SSN (Online) 2456 -1304

International Journal of Science, Engineering and Management (IJSEM)
Vol 3, Issue 4, April 2018

Strong (G, D)-number of Product Graphs

[1] S.Velammal, [2] S.Rajalakshmi, [3] K.Palani, [4] S.Kalavathi
[1, 2] PG Mathematics, [3] Department of Mathematics, [4] Research scholar
[1][2][3] A.P.C.Mahalaxmi College for Women, Thoothukudi
[4] Manonmaniam sundaranar university, Tirunelveli

Abstract- **Strong (G,D)-number of Graphs was introduced by Palani K and Santhaana Gomathi C. Let G be a (V,E) graph. A dominating set is said to be a strong dominating set of G if it strongly dominates all the vertices of its complement. A (G,D)-set D of G is said to be a strong (G,D)-set of G if it strongly dominates all the vertices of V-D. In this paper, we find the strong (G,D)-number of product graphs of some standard graphs.**

I. INTRODUCTION

"Graph Theory" is an important branch of Mathematics. It has grown rapidly in recent times with a lot of research activities. In 1958, domination was formulized as a theoretical area in graph theory by C. Berge. He referred to the domination number as the coefficient of external stability and denoted as β(G). In 1962, Ore [6] was the first to use the term 'Domination' number by δ(G) and also he introduced the concept of minimal and minimum dominating set of vertices in graph. In 1977, Cockayne and Hedetniemi [5] introduced the accepted notation γ(G) to denote the domination number. Let G = (V,E) be any graph. A dominating set of a graph G is a set D of vertices of G such that every vertex in V-D is adjacent to atleast one vertex in D. The minimum cardinality among all dominating sets of G is called the domination number of G. It is denoted by γ(G).The concept of geodominating (or geodetic) set was introduced by Buckley and Harary in [1] and Chartrand, Zhang and Harary in [2, 3, 4]. Let u, v ∈V (G). A u-v geodesic is a u-v path of length d(u, v). A vertex x is said to lie on a u-v geodesic p if x is any vertex on p. A set S of vertices of G is a geodominating (or geodetic) set if every vertex of G lies on an x-y geodesic for some x,y in S. The minimum cardinality of geodominating set is the geodomination (or geodetic) number of G. It is denoted by g(G). K. Palani et.al[7,8,9] introduced the new concept (G,D)- set of graphs. A (G,D)- set of graph G is a subset S of vertices of G which is both dominating and geodominating (or geodetic) set of G. A (G,D)- set of G is said to be a minimal (G,D) set of G if no proper subset of S is a (G,D)- set of G. The minimum cardinality of all minimal (G,D)-set of G is called the (G,D)- number of G. It is denoted by γG(G). In [10] C. SanthaanaGomathi, K. Palani and S.Kalavathi initiated the study of strong (G,D)-number of a graph. The product (Cartesian product) of two graphs G_(1)& G_(2)denoted by G_1× G_2 has the vertex set V_1× V_2 and two vertices u=(u_(1 ,) u_2) and v=(v_(1 ,) v_2) are adjacent in G_1× G_(2) whenever [u1=v1 and u2 is adjacent to v2 in G2] or [u2 = v2 and u1 is adjacent to v1 in G1].A strong (G,D)-set is a (G,D)-set D which strongly dominate all the vertices of V-D. K. Palani et.al [11,12] investigate the (G,D)- number of Middle and Inflated Graphs of some standard graphs.

The following theorems are from [10]:

a. Theorem: sY_G (P_n) = 2+⌈(n-2)/3⌉
b. Theorem: sY_G (C_n) =⌈n/2⌉
c. Theorem: Any strong (G,D)-set contains all the extreme vertices of G. In particular, all the end vertices of G.

II STRONG (G, D) NUMBER OF PRODUCT GRAPHS:

2.1 Theorem: $s\gamma_G\ (K_m \cup K_n)$ = m + n

Proof: Let S_1 and S_2 be minimum strong (G, D) sets of K_m and K_n respectively. Then, $S1 \cup S_2$ is a strong (G, D) set of $K_m \cup K_n$. Further, $S1 \cup S_2$ is minimum strong (G, D) set of $K_m \cup K_n$

Hence by 1.1,$s\gamma_G\ (K_m \cup K_n)$ =m + n

2.2 THEOREM: $S\gamma_G(K_m + K_n) = m + n$

Proof: $K_m + K_n$ is isomorphic toK_{m+n}.

