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On Zero-Symmetric Semicentral Γ-Near Rings

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Abstract:- **In this paper, we define zero-symmetric part in semicentral Γ-near rings. The definitions of left semicentral, right semicentral gamma near rings are defined along with the equivalence conditions for an idempotent e in the Γ-near ring to be both left semicentral and right semicentral. Any reduced gamma near ring with unity is both left and right semicentral Γ-near ring if it is zero-symmetric. Any left(right) regular unital Γ-near ring is right(left) semicentral if it is zero-symmetric.**

Key Words: Semi central, idempotent element in Γ-near ring, unital, zero-symmetric, left(right) regular, reduced.

1. INTRODUCTION

For basic definition of near ring we refer Pilz [13, 14]. A generalization of both the concepts near-rings and Γ-rings, namely Γ-near rings was introduced by Satyanarayana Bhavanari [2, 15]. Yong Uk Cho [17] obtained some basic concepts and properties of Γ-near ring through regularity condition. Mason [7] give some characterization of left regular unital near-rings. Naoki kimura [12] gives some structures of idempotent semigroup. J Han, Y Lee, S Park [8] works on semicentral idempotents in ring. Further, Henry Heatherly, Ralch P. Tucci [9] works on central & semicentral idempotents in rings. Our main aim of this paper is to introduce the concept of semicentral gamma near rings.

II. PRELIMINARIES:

DEFINITION: II.1 [1]

Let (M, +) be a group (need not be abelian) and Γ be a non-empty set. Then M is said to be *Γ-near ring* if there exists a mapping M $\times \Gamma \times$ M $\longrightarrow$ M (the image of (a, α, b) is denoted by $a\alpha b$) satisfying the following condition:

(i) $(a+b)\alpha c = a\alpha c + b\alpha c$
(ii) $(a\alpha b)\beta c = a\alpha(b\beta c) \forall\, a, b, c \in M$ and $\alpha, \beta \in \Gamma$

DEFINITION: II.2 [16]

A gamma near ring M is said to be *unital Γ-near ring*, if M has an unit element.

DEFINITION: II.3 [1]

A Γ-near ring M is said to be *zero symmetric Γ-near ring* if $a\gamma 0 = 0 \,\forall\, a \in M \,\&\, \gamma \in \Gamma$.

DEFINITION: II.4 [16]

An element d in the gamma near ring M is called *distributive* if $d\gamma(a+b) = d\gamma a + d\gamma b \,\forall\, a, b \in M \,\&\, \gamma \in \Gamma$.

NOTATION: II.5

$M_0 - \{a \in M \mid a\gamma 0 = 0\}$ which is called the zero symmetric part of M.

$M_c - \{a \in M \mid a\gamma 0 = a\}$ which is called the constant part of M.

$M_d -$ set of all distributive elements in M.

DEFINITION: II.6 [10]

An element e in the gamma near ring, M is called an *idempotent* if $e\gamma e = e$ for all γ in Γ.

DEFINITION: II.7 [17]

A gamma near ring M is called *regular Γ-near ring* if for any element $a \in$ M, there exists an element x in M such that $a = a\gamma_1 x\gamma_2 a$ for every non-zero pair of elements $\gamma_1, \gamma_2 \in \Gamma$. And the element a is called *regular*.

DEFINITION: II.8

A Γ-near ring M is called *π-regular gamma near ring* if for any element a in M, there exists a positive integer n such that a^n is regular.

In these cases, $a\gamma x, x\gamma a, a^n\gamma x, x\gamma a^n$ are all idempotents.

DEFINITION: II.9 [16]

A Γ-near ring M is said to be *left regular gamma near ring* if for each $a \in M$ there exists $x \in$ M such that $a = x\gamma_1 a\gamma_2 a$ for every non-zero pair of elements $\gamma_1, \gamma_2 \in \Gamma$.

DEFINITION: II.10 [16]

A gamma near ring M is said to be *right regular Γ-near ring* if, for each $a \in M,$ there exists $x \in M$ such that $a = a\gamma_1 a\gamma_2 x$ for every non-zero pair of elements γ_1, γ_2 in Γ.

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DEFINITION: II.11

An idempotent e in Γ-near ring M is *central* in M if $e\gamma x = x\gamma e$ for all x in M.

DEFINITION: II.12

A Γ-near ring M in which every idempotent is central is called *central Γ-near ring.*

DEFINITION: II.13 [2]

An element $0 \neq a$ in the gamma near ring M is called *nilpotent* if $\{a\}^n = 0$ for some $n \geq 2$.

