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On Contra g*α-continuous functions

[1] Dr. A. Punitha Tharani, [2] T. Delcia
[1] Associate Professor, [2] Research Scholar
Department of Mathematics, St.Mary's College (Autonomous),Thoothukudi, Tamil Nadu, India

Abstract: -- **In this paper,we introduce a new class of functions called contra g*α-continuous functions in topological spaces.some characterizations and several properties concerning contra g*α-continuous functions are obtained.**

Keywords: -- **contra g*α-continuous,almost contra g*α-continuous,contra g*α-irresolute,g*α-locally indiscrete**

I. INTRODUCTION

In 1996,Dontchev presented a new notions of continuous functions called contra-continuity.This notion is stronger form of LC-continuity.The purpose of this paper is to introduce a new class of generalized continuous functions called contra g*α- continuous functions and almost contra g*α-continuous functions and investigate their relationship with other functions.

II. PRELIMINARIES

In this paper the spaces X and Y always mean topological spaces (X,τ) and (Y,σ) respectively.For a subset A of a space,cl(A) and int(A)represent closure of A and interior of A resopectively.

Definition 2.1:A Subset A of (X,τ) is called

(1) a preopen set **[6]** if A ⊆ int cl(A) and preclosed set if cl(int(A)) ⊆ A

(2) a regular open set **[13]** if A=int cl(A) and regular closed set if A=cl(int(A))

(3) a α-open set **[7]** if A⊆int(cl(int(A)) and α-closed if cl(int(cl(A))⊆A

Definition 2.2: A Subset A of (X,τ) is called

(1) generalized closed set(briefly g-closed) **[5]** if cl(A) ⊆ U whenever A⊆U and U is open in X.

(2) g*-closed **[14]** if cl(A) ⊆ U whenever A⊆U and U is g-open in X.

(3) regular generalized closed(briefly rg-closed) **[8]** if cl(A) ⊆ U whenever A⊆U and U is regular open in X.

(4) generalized preregular closed set(briefly gpr-closed)**[4]** if pcl(A)⊆U

(5) whenever A⊆U and U is regular open in X.

(6) $g^{\#}$-closed **[15]** if cl(A) ⊆ U whenever A⊆U and U is αg-open in X.

The complements of the above mentioned closed sets are their respective open sets.

Definition 2.3[9]: A subset A of (X,τ) is called g*α-closed if αcl(A)⊆U whenever A⊆U and U is g*-open in X.The complement of g*α-closed set is g*α-open set.The family of g*α-closed sets and g*α-open sets are denoted by G*α-C(X) and G*α-O(X)

Definition 2.4: A function f: (X,τ)→(Y,σ) is said to be

(1) g*α-continuous **[9]** if f^{-1}(V) is g*α-closed in (X,τ) for every closed set V of (Y,σ).

(2) g*α-irresolute**[9]** if f^{-1}(V) is g*α-closed in (X,τ) for every g*α-closed set V of (Y,σ).

Definition 2.5: A function f: (X,τ)→(Y,σ) is called

(1) a contra continuous **[1]** if f^{-1}(V) is closed in (X,τ) for every open set V of (Y,σ).

(2) a contra g*-continuous **[11]** if f^{-1}(V) is g*-closed in (X,τ) for every open set V of (Y,σ).

(3) a contra $g^{\#}$-continuous **[15]** if f^{-1}(V) is $g^{\#}$- closed in (X,τ) for every open set V of (Y,σ)

Definition 2.6:A space X is called

(i) ${}_{\alpha}T^{**}_{1/2}$-space**[9]** if every g*α-closed set in it is closed.

(ii) locally indiscrete**[12]** if every open subset of X is closed in X

III. CONTRA g*α-CONTINUOUS FUNCTION

In this section, we introduce the notions of contra g*α-continuous, contra g*α-irresolute and almost contra g*α-continuous functions in topological spaces and study some of their properties.

Definition 3.1:A function f: (X,τ)→(Y,σ) is called Contra g*α-continuous if f^{-1}(V) is g*α-closed set in X for each open set V in Y.

Example 3.2: : Let X = {1,2,3} =Y with τ={∅,X,{1}} and σ = {∅,Y,{2}}.Define f: (X,τ)→(Y,σ) by f(1)=1,f(2)=3 and f(3)=2.clearly f is contra g*α-continuous function.

Example 3.3: : Let X = {1,2,3,4} =Y with τ={∅,X,{1},{2},{1,2},{1,2,3}} and σ = {∅,Y,{3},{1,3,4}}.Define f: (X,τ)→(Y,σ) by f(1) =1,f(2)=2, f(3)=4 and f(4)=3.clearly f is contra g*α-continuous function.