Let $G = K_m + K_n$. Therfore, the set V(G) is the unique $(G, D) -$ Set of $K_m + K_n$ which is also strong dominating Therefore V(G) is the unique strong (G, D) - set of G.

Hence$s\gamma_G\ (K_m + K_n)$ = $s\gamma_G\ (K_{m+n})$ = m+n

2.3 ILLUSTRATION: $S\gamma_G(K_3 + K_4) = 7$

Solution:

Here, m = 3 and n = 4

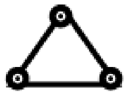
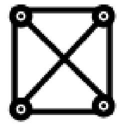
Let G_1 & G_2 be two complete graphs k_3 & k_4 respectively

All Rights Reserved © 2018 IJSEM

IFERP connecting engineers... developing research

ISSN (Online) 2456 -1304

International Journal of Science, Engineering and Management (IJSEM)
Vol 3, Issue 4, April 2018

$G_1: k_3 =$  $G_2: k_4 =$ 

$s\gamma_G\ (K_3 = 3$
$s\gamma_G\ (K_4) = 4 \quad (\boldsymbol{G_1 + G_2}) = \boldsymbol{k_7}$

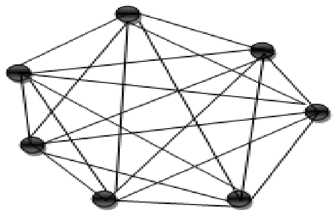


Figure 2.1

Hence,$s\gamma_G\ (K_3 + K_4) = s\gamma_G\ (K_{3+4}) = 7$

2.4 Theorem:

$$\boldsymbol{s\gamma_G}\ (\mathbf{P_2 \times P_n}) = \begin{cases} \boldsymbol{2k+2\ if\ n = 4k\ or\ 4k+1} \\ \boldsymbol{2k+3\ if\ n = 4k+2\ or\ 4k+3} \end{cases}$$

Proof:
Label the vertices of $(P_2 \times P_n)$ as in figure2.2

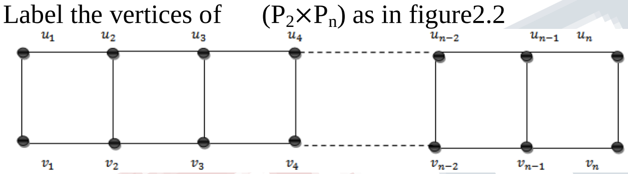


Figure: 2.2

To find the strong (G,D) number, we proceed in the following cases.

Case 1: n= 4k

Let S = $\{v_1, u_n\}$, $S_1 = \{u_2, u_6, \dots u_{4(K-1)+2}\}$ $S_2 = \{v_4, v_8, \dots\dots\dots\dots v_{4(K-1)}\}$, $S_3 = \{u_{n-1}\}$

Obviously $S \cup S_1 \cup S_2 \cup S_3$ is a minimum strong (G, D) set of $(P_2 \times P_n)$

S, S_1, S_2, S_3 Have no common point

Also, $|S| = 2, |S_1| = k, |S_2| = k-1, |S_3| = 1$

Hence $\boldsymbol{s\gamma_G}\ (P_2 \times P_n) = |S| + |S_1| + |S_2| + |S_3|$
$= 2k+2$

case 2: n=4k+1

Let S = $\{v_1, u_n\}$, $S_1 = \{u_2, u_6, \dots u_{4(K-1)+2}\}$ $S_2 = \{v_4, \dots\dots\dots\dots v_{4k}\}$,

Here, ,S∪ $S_1 \cup S_2$ is a minimum strong (G,D) set of $(P_2 \times P_n)$

Also, $|S| = 2$,| S_1 |=k, $|S_2|$ = k

$\boldsymbol{s\gamma_G}\ (P_2 \times C_n) = |S| + |S_1| + |S_2| + 1 = 2 + k + k = 2k+2$

case 3: n=4k+2

Let S = $\{v_1, u_n\}$, $S_1 = \{u_2, u_6, \dots u_{4(K-1)+2}\}$ $S_2 = \{v_4, \dots\dots\dots\dots v_{4k}\}$, $S_3 = \{u_{4k+1}\}$

