamma_4 c => a\gamma_2 b\gamma_3 c =$ $a\gamma_2 c\gamma_3 b\gamma_4 c[c\gamma_4 c = c]$. Hence S is left quasi-normal.

3.6 Theorem:-
An idempotent commutative Γ-semigroup S is left regular implies it is left and right normal, when it is normal.

Proof:-
Let S be a normal idempotent commutative Γ-semigroup. Assume that S is left regular, then $a\gamma_1 b\gamma_2 a = a\gamma_1 b$ $=> a\gamma_1 b\gamma_2 a\gamma_3 c = a\gamma_1 b\gamma_2 c => a\gamma_1 a\gamma_2 b\gamma_3 c =$ $a\gamma_1 c\gamma_2 b$ $[b\gamma_2 a = a\gamma_2 b \& b\gamma_2 c = c\gamma_2 b] => a\gamma_1 b\gamma_2 c =$ $a\gamma_1 c\gamma_2 b$ $[a\gamma_1 a = a]$. Therefore S is left normal.

Assume that S is left regular then, $a\gamma_1 b\gamma_2 a =$ $a\gamma_1 b => a\gamma_1 b\gamma_2 a\gamma_3 c = a\gamma_1 b\gamma_2 c => a\gamma_1 a\gamma_2 b\gamma_3 c =$ $b\gamma_1 a\gamma_2 c$ $[b\gamma_2 a = a\gamma_2 b \& a\gamma_1 b = b\gamma_1 a]$ $=> a\gamma_2 b\gamma_3 c =$ $b\gamma_1 a\gamma_2 c$ $[a\gamma_1 a = a]$. Therefore S is right normal.

CONCLUSION:-

In this paper, it is proved that any left quasi-normal idempotent commutative gamma semigroup is both left and right semi-normal. Left regularity will be both left and right normality. A left quasi normal idempotent commutative gamma semigroup is both left and right semi-regular. A semi-normal gamma semigroup is both left and right semigroup, if it is a left semigroup.

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