Theorem 3.4: Every contra continuous function is conta

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g*α-continuous.

Proof: It follows from the fact that every closed set is g*α-closed.

The converse of the above theorem is not true as seen from the following example.

Example 3.5: Let X = {1,2,3,4} =Y with τ={∅,X,{1},{2,3},{1,2,3}} and σ ={∅,Y,{1,3,4}}.Define f: (X,τ)→(Y,σ) by f(1) =4,f(2)=3, f(3)=2and f(4)=1.clearly f is contra g*α-continuous but not contra continuous since f^{-1}({1,3,4})={1,2,4}is g*α-closed but not closed in X.

Theorem 3.6: If a function f:X→Y is contra g*α-continuous and X is $_\alpha T^{**}_{1/2}$-space, then f is contra continuous.

Proof: Let V be an open set in Y.Since f is conta g*α-continuous, f^{-1}(V) is g*α -closed in X.Hence V is closed in X since X is $_\alpha T^{**}_{1/2}$-space.Thus f is contra continuous.

Corallary3.7: If X is $_\alpha T^{**}_{1/2}$-space then for a function f:X→Y the following are equivalent.

(i)f is contra continuous

(ii) f is contra g*α-continuous

Proof: It is obvious.

Remark 3.8: The concept of g*α-continuity and contra g*α- continuity are independent as shown in the following example.

Example 3.9: Let X = {1,2,3,4} =Y with τ={∅,X,{1},{1,4},{1,2,4}} and σ ={∅,Y,{2},{3,4},{2,3,4}}.Define f: (X,τ)→(Y,σ) by f(1) =1,f(2)=3, f(3)=4 and f(4)=2.clearly f is contra g*α-continuous but f^{-1}({1,2})={1,4} is not g*α-closed in X. Therefore f is not g*α-continuous.

Example 3.10: Let X = {1,2,3,4} =Y with τ={∅,X,{1,3,4}} and σ ={∅,Y,{1,4}}.Define f: (X,τ)→(Y,σ) by identity mapping.clearly f is g*α-continuous but not contra g*α-continuous since f^{-1}({1,4})={1,4} is not g*α-closed in X. Therefore f is not g*α-continuous.

Theorem 3.11: Every contra g-continuous is conta g*α-continuous.

Proof:since every g-closed set is g*α-closed ,the proof follows.

The converse of the above theorem is not true as seen from the following example.

Example 3.12: Let X = {1,2,3} =Y with τ={∅,X,{1},{1,2}} and σ = {∅,Y,{2}}.Define f: (X,τ)→(Y,σ) by f(1) =3,f(2)=2 and f(3)=1.clearly f is contra g*α-continuous but not contra g-continuous.

Theorem 3.13:(i)Every contra g*-continuous is contra g*α-continuous.

(ii)Every contra g*α-continuous is contra rg-continuous.

(iii)Every contra g*α-continuous is contra gpr-continuous

(iv)Every contra $g^{\#}$-continuous is contra g*α-continuous.

Proof:(i) & (iv) proof follows from the fact that every g*-closed and $g^{\#}$- closed is g*α-closed.

(ii) & (iii) since every g*α-closed set is rg-closed and gpr-closed,the proof follows.

Remark 3.14: The converse of the above theorem need not be true as seen from the following examples.

Example 3.15: Let X = {1,2,3} =Y with τ={∅,X,{1},{1,2},{1,3}} and σ = {∅,Y,{2}}.Define f: (X,τ)→(Y,σ) by f(1) =2,f(2)=3 and f(3)=1.clearly f is contra rg-continuous and contra gpr-continuous but not contra g*α-continuous.

Example 3.16: Let X = {1,2,3} =Y with τ={∅,X,{1}} and σ = {∅,Y,{2,3}}.Define f: (X,τ)→(Y,σ) by f(1) =2,f(2)=1and f(3)=3.Here f is contra g*α-continuous but not contra $g^{\#}$-continuous and contra g*-continuous.

Remark 3.17: The composition of two contra g*α-continuous functions need not be contra g*α-continuous as seen from the following example.

Example 3.18: Let X = {1,2,3,4} =Y with τ={∅,X,{3},{1,3,4}} and σ={∅,Y,{1},{2},{1,2}{2,3},{1,2,3}} and η = {z, ∅,{1,3,4}Let f: (X,τ)→(Y,σ) and g: (Y,τ)→(η,σ) by identity mapping.Here f and g are contra g*α-continuous.But g°f:X→Z is not contra g*α-continuous,since (g°f)$^{-1}$({1,3,4}}=f^{-1}(g^{-1}({1,3,4}))= f^{-1}({1,3,4})={1,3,4} which is not g*α-closed in X.