Obviously, S∪ $S_1 \cup S_2 \cup S_3$ is minimum strong (G, D) set of $(P_2 \times P_n)$

Also, $|S| = 2$, | S_1|=k, $|S_2|$ = k, $|S_3| = 1$

$\boldsymbol{s\gamma_G}\ (P_2 \times P_n) = |S| + |S_1| + |S_2| + |S_3| = 2K+3$

Case 4: n=4k+3

Let S = $\{v_1, u_n\}$, $S_1 = \{u_2, u_6, \dots u_{4(K)+2}\}$ $S_2 = \{v_4, \dots\dots\dots\dots v_{4k}\}$

Here ,S∪ $S_1 \cup S_2$ is a minimum strong (G,D) set of $(P_2 \times P_n)$

Also, $|S| = 2$,| S_1 |=k + 1, $|S_2|$ = k

$\boldsymbol{s\gamma_G}\ (P_2 \times C_n) = |S| + |S_1| + |S_2| = 2k+3$

2.5 ILLUSTRATION: $\boldsymbol{s\gamma_G}\ (P_2 \times P_8) = 6 = 2k+2$

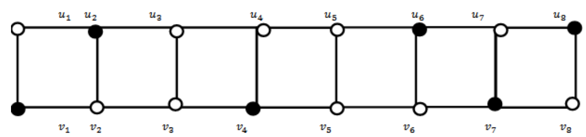


Figure 2.3

Here k=2 ,S ={ u_2 , u_6 , v_1, $v_{4,}$, v_7, u_8 } is a minimum strong (G,D) set.

Hence $\boldsymbol{s\gamma_G}\ (P_2 \times P_8) = |S| = 6 = 2k+2$

2.6 ILLUSTRATION: $\boldsymbol{s\gamma_G}\ (P_2 \times P_9) = 6 = 2k+2$

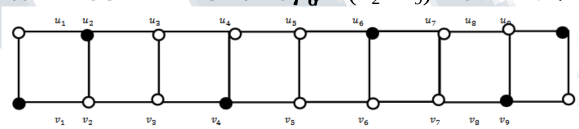


Figure 2.4

Here k=2,S ={ u_2 , u_6 , v_1, $v_{4,}$, v_8, u_9 } is a minimum strong (G,D) set.

Hence$\boldsymbol{s\gamma_G}$ $(P_2 \times \mathbf{P_9})$ = $|\boldsymbol{S}|$ **= 6 = 2k+2**

2.7 ILLUSTRATION:. $s\gamma_G(P_2 \times P_{10})$ = 7= 2k+3

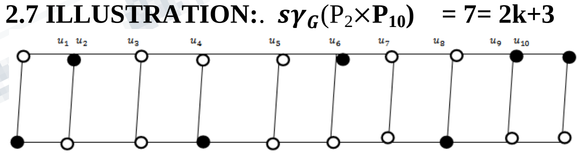


Figure 2.5

Here k=2,S ={ u_2 , u_6 , v_1, $v_{4,}$, v_8, u_9, u_{10} } is a minimum strong (G,D) set .

Hence $\boldsymbol{s\gamma_G}(P_2 \times \mathbf{P_{10}})$ = $|\boldsymbol{S}|$ **= 7 = 2k+3**

2.8 ILLUSTRATIO:. $s\gamma_G(P_2 \times P_{11})$ = 7 = 2k+3

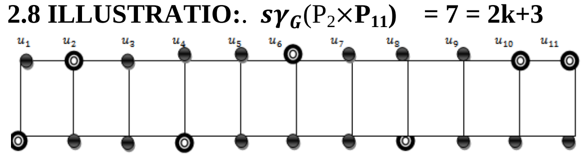


Figure 2.6

Here k=2,S ={ u_2 , u_6 , v_1, $v_{4,}$, v_8, $u_{10}\ u_{11}$ } is a minimum strong (G,D) set.

