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Solution of Differential Equations with Applications for Engineers

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Abstract: **- Over the last hundred years, many techniques have been developed for the solution of ordinary differential equations and partial differential equations. While a major portion of the techniques is only useful for academic purposes, there are some which are important in the solution of real problems. In this paper only few methods for solving ordinary differential and partial differential equations are discussed, as it is impossible to cover all the available techniques.. At the end some motivating examples are discussed which are commonly used by the engineers for the solution of real engineering problems.**

I. INTRODUCTION

Classification of ordinary and partial equations:

An ordinary differential equation (ODE) contains derivatives with respect to only one variable, partial differential equations (PDE) contain differentials with respect to several independent variables. The differential equation can also be classified as linear or nonlinear and homogeneous &non homogeneous . In a differential equation, when the variables and their derivatives are only multiplied by constants, then the equation is linear. The variables and their derivatives must always appear as a simple first power.

In(1), if h(x) is 0, then we term this equation as homogeneous. The general solution of non-homogeneous ordinary differential equation (ODE) or partial differential equation (PDE) is the sum of the Complementary function of the corresponding homogenous equation (i.e. with h(x) = 0) plus the particular solution of the non-homogeneous ODE or PDE. On the other hand, Non-linear differential equations are formed by the products of the unknown function and its derivatives are allowed and its degree is > 1. There are very few methods of solving nonlinear differential equations .. But nonlinear equation can be approximated as linear equation for many practical problems, either in an analytical or numerical form. The nonlinear nature of the problem is then approximated as series of linear differential equation by simple increment..

This approach is adopted for the solution of many non-linear engineering problems. Without such procedure, most of the non-linear differential equations cannot be solved. Differential equation can further be classified by the order of differential equation. In general, higher-order differential equations are difficult to solve, and analytical solutions are not available for many higher differential equations. A linear differential equation is generally governed by an equation form as Eq. (1).

$(\frac{d^n z}{dx^n}+c_1(x)\ \frac{d^{n-1} z}{dx^{n-1}}+\ \ldots\ldots\ldots + c_n(x)z = h(x)$---- (1)-

Non-linear" differential equation can generally be further classified as

1. *Truly nonlinear* in the sense that G is nonlinear in the derivative terms.

$G(t_1,t_2,t_3,u, \frac{\partial u}{\partial t1},\frac{\partial u}{\partial t2}, \frac{\partial^2 u}{\partial,t1\ \partial,t2}) = 0$

2. *Quasi-linear 1st* PDE if nonlinearity in *F* only involves *u* but not its derivatives

$A(y_1,y_2,u)\frac{\partial z}{\partial y1} + B(y_1,y_2,u)\frac{\partial z}{\partial y2} = C(y_1,y_2,u)$

3. *Quasi-linear 2nd* PDE if nonlinearity in *G* only involves *u* and its first derivative but not its second-order derivatives

$C_{11}(t_1,t_2,,u, \frac{\partial u}{\partial,t1},\frac{\partial u}{\partial,t2},)\ \frac{\partial^2 u}{\partial t1\ ^2} + C_{12}(t_1,t_2,,u, \frac{\partial u}{\partial t1},\frac{\partial u}{\partial t2},)\ \frac{\partial^2 u}{\partial,t1\ \partial,t2} = G(t_1,t_2,,u,u\ \frac{\partial u}{\partial,t1},\frac{\partial u}{\partial,t2},)$

Examples of differential equations

- dy/dx=7x-9 first-order ODE
- (x-3y)dy -7ydx=0 ; first-order ODE (nonlinear)/homogenous
- $\frac{d^2z}{dt^2}+\ t^2z(\frac{dz}{dt})^4\ +z=o$ second -order ODE (nonlinear)/homogenous.

$8x\frac{\partial^2 u}{\partial x^2}\ +4u\frac{\partial^2 u}{\partial y^2}\ +7\ u^2\ =0$ second order PDE(linear)/homogeneous

II SOME PARTIAL DIFFERENTIAL EQUATIONS IN ENGINEERING PROBLEMS

Many engineering problems are governed by different types of partial differential equations, and some of the more important types are given below.

1.Tricomi equation uxx + xuyy =0 it is eliptic hyperbolic

2. Laplace equation:$\nabla^2\Phi=0$

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3. Schrodinger's equation $\frac{\partial^2 \varphi}{\partial x^2} + \frac{8\pi^2\ m}{h^2}$ (E-V)φ=0 *4. 4. Transverse vibrations equation :* $\alpha^2 \frac{\partial^4 u}{\partial x^4} + \frac{\partial^2 u}{\partial t^2} = 0$

5. Poisson's equation $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = f(x,y,z)$

6. Helmholtz's equation uxx+uyy+uzz+k2u=0

7. Telegraph equation $\frac{\partial^2 u}{\partial x^2} = RC\frac{\partial i}{\partial t,}$

III. SEPARABLE DIFFERENTIAL EQUATIONS

A differential equation is considered separable if the two variables can be moved to opposite sides of the equation. This facilitates solving a homogenous differential equation, which can be difficult to solve without separation. Consider the equation

dy/dx p(y)=q(x)This equation can be rearranged to .dyP(y)=q(x)dx. Any equation that can be manipulated this way is separable. The equation is solved by integrating both sides, resulting in an implicit solution. If an initial condition is provided, you can solve the implicit solution for an explicit solution, and determine the interval of validity, the range of x where the solution is valid. The interval of validity must be continuous and must contain the x-value given in the original condition.