Theorem 3.19:If f: (X,τ)→(Y,σ) is contra g*α-continuous and g:Y→Z is continuous then g°f:X→Z is contra g*α-continuous.

Proof: Let V be open in Z.Since g is continuous g^{-1}(V) is open in Y.Then f^{-1}(g^{-1}(V)) is g*α-closed in X since f is contra g*α-continuous. Thus (g° f)$^{-1}$(V) is g*α-closed in X.Hence g° f is contra g*α-continuous.

Corallary 3.20: If f: (X,τ)→(Y,σ) is g*α-irresolute and g:Y→Z is contra continuous function then g°f:X→Z is contra g*α-continuous.

Proof: Using the fact that every contra continuous is contra g*α-continuous.

Theorem 3.21: Let f: X→Y be surjective,g*α-irresolute and g*α-open and is continuous and g:Y→Z be any function then g° f is contra g*α-continuous iff g is contra g*α-continuous.

Proof: Suppose g°f is contra g*α-continuous.Let V be a closed set in Z.Then (g° f)$^{-1}$(V) = f^{-1}(g^{-1}(V)) is g*α-open in X.Since f is g*α-open and surjective,g*α-irresolute f($f^{-1}(g^{-1}$(V)) is g*α-open in Y.(ie) g^{-1}(V) is g*α-open in Y.Hence g is contra g*α-continuous.Conversely suppose that g is contra g*α-continuous.Let V be closed in Z.Then g^{-1}(V) is g*α-open in Y.Since f is g*α-irresolute $f^{-1}(g^{-1}$(V)) is g*α-open (ie) (g° f)$^{-1}$(V) is g*α-open in X.Hence g° f is contra g*α-continuous.

Theorem 3.22: Let f:X→Y be a map.Then the following are equivalent.

(i)f is contra g*α-continuous

(ii)The inverse image of each closed set in Y is g*α-open in X.

Proof:(i)⟹(ii) and (ii)⇒(i) are obvious.

Definition 3.23:A space (X,τ) is called locally g*α-indiscrete if every g*α-open set of X is closed in X.

Theorem 3.24: Let X be locally g*α-indiscrete.If f:X→Y is contra g*α-continuous then it is continuous.

Proof: Let V∈ O(Y).Then f^{-1}(V) is g*α-closed in X.Since X is locally g*α-indiscrete space,f^{-1}(V) is open in X.Hence f is continuous.

Theorem 3.25: If a function f:X→Y is g*α-continuous and the space (X,τ) is g*α-locally indiscrete then f is contra continuous.

Proof: Let V∈ O(Y).Since f is g*α-continuous,$f^{-1}(V)$ is g*α-open in X.Since X is locally g*α- indiscrete, f^{-1}(V) is closed in X.Hence f is contra continuous.

Definition 3.25:A function f: (X,τ)→(Y,σ) is called almost g*α-continuous if f^{-1}(V) is g*α-open set in X for every regular open set V of Y.

Definition 3.26: A function f: (X,τ)→(Y,σ) is called almost Contra g*α-continuous if f^{-1}(V) is g*α-closed set in X for every regular open set V of Y.

Theorem 3.26: Every contra g*α-continuous function is almost contra g*α-continuous.

Proof: Since every regular open set is open the proof follows.

Theorem 3.27: Every regular set connected function is almost contra g*α-continuous but not conversely.

Proof: Proof is straight forward.

Example 3.28:X=Y={1,2,3,4} with τ={∅,X,{1},{1,2},{1,4},{1,2,3},{4},{1,2,4}}and σ={∅,Y,{3},{4},{3,4},{2,4},{1,3,4},{2,3,4}}.Let f be an identity map.The inverse image of regular open set {2,4} is not clopen in X.But the inverse image of regular open set in Y is g*α-closed in X.Hence f is almost contra g*α-continuous but not regular set connected.

Theorem 3.27: Let f: X→Y and g:Y→Z be two functions.Then the following properties hold.

a)If f is almost contra g*α-continuous and g is regular set connected,then g∘f:X→Z is almost contra g*α-continuous and almost g*α-continuous.

b)If f is almost contra g*α-continuous and g is perfectly continuous g∘f:X→Z is g*α-continuous and contra g*α-continuous.

Proof:(a) Let VϵRO(Z).Since g is regular set connected,g^{-1}(V) is clopen in Y.since f is almost contra g*α-continuous,$f^{-1}(g^{-1}$(V))=(g∘f)$^{-1}$(V) is g*α-open and g*α-closed in X.Therefore g∘f is almost contra g*α-continuous and almost g*α-continuous.