Hence $\boldsymbol{s\gamma_G}(P_2 \times \mathbf{P_{10}})$ = $|\boldsymbol{S}|$ **= 7 = 2k+3**

2.9 Theorem: For all n≥3 ,

All Rights Reserved © 2018 IJSEM

IFERP connecting engineers... developing research
ISSN (Online) 2456 -1304

International Journal of Science, Engineering and Management (IJSEM)
Vol 3, Issue 4, April 2018

$$s\gamma_G \ (P_2\times C_n)=\begin{cases} 2k \ \ if \ \ n=4k \\ 2k+1 \ \ if \ \ n=4k+1 \\ 2(k+1) \ \ if \ \ n=4k+2 \ \ or \ \ 4k+3 \end{cases}$$

Proof:
Label the vertices of $P_2\times C_n$ as in figure2.7

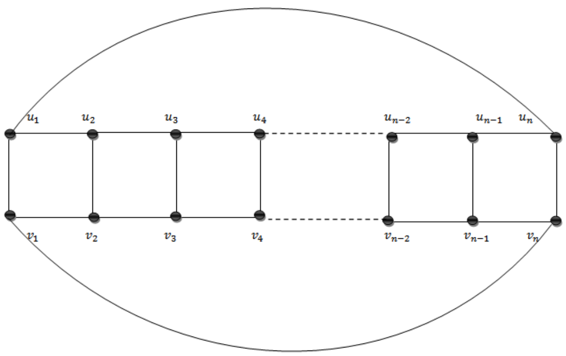


Figure: 2.7

To find the strong (G, D) number we proceed in the following cases.

Case 1: n= 4k

Let $S_1=\{u_1,u_5,\ldots\ldots\ldots,u_{4(k-1)+1}\}$
$S_2=\{v_3,v_7,\ldots\ldots\ldots,v_{4(k-1)+3}\}$
Obviously, $S1\cup S_2$ is a minimum strong (G, D) set of $P_2\times C_n$
Also, $|S_1|=k$, $|S_2|=k$ Further $S_1 \cap S_2 = \emptyset$
$s\gamma_G$ $(P_2\times C_n)= |S_1|+; |S_2| =2k$

Case 2: n=4k+1

Here, $S1\cup S_2$ where $S_1=\{u_1, u5,\ldots\ldots\ldots,u_{4k+1}\}$
And $S_2=\{v_3 ,v_7,\ldots\ldots\ldots,v_{4(k-1)+3}\}$
is a minimum strong (G,D) set and hence
$s\gamma_G$ $(P_2\times C_n)= |S_1|+ |S_2| =k+1+k=2k+1$

Case 3: n=4k+2

Let $S_1=\{u_1,u_5,\ldots\ldots\ldots,u_{4k+1}\}$
$S_2=\{v_3,v_7,\ldots\ldots\ldots,v_{4(k-1)+3}\}$
Here , $S_1\cup S_2\cup\{v_{4k+2}\}$ is a minimum strong (G,D) set
Also, $|S_1|=k+1$; $|S_2| =k$
$s\gamma_G$ $(P_2\times C_n)= |S_1|+ |S_2|+1 =k+1+k+1=2(k+1)$

case 4: n=4K+3

let $S_1=\{u_1,u_5,\ldots\ldots\ldots,u_{4k+1}\}$
$S_2=\{v_3,v_7,\ldots\ldots\ldots,v_{4k+3}\}$
Obviously, $S1\cup S_2$ is minimum strong (G, D) set
Also, $|S_1|=k+1$; $|S_2| =k+1$
$s\gamma_G$ $(P_2\times C_n)= |S_1|\cup |S_2|= |S_1|+|S_2|=k+1+k+1$

$=2(k+1)$

2.10 ILLUSTRATION:
$s\gamma_G$ **$(P_2\times C_8)$=4**

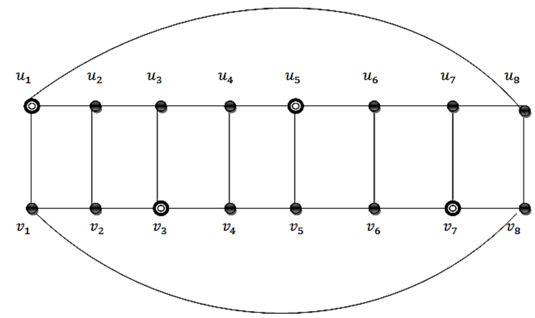


Figure: 2.8

Here k=2 ,S $=\{u_1, u_5, v_3, v_7\}$ is a minimum strong (G,D) set.
Hence $s\gamma_G$ $(P_2\times C_{8)} = |S| = 4 = 2K$