Example:

1. Solve the equation $2\,y\,dy = (x^2 + 1)\,dx$.

Since this equation is already expressed in "separated" form, just integrate:

$2\,y\,dy = (x^2 + 1)\,dx$.

$\int 2y dy = (x^2+1)dx$

$y2 = \frac{1}{3}x^3+x+c$

2. Solve the IVP $\frac{dy}{dx} = \frac{x(e^{x^2}+2)}{6y^2}$, Y(0)=1

The equation can be rewritten as follows. $6y^2dy=x(e^{x^2}+2)dx$ integrating both sides

$\int 6y^2 dy = \int x(e^{x^2}+2)dx$

$2y^3=\frac{1}{2}e^{x^2}+x^2+c$ Since the initial condition states that $y = 1$ at $x = 0$, the parameter can be evaluated We get c=1.5

The solution of the IVP is therefore

$2y^3=0.5e^{x^2}+x^2+1.5$

$4y^3=e^{x^2}+2x^2+3$

IV. VARIATION OF PARAMETERS

The method of Variation of Parameters is a much more general method that can be used in many more cases. However, there are two disadvantages to the method. First, the complementary solution is absolutely required to do the problem. This is in contrast to the method of undetermined coefficients where it was advisable to have the complementary solution on hand but was not required. Second, as we will see, in order to complete the method we will be doing a couple of integrals and there is no guarantee that we will be able to do the integrals. So, while it will always be possible to write down a formula to get the particular solution, we may not be able to actually find it if the integrals are too difficult or if we are unable to find the complementary solution.

To derive the formula for variation of parameters we can consider y''+ q(x)y'+ r(x)y =0.----(2)Then we know that $y=c_1y_1+c_2y_2$

is a solution of (2) for any constants c_1 and c_2. We now ``vary" c_1 and c_2 to functions of x, so that

$y = u(x)\,y_1+v(x)\,y_2$, $x \in I$ is the solution of equation y''+ q(x)y'+ r(x)y =0.

where *f* is a piecewise continuous function defined on I. The details are given in the following theorem.

THEOREM (Method of Variation of Parameters)

Let q(x) and r(x) be continuous functions defined on I and let f be a piecewise continuous function on I Let y_1 and y_2 be two linearly independent solutions of (1) on I, Then a particular solution y_p of (1) is given by

$y_p = -y_2\int \frac{y2f(x)}{W}dx + y_2\int \frac{y1f(x)}{W}$

where $W=W(y_1, y_2)$is the Wronskian of y_1 and y_2

V PARTIAL DIFFERENTIAL EQUATIONS

In many engineering or science problems, such as heat transfer, elasticity, quantum mechanics, water flow and others, the problems are governed by partial differential equations. By nature, this type of problem is much more complicated than the previous ordinary differential equations. There are several major methods for the solution of PDE, including separation of variables, method of characteristic, integral transform, superposition principle, change of variables, Lie group method, semi analytical methods as well as various numerical methods. Although the existence and uniqueness of solutions for ordinary differential equation is well established with the Picard-Lindelöf theorem, but that is not the case for many partial differential equations. In fact, analytical solutions are not available for many partial differential equations, which is a well-known fact, particularly when the solution domain is not regular or homogeneous, or the material properties change with the solution steps.

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VI CLASSIFICATION OF SECOND-ORDER PDE

Refer to the following general second-order partial differential equation

$A\frac{\partial^2 u}{\partial v^2}+ B\frac{\partial^2 u}{\partial v\partial Y}C\frac{\partial^2 u}{\partial v^2}+D\frac{\partial u}{\partial x}+E\frac{\partial u}{\partial y}+Fu+G=0$ The conic curve can be classified with following criterion

$$B^2 \_ 4AC\begin{cases} > 0 \, hyperbola \\ = 0 \, parabola \\ < ellipse \end{cases}$$

Following the conic curves, the general partial differential is also classified according to similar

criterion as

$$\text{Classification } B^2 \_ 4AC\begin{cases} > hyperbolic \\ = 0 \, parabolic \\ < elliptic \end{cases}$$

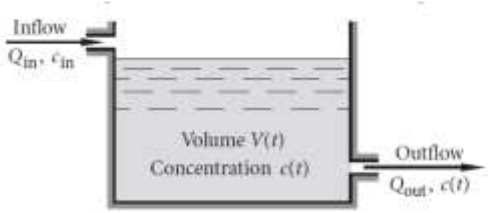
This classification was proposed by Du Bois-Reymond in 1839. In this section, only some of the more common techniques are discussed. Some commonly used examples for PDE are Laplace equation, One dimensional Wave equation and One dimensional heat equation .