(b) Let v be open in Z.Since g is perfectly continuous, g^{-1}(V) is clopen in Y.since f is almost contra g*α-continuous, $f^{-1}(g^{-1}$(V))=(g∘f)$^{-1}$(V) is g*α-open and g*α-closed in X. Therefore g∘f is g*α-continuous and contra g*α-continuous.

Definition 3.25:A function f: (X,τ)→(Y,σ) is called Contra g*α-irresolute if f^{-1}(V) is g*α-closed set in X for every g*α- open set V in Y.

Definition 3.26:A function f: (X,τ)→(Y,σ) is called perfectly Contra g*α-irresolute if f^{-1}(V) is g*α-closed and g*α-open in X for every g*α- open set V in Y.

Theorem 3.27:If f: X→Y is perfectly contra g*α-irresolute iff f is contra g*α-irresolute and g*α-irresolute.

Proof: It directly follows from the definitions.

Remark 3.28: The following example shows that the concepts of g*α-irresolute and contra g*α-irresolute are indepdent of each other.

Example 3.29: Let X=Y={1,2,3} with τ={∅,X,{1},{2},{1,2}} and σ={∅,Y,{1,2}}.Define f:X→Yby f(1)=2,f(2)=1 and f(3)=3.Clearly f is g*α-irresolute but not contra g*α-irresolute since f^{-1}({1}) ={2}is not g*α-closed in X.

Example 3.30: Let X=Y={1,2,3} with τ={∅,X,{1},{1,2}} and σ={∅,Y,{2,3}}.Define f:X→Y by f(1)=1,f(2)=3 and f(3)=2.Clearly f is contra g*α-irresolute but not g*α-irresolute since f^{-1}({1,3})={1,2}is not g*α-closed in X.

Remark 3.31: Every contra g*α-irresolute function is contra g*α-continuous .But the converse need not be true as seen from the following example.

Example 3.32: Let X=Y={1,2,3,4} with τ={∅,X,{1},{1,2},{1,4},{1,2,3},{4},{1,2,4}} and σ={∅,Y,{3},{1,3,4}}.Define f:X→Y by an identity mapping.Clearly f is contra g*α-continuous but not contra g*α-irresolute.

Theorem 3.33: Let f:X→Y and g:Y→Z be a function.Then

(i)if g is g*α-irresolute and f is contra g*α-irresolute then g°f is contra g*α-irresolute.

(ii)if f is g*α-irresolute and g is contra g*α-irresolute then g°f is contra g*α-irresolute.

Proof(i) Let U be g*α-open in Z.Since g is g*α-irresolute , g^{-1}(U) is g*α-open in Y.Thus $f^{-1}(g^{-1}$(U)) is g*α-closed in X since f is contra g*α-irresolute.(ie)$(g°f)^{-1}$(U) is g*α-closed in X.This implies that g°f is contra g*α-irresolute.

(ii) Let U be g*α-open in Z.Since g is contra g*α-irresolute , g^{-1}(U) is g*α-closed in Y.Thus f (g^{-1}(U)) is g*α-closed in X since f is g*α-irresolute (ie)$(g°f)^{-1}$(U) is g*α-closed in X.This implies that g°f is contra g*α-irresolute.

Theorem 3.34: If f:X→Y is contra g*α-irresolute and g:Y→Z is g*α-continuous then g°f is contra g*α-continuous.

Proof: Let U be an open set in Z.Since g is g*α-continuous,g^{-1}(U) is g*α-open in Y.Thus $f^{-1}(g^{-1}$(U)) is g*α-closed in X Since f is contra g*α-irresolute.(ie)$(g°f)^{-1}$(U) is g*α-closed in X.This implies that g°f is contra g*α-continuous.

Remark 3.35: Every perfectly contra g*α-irresolute function is contra g*α-irresolute and g*α-irresolute.

The following two examples show that a contra g*α-irresolute function may not be perfectly contra g*α-irresolute and a g*α-irresolute function may not be perfectly contra g*α-irresolute.

Example 3.36: Let X=Y={1,2,3} with τ={∅,Y,{1}} and σ={∅,X,{2},{2,3}}.Define f:X→Y by an identity mapping.Clearly f is contra g*α-irresolute but not perfectly contra g*α-irresolute.

Example 3.37: Let X=Y={1,2,3,4} with τ={∅,X,{3},{1,2},{1,2,3}} and σ={∅,Y,{1},{2,3},{1,2,3}}.Define f:X→Y by an identity mapping.Clearly f is g*α-irresolute but not perfectly contra g*α-irresolute.

Theorem 3.38:A function is perfectly contra g*α-irresolute iff f is contra g*α-irresolute and g*α-irresolute.

Proof: It follows from the definitions

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