2.11 ILLUSTRATION:
$s\gamma_G$ **$(P_2\times C_9)$ = 5**

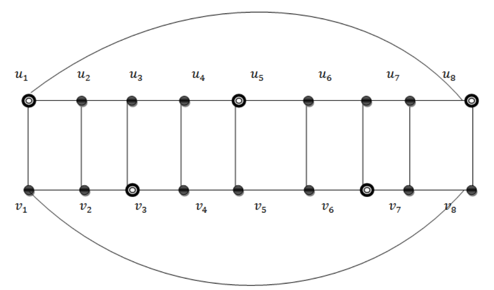


Figure: 2.9

here k=2 ,S $=\{u_1, u_5, u_9 \ v_3, v_{7,}\}$ is a minimum strong (G,D) set.
Hence $s\gamma_G$ $(P_2\times C_9) = |S| = 5 = 2k+1$

2.12 ILLUSTRATION:
$s\gamma_G$ **$(P_2\times C_{10})$ = 6**

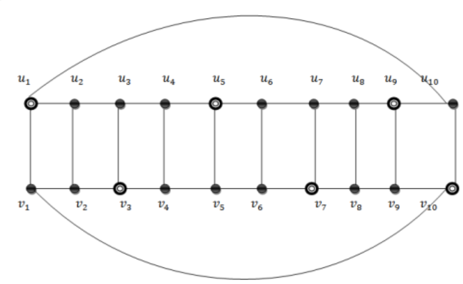


Figure 2.10

here k=2 ,S $=\{u_1, u_5, u_9, v_3, v_7, v_{10\}}$ is a minimum strong (G,D) set.
Hence $s\gamma_G$ $(P_2\times C_{10}) = |S| = 6=2(k+1)$

2.13 ILLUSTRATION:
$s\gamma_G$**$(P_2\times C_{11})$ = 6**

All Rights Reserved © 2018 IJSEM

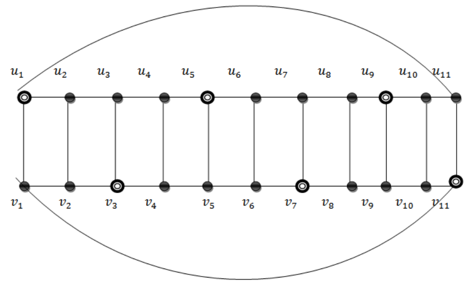


Figure: 2.11

Here k=2 ,S ={ u_1 , u_5 ,u_9, v_3, v_7,v_{11}} is a minimum strong (G,D) set.

Hence $s\gamma_G$ ($P_2 \times C_8$)= $|S|$ = 6= 2(k+1)

2.14 THEOREM:

$s\gamma_G$ **($K_{1,m} \times P_n$)= m+n-1**

Proof:

Label the vertices of K1, m×P_n **as** in figure 2.12

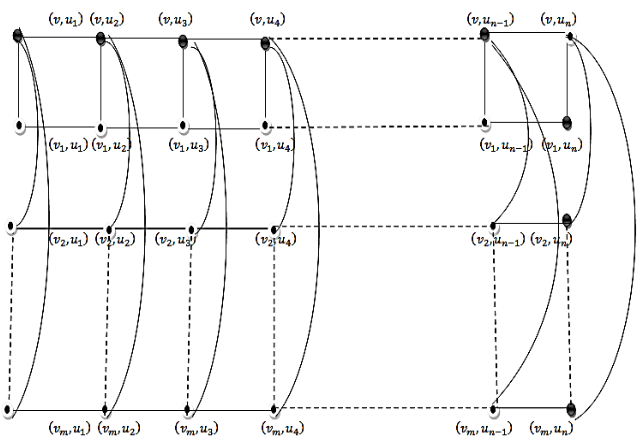


Figure 2.12

let V($K_{1,m}$) = {v,v_1,v_2,v_3,…………v_m} and V(P_n)= {u_1,u_2,u_3,………u_n}

Obviously, the set S = $\{(v,u_i),(v_j,u_n)$ / 1≤ i ≤ n-1, 1≤ j ≤m};

Strong dominates all the vertices of **$K_{1,m} \times P_n$** . Further, every vertex of V($K_{1,m} \times P_n$ –S) of the form (v,u_i) and (v_i,u_i) for I = 1 to n lie in the geodesic joining (v,u_1) and (v_1,u_n).

Also, any element of V($K_{1,m} \times P_n$ –S) of the form (v_k,u_i) for i= 1 to n lie in the geodesic joining (v,u_1) and (v_k,u_n) for k=2 to m.

∴ S is strong (G, D) set of ($K_{1,m} \times P_n$)

$s\gamma_G$ ($K_{1,m} \times P_n$)≤ $|S|$= m+n-1 ……….. (1)

Also, no set of less than $|S|$ elements is a strong (G, D) set of ($K_{1,m} \times P_n$)

∴ $s\gamma_G$ ($K_{1,m} \times P_n$)≥ $|S|$= m+n-1………….. (2)

From (1) and (2), we get,

$s\gamma_G$ ($K_{1,m} \times P_n$)= $|S|$= m+n-1

2.15 ILLUSTRATION:

$s\gamma_G$($K_{1,4} \times P_5$)= $|S|$ =m+n-1

Proof:

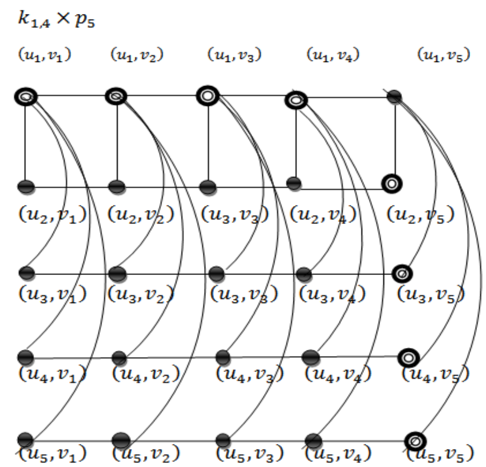


Figure 2.13

The S = { $(u_1v_1),(u_1,v_2),(u_1,v_3),(u_1,v_4),(u_2,v_5),(u_3,v_5),(u_4,v_5),(u_5,v_5)\}$

∴ $s\gamma_G$ ($K_{1,4} \times P_5$)= $|S|$= 8=m+n-1

REFERENCES

[1] Buckley F, Harary F and Quintas V L, Extremal results on the geodetic number of a graph, Scientia, volume A2 (1988), 17-26.

[2] Chartrand G, Harary F and Zhang P, Geodetic sets in graphs, DiscussionesMathematicae Graph theory, 20 (2000), 129-138e.

[3] Chartrand G, Harary F and Zhang P, On the Geodetic number of a graph, Networks, Volume 39(1) (2002), 1-6.

[4] Chartrand G, Zhang P and Harary F, Extremal problems in Geodetic graph Theory, CongressusNumerantium 131 (1998), 55-66.

[5] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Fundamentals Of Domination in graphs, Marcel Decker, Inc., New York 1998.

[6] Ore .O Theory of Graphs, American Mathematical Society Colloquium Publication 38 (American Mathematical Society Providence RI) 1962.

All Rights Reserved © 2018 IJSEM

[7] Palani .K and Kalavathi. S, (G,D) – Number of some special graphs, International Journal Of Engineering and Mathematical Sciences January-June 2014, Volume 5,Issue-1, pp.7-15ISSN(Print) – 2319 – 4537, (Online) – 2319 – 4545.

[8] Palani. K and Nagarajan. A (G,D) – number of graphs,International Journal Of Engineering and Mathematics Research. ISSN 0976 – 5840 Volume 3(2011), pp 285 -299.

[9] Palani. K, Nagarajan. A and Mahadevan. G, Results connecting domination, geodetic and (G,D)– number of graph, International Journal Of Combinatorial graph theory and applications, Volume 3, No.1, January – June (2010)(pp.51 -59).

[10] C. SanthaanaGomathi, Palani K and Kalavathi S, Strong (G,D)-Number of a graph – communicated.

[11] M. Mahalakshmi, A. Sony and K. Palani, Strong (G, D)-Number of Middle graphs -communicated.

[12] G.Susi vinnarasi, V. Selvalakshmi and K. Palani, Strong (G, D)-Number of Inflated graphs