DEFINITION: II.14 [11]

A gamma near ring M is called *reduced Γ-near ring* if M has no non-zero nilpotent elements. (i.e.) for each $a \in M, (a\gamma)^n a = 0$, for some positive integer n implies $a = 0$.

LEMMA: II.15

Let M be a left regular gamma near ring. If for any a, b in M with $a\gamma b = 0$ for all γ in Γ, then $(b\gamma a)^n = b\gamma 0$ for all $\gamma \in \Gamma$ & for all positive integer n.
In particular, $b\gamma a = b\gamma 0 \ \forall \ \gamma \in \Gamma$.

Proof:

Let M be a left regular gamma near ring and let for any $a, b \in M$ with $a\gamma b = 0$ for all $\gamma \in \Gamma$.

(i) $n = 1$, for $b\gamma a \in$ M, by the left regularity, there exists $x \in M$ such that $b\gamma a = x\gamma_1(b\gamma a)\gamma_2(b\gamma a)$. From this equality, we obtain that $b\gamma a = x\gamma_1 b\gamma a\gamma_2 b\gamma a = x\gamma_1 b\gamma 0\gamma a = x\gamma_1 b\gamma 0 = x\gamma_1 b\gamma 0\gamma_2 b = b\gamma a\gamma_2 b = b\gamma 0$. Thus the statement is true for $n = 1$.

(ii) Assume that statement is true for $n = k$, so $(b\gamma a)^k = b\gamma 0$ for all $\gamma \in \Gamma$. Then $(b\gamma a)^{k+1} = (b\gamma a)^k\gamma_1(b\gamma a) = (b\gamma 0)\gamma_1(b\gamma a) = b\gamma 0\gamma_1 b\gamma a = b\gamma 0$. Hence the statement with left regular case is true for all positive integer n.

COROLLARY: II.16

Let M be a right regular Γ-near ring. If for any $a, b \in M$ with $a\gamma b = 0$ for all γ in Γ, then $(b\gamma a)^n = b\gamma 0$, for all γ in Γ and for all positive integer n.
In particular, $b\gamma a = b\gamma 0 \ \forall \ \gamma \in \Gamma$.

LEMMA: II.17

Let M be a left regular Γ-near ring. If for any $a, b \in M$, with $a\gamma b = 0$ and $a^n = a\gamma 0$ for all positive integer $n \geq 2$ and $\forall \ \gamma \in \Gamma$, then $a = 0$.
In particular, if for any $a, b \in M$ with $a\gamma b = 0$ and $a^2 = a\gamma 0 \ \forall \ \gamma \in \Gamma$, then $a = 0$.

Proof:

Let M be a left regular gamma near ring and let for any $a, b \in M$ with $a\gamma b = 0$ & $a^n = a\gamma 0$ for all positive integer $n \geq 2$ and $\forall \ \gamma \in \Gamma$. Then for $a \in M$, by left regularity, there exists $x \in M$ such that $a = x\gamma_1 a^2 = x\gamma_1 a\gamma_2 a = x\gamma_1(x\gamma_1 a\gamma_2 a)\gamma_2 a = x^2\gamma_1 a^3 = \cdots \ldots \ldots . = x^{n-1}\gamma_1 a^n = x^{n-1}\gamma_1 a\gamma 0$. On the other hand, $a\gamma 0 = a^n = a\gamma a^{n-1} = x^{n-1}\gamma_1 a\gamma 0\gamma a^{n-1} = x^{n-1}\gamma_1 a\gamma 0 = a$. Hence $a = a\gamma 0 = a\gamma 0\gamma b = a\gamma b = 0$.

COROLLARY: II.18

Let M be a right regular gamma near ring. If for any $a, b \in M$ with $a\gamma b = 0$ and $a^n = a\gamma 0$ for all positive integer $n \geq 2$ and for all γ in Γ, then $a = 0$.

LEMMA: II.19

Let M be a zero-symmetric left regular Γ-near ring with $a\gamma b = 0$, for any $a, b \in M$ & $\gamma \in \Gamma$. Then M is reduced.

Proof:

Let M be a zero-symmetric left regular gamma near ring with $a\gamma b = 0$ for any $a, b \in M$ & $\gamma \in \Gamma$. By lemma: II.15, it follows that $a^n = a\gamma(a^{n-1}) = a\gamma 0$ for all positive integer n & $\forall \ \gamma \in \Gamma$. Then by lemma: II.17, $a = 0$. Therefore, $a^n = 0 \Longrightarrow a = 0$. Hence M is reduced.

COROLLARY: II.20

Let M be a zero-symmetric right regular gamma near ring with $a\gamma b = 0$, for any $a, b \in M$ & $\gamma \in \Gamma$. Then M is reduced.

III. SEMICENTRAL GAMMA NEAR RINGS:

Throughout this paper, let M stands for unitary gamma near ring.

DEFINITION: III.1

An idempotent e in Γ-near ring M is called *left semicentral* in M if $M\Gamma e = e\Gamma M\Gamma e$.

DEFINITION: III.2

An idempotent e in gamma near ring M is called *right semicentral* in M if $e\Gamma M = e\Gamma M\Gamma e$.

DEFINITION: III.3

A gamma near ring M in which every idempotent is left semicentral is called *left semicentral Γ-near ring.*

DEFINITION: III.4

A Γ-near ring M in which every idempotent is right semicentral is called *right semicentral gamma near ring.*

DEFINITION: III.5

Two idempotents e, f in the gamma near-ring M are said to be isomorphic if there exist $a, b \in$ M such that $e = a\gamma b, f = b\gamma a$ for all γ in Γ.

NOTATION: III.6

I(M) – set of all idempotents of M

$S_l(M)$ – set of all left semicentral idempotents in M

$S_r(M)$ – set of all right semicentral idempotents in M

$C(M)$ – set of all central idempotents in M

Now, we give the characterizations of left semicentral and right semicentral conditions in Γ-near ring M.

PROPOSITION: III.7

For any idempotent e in the gamma near ring M, the following conditions hold.

(i) *e is left semicentral* $\Leftrightarrow a\gamma e = e\gamma_1 a\gamma e\ \forall\ a \in M\ \&\ \gamma, \gamma_1 \in \Gamma$

(ii) *e is right semicentral* $\Leftrightarrow e\gamma a = e\gamma a\gamma_1 e\ \forall\ a \in M\ \&\ \gamma, \gamma_1 \in \Gamma$

Proof:

(i) (=>) Let $e \in$ M be left semicentral. Then for any $a \in M$, there exists $b \in M$ such that $a\gamma e = e\gamma b\gamma_1 e$. Multiplying e to the left on both sides of the equation, we get $e\gamma_1 a\gamma e = e\gamma_1 e\gamma b\gamma_1 e = e\gamma b\gamma_1 e$. Thus $a\gamma e = e\gamma b\gamma_1 e = e\gamma_1 a\gamma e$. Therefore, $a\gamma e = e\gamma_1 a\gamma e\ \forall\ a \in M\ \&\ \gamma, \gamma_1 \in \Gamma$.

(<=) Let $x \in M\Gamma e$. Then $x = a\gamma e$ for some $a \in M\ \&\ \gamma \in \Gamma$. Thus $x = a\gamma e = e\gamma_1 a\gamma e \in e\Gamma M\Gamma e$ and $M\Gamma e \subset e\Gamma M\Gamma e$. Let $y \in e\Gamma M\Gamma e$. Then $y = e\gamma_1 a\gamma e = a\gamma e \in M\Gamma e$ and so $e\Gamma M\Gamma e \subset M\Gamma e$. Hence e is left semicentral.

(ii) (=>) Let $e \in M$ be right semicentral. Then for any $a \in M$, there exists $b \in M$ such that $e\gamma a = e\gamma_1 b\gamma e$. Multiplying e to the right on both sides of the equation, we get $e\gamma a\gamma_1 e = e\gamma_1 b\gamma e\gamma_1 e = e\gamma_1 b\gamma e$. Thus $e\gamma a = e\gamma_1 b\gamma e = e\gamma a\gamma_1 e$. Therefore, $e\gamma a = e\gamma a\gamma_1 e\ \forall\ a \in M\ \&\ \gamma, \gamma_1 \in \Gamma$.

(<=) Let $x \in e\Gamma M$. Then $x = e\gamma a$ for some $a \in M\ \&\ \gamma \in \Gamma$. Thus $x = e\gamma a = e\gamma a\gamma_1 e \in e\Gamma M\Gamma e$ and so $e\Gamma M \subset e\Gamma M\Gamma e$. Let $y \in e\Gamma M\Gamma e$. Then $y = e\gamma a\gamma_1 e = e\gamma a \in e\Gamma M$ and so $e\Gamma M\Gamma e \subset e\Gamma M$. Hence e is right semicentral.

PROPOSITION: III.8

Let $e \in C(M)$. *Then* $e \in S_l(M)$ *and* $e \in S_r(M)$.

Proof:

Let $e \in C(M)$ and $r \in M$. Then $e\gamma_1 r\gamma e = (e\gamma_1 r)\gamma e = (r\gamma_1 e)\gamma e = r\gamma_1(e\gamma e) = r\gamma_1 e$ and $e\gamma r\gamma_1 e = e\gamma(r\gamma_1 e) = e\gamma(e\gamma_1 r) = (e\gamma e)\gamma_1 r = e\gamma_1 r$. Then by proposition: III.7, $e \in S_l(M)$ and $e \in S_r(M)$.

Now, we state semicentral and central properties of zero-symmetric reduced gamma near ring.

PROPOSITION: III.9

Let M be a zero-symmetric reduced Γ-near ring with unity. Then

(i) *right semicentral Γ-near ring*

(ii) *left semicentral Γ-near ring*

(iii) *central*

Proof:

Let M be a zero-symmetric reduced gamma near ring with unity.

(i) Let $e \in M$ be an idempotent element and x be an unit element in M. Then $(e\gamma x - e\gamma x\gamma_1 e)\gamma_1 e = e\gamma x\gamma_1 e - e\gamma x\gamma_1 e\gamma_1 e = e\gamma x\gamma_1 e - e\gamma x\gamma_1 e = 0$, $(e\gamma x - e\gamma x\gamma_1 e)\gamma_1 e\gamma x = e\gamma x\gamma_1 e\gamma x - e\gamma x\gamma_1 e\gamma_1 e\gamma x = e\gamma x\gamma_1 e\gamma x - e\gamma x\gamma_1 e\gamma x = 0$ and $(e\gamma x - e\gamma x\gamma_1 e)\gamma_1 e\gamma x\gamma_1 e = e\gamma x\gamma_1 e\gamma x\gamma_1 e - e\gamma x\gamma_1 e\gamma_1 e\gamma x\gamma_1 e = e\gamma x\gamma_1 e\gamma x\gamma_1 e - e\gamma x\gamma_1 e\gamma x\gamma_1 e = 0$. Also, $e\gamma x\gamma_1(e\gamma x - e\gamma x\gamma_1 e) = e\gamma x\gamma_1 e\gamma x - e\gamma x\gamma_1 e\gamma x\gamma_1 e = e\gamma x\gamma_1 e - e\gamma x\gamma_1 e\gamma e = e\gamma x\gamma_1 e - e\gamma x\gamma_1 e = 0$ and $e\gamma x\gamma_1 e\gamma_1(e\gamma x - e\gamma x\gamma_1 e) = e\gamma x\gamma_1(e\gamma_1 e\gamma x - e\gamma_1 e\gamma x\gamma_1 e) = e\gamma x\gamma_1(e\gamma x - e\gamma x\gamma_1 e) = 0$. Hence, $(e\gamma x - e\gamma x\gamma_1 e)\gamma_1(e\gamma x - e\gamma x\gamma_1 e) = e\gamma x\gamma_1(e\gamma x - e\gamma x\gamma_1 e) - e\gamma x\gamma_1 e\gamma_1(e\gamma x - e\gamma x\gamma_1 e) = 0$. Since M is reduced, $e\gamma x - e\gamma x\gamma_1 e = 0$ (i.e.) $e\gamma x = e\gamma x\gamma_1 e$. Consequently, $e\Gamma M = e\Gamma M\Gamma e\ \forall\ e \in M$. Thus M is right semicentral Γ-near ring.

(ii) Let $e \in M$ be an idempotent element and $x \in M$ be an unit element in M. Then $e\gamma_1(x\gamma e - e\gamma_1 x\gamma e) = e\gamma_1 x\gamma e - e\gamma_1 e\gamma_1 x\gamma e = e\gamma_1 x\gamma e - e\gamma_1 x\gamma e = 0$, $x\gamma e\gamma_1(x\gamma e - e\gamma_1 x\gamma e) = x\gamma e\gamma_1 x\gamma e - x\gamma e\gamma_1 e\gamma_1 x\gamma e = x\gamma e\gamma_1 x\gamma e - x\gamma e\gamma_1 x\gamma e = 0$ and $e\gamma_1 x\gamma e\gamma_1(x\gamma e - e\gamma_1 x\gamma e) = e\gamma_1 x\gamma e\gamma_1 x\gamma e - e\gamma_1 x\gamma e\gamma_1 e\gamma_1 x\gamma e = e\gamma_1 x\gamma e\gamma_1 x\gamma e - e\gamma_1 x\gamma e\gamma_1 x\gamma e = 0$. Also, $(x\gamma e - e\gamma_1 x\gamma e)\gamma_1 x\gamma e = x\gamma e\gamma_1 x\gamma e - e\gamma_1 x\gamma e\gamma_1 x\gamma e = e\gamma_1 x\gamma e - e\gamma e\gamma_1 x\gamma e = e\gamma_1 x\gamma e - e\gamma_1 x\gamma e = 0$ and $(x\gamma e - e\gamma_1 x\gamma e)\gamma_1 e\gamma_1 x\gamma e = (x\gamma e\gamma_1 e - e\gamma_1 x\gamma e\gamma_1 e)\gamma_1 x\gamma e = (x\gamma e - e\gamma_1 x\gamma e)\gamma_1 x\gamma e = 0$. Hence $(x\gamma e - e\gamma_1 x\gamma e)\gamma_1(x\gamma e - e\gamma_1 x\gamma e) = x\gamma e\gamma_1(x\gamma e - e\gamma_1 x\gamma e) - e\gamma_1 x\gamma e\gamma_1(x\gamma e - e\gamma_1 x\gamma e) = 0$. Since M is reduced, $x\gamma e - e\gamma_1 x\gamma e = 0$ (i.e.) $x\gamma e = e\gamma_1 x\gamma e$. Consequently, $M\Gamma e = e\Gamma M\Gamma e\ \forall\ e \in M$. Therefore, M is left semicentral gamma near ring.

(iii) By (i) & (ii), $e\Gamma M = e\Gamma M\Gamma e$ & $M\Gamma e = e\Gamma M\Gamma e$ $\forall\, e \in M$. Thus $e\Gamma M = M\Gamma e = e\Gamma M\Gamma e$. For $m \in M$, there exists $x, y \in M$ such that $m\gamma_2 e = e\gamma_1 x\gamma_2 e$ and $e\gamma_1 m = e\gamma_1 y\gamma_2 e$. Now $e\gamma_1 m\gamma_2 e = e\gamma_1(m\gamma_2 e) = e\gamma_1(e\gamma_1 x\gamma_2 e) = e\gamma_1 x\gamma_2 e = m\gamma_2 e$ and $e\gamma_1 m\gamma_2 e = (e\gamma_1 m)\gamma_2 e = (e\gamma_1 y\gamma_2 e)\gamma_2 e = e\gamma_1 y\gamma_2 e = e\gamma_1 m$. Thus $e\gamma_1 m = e\gamma_1 m\gamma_2 e = m\gamma_2 e$ $\forall\, m \in M$ & $\gamma_1, \gamma_2 \in \Gamma$. Therefore, every idempotent in M is central and hence M is central.

PROPOSITION: III.10

Let M be a zero-symmetric left regular unital gamma near ring with $\alpha\gamma b = 0$ for any $a, b \in M$ and $\gamma \in \Gamma$. Then M is right semicentral Γ-near ring.

Proof:

Let M be a zero-symmetric left regular Γ-near ring with $a\gamma b = 0$ for any $a, b \in M$ & $\gamma \in \Gamma$. By lemma: II.19, M is reduced and by proposition: III.8, M is right semicentral gamma near ring.

COROLLARY: III.11

Let M be a zero-symmetric right regular unital Γ-near ring with $a\gamma b = 0$ for any $a, b \in M$ & $\gamma \in \Gamma$. Then M is left semicentral Γ-near ring.

Proof:

Let M be a zero-symmetric right regular Γ-near ring with $a\gamma b = 0$ for any $a, b \in M$ & $\gamma \in \Gamma$. By corollary: II.20, M is reduced and by proposition: III.8, M is left semicentral gamma near ring.

PROPOSITION: III.12

For an idempotent e of an Γ-near ring M the following conditions are equivalent:

(i) $e \in M$ is left semicentral

(ii) $a\gamma_2 e = e\gamma_1 a\gamma_2 e$ for all units $a \in M$, $\gamma_1, \gamma_2 \in \Gamma$

(iii) $a\gamma_2 e = e\gamma_1 a\gamma_2 e$ for all nilpotents $a \in M$, $\gamma_1, \gamma_2 \in \Gamma$

(iv) $f\gamma e$ is an idempotent for all idempotents $f \in M$ & γ in Γ

(v) $f\gamma_2 e = e\gamma_1 f\gamma_2 e$ for all idempotents $f \in M$ & $\gamma_1, \gamma_2 \in \Gamma$ which are isomorphic to e

(vi) $(f\gamma_2 e)^n = (e\gamma_1 f\gamma_2 e)^n$ for all idempotents $f \in M$ & $\gamma_1, \gamma_2 \in \Gamma$ which are isomorphic to e where n is some positive integer.

Proof:

(1)⟹(2): Suppose $e \in M$ is left semicentral, then $r\gamma_2 e = e\gamma_1 r\gamma_2 e$ $\forall\, r \in M$ & $\gamma_1, \gamma_2 \neq 0 \in \Gamma$ by proposition: III.7. Therefore, it is true for all units $r \in M$.

(1)⟹(3) is obvious by proposition: III.7

(2) ⟹(3): Suppose that the condition (2) holds. Let r be an nilpotent element of M. Then, $1 + r$ is an unit of M. By assumption (2), $(1 + r)\gamma_2 e = e\gamma_1(1 + r)\gamma_2 e$. This implies, $1\gamma_2 e + r\gamma_2 e = e\gamma_1 1\gamma_2 e + e\gamma_1 r\gamma_2 e = 1\gamma_2 e + e\gamma_1 r\gamma_2 e$. Thus, $r\gamma_2 e = e\gamma_1 r\gamma_2 e$. Hence (3) holds.

(3) ⟹ (1): Suppose that the condition (3) holds. Let $a \in M$ be arbitrary. Consider the element $r = (1 - e)\gamma a\gamma e \in M$. Since $r\gamma r = 0$ & $r\gamma_2 e = e\gamma_1 r\gamma_2 e$, $(1 - e)\gamma a\gamma e = 0$. Thus, $a\gamma e = e\gamma a\gamma e$ and so e is left semicentral.

(1)⟹(4): Suppose that $e \in M$ is left semicentral. Then $(f\gamma e)\gamma_1(f\gamma e) = f\gamma(e\gamma_1 f\gamma e) = f\gamma f\gamma e = f\gamma e$. Thus $f\gamma e$ is an idempotent for all idempotents $f \in M$

(4) ⟹ (5): Suppose that the condition (4) holds. Since $1 - f \in M$ are idempotents for all idempotents $f \in M$, $(1 - f)\gamma e = [(1 - f)\gamma e]^2$ by assumption. Thus, $e - f\gamma e = (1 - f)\gamma e = [(1 - f)\gamma e]^2 = (e - f\gamma e)\gamma(e - f\gamma e) = e\gamma e - e\gamma f\gamma e - f\gamma e\gamma e + f\gamma e\gamma f\gamma e = e - e\gamma f\gamma e - f\gamma e + (f\gamma e)^2 = e - e\gamma f\gamma e - f\gamma e + f\gamma e = e - e\gamma f\gamma e$, so $f\gamma e = e\gamma f\gamma e$ for all idempotents $f \in M$.

(5) ⟹ (6) & (6) ⟹ (7) is obvious.

(7) ⟹ (1): Since the condition (6) holds and assume that e is not left semicentral. Then there is $a \in M$ such that $a\gamma_2 e - e\gamma_1 a\gamma_2 e \neq 0$. Consider $f = e + a\gamma_2 e - e\gamma_1 a\gamma_2 e$. Then $f^2 = f\gamma f \neq e, f\gamma e = (e + a\gamma_2 e - e\gamma_1 a\gamma_2 e)\gamma e = e\gamma e + a\gamma_2 e\gamma e - e\gamma_1 a\gamma_2 e\gamma e = e + a\gamma_2 e - e\gamma_1 a\gamma_2 e = f$ & $e\gamma f = e\gamma[e + a\gamma_2 e - e\gamma_1 a\gamma_2 e] = e\gamma e + e\gamma a\gamma_2 e - e\gamma e\gamma_1 a\gamma_2 e = e + e\gamma a\gamma_2 e - e\gamma_1 a\gamma_2 e = e$, so e & f are isomorphic idempotents. Therefore, $e = e\gamma f = e\gamma f\gamma_1 e = (e\gamma f\gamma_1 e)^n = (f\gamma_1 e)^n = f v e = f$ for any positive integer n