VII MOTIVATING EXAMPLES:

A few examples are presented to illustrate how practical problems are modeled mathematically and how differential equations arise in them.

EXAMPLE:1

A tank contains liquid of volume V(t) which is polluted with pollutant concentration in percentage of (t) at time t. To reduce the pollutant concentration an inflow of rate Qin is injected to the tank. Unfortunately, the inflow is also polluted but to a lesser degree with a pollutant concentration cin. It is assumed that the inflow is perfectly mixed with the liquid in the tank instantaneously. An outflow of rate Qout is removed from the tank as shown. Suppose that, at time t = 0, the volume of the liquid is V0 with a pollutant concentration of c0.

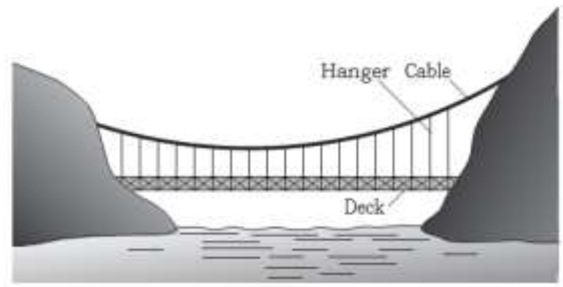


The equation governing the pollutant concentration c(t) is given by

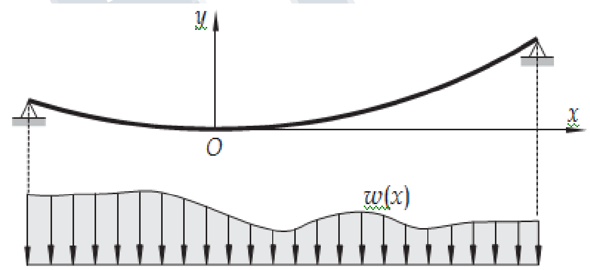
with initial condition c(0) = c0. This is a first-order ordinary differential equation

$$[V_0 + (Q_{in} - Q_{out})t]\frac{dc(t)}{dt} + Q_{in}c(t) = Q_{in}c_{in},$$

EXAMPLE:2



Consider the suspension bridge as shown, which consists of the main cable, the hangers, and the deck. The self-weight of the deck and the loads



applied on the deck are transferred to the cable through the hangers.

Set up the Cartesian coordinate system by placing the origin O at the lowest point of the cable. The cable can be modeled as subjected to a distributed load w(x). The equation governing the shape of the cable is given by

$\frac{d^2y}{dx^2}= \frac{w(x)}{T}$ Here T is the tension in the cable at the lowest point. This is ODE of second order.

EXAMPLE:3

As shown in Figure 5.8, jet engines are supported by the wings of the airplane. To study the horizontal motion of a jet engine, it is modeled as a rigid body supported by an elastic beam. The mass of the engine is m and the moment of inertia about its centroidal axis C is J. The elastic beam is further modeled as a mass less bar hinged

at A, with the rotational spring κ providing restoring moment equal to κθ, where θ is the angle between the bar and the vertical line as shown in Figure

For small rotations, i.e. $|\boldsymbol{\theta}|<1$, set up the equation of motion for the jet engine terms of θ. Find the natural frequency of oscillation.

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CONCLUSION

The purpose of this paper is to relate differential equations, which has very huge number of applications, to engineering problems. In this paper I have give only three motivating problems .Even though there are many problems & methods that are not discussed here,it is expected that the problems presented in this paper can motivate engineering students to understand mathematics better.

REFERENCES

[1] Solution of Differential Equations with applications to Engineering Problems by Cheng Yung Ming Department of Civil and Environmental Engineering, Hong Kong Polytechnic University, Hong Kong SAR, China

[2] Differential equations for engineers by Wei-Chau Xie University of Waterloo, Cambridge.

[3] Bronson R. and Costa G.B. Differential equations, McGraw-Hill, USA, 2006.

[4] Braun, M., Differential Equations and Their Applications, Springer Science+Business Media. New York,1993

[5] An application of differential equations in the study of elastic columns by Krystal Caronongan B.S., Southern Illinois University, 2008.

[6] Differential Equations with Applications to Industry Article in International Journal of Differential Equations 2012 • December 2012.

[7] First order ordinary differential equations from book Engineering Differential Equations: Theory and Applications (pp.57-90).

[8] Differential Equations in Engineering Courses D. C. Francis pages 263-268, Published : 09 Jul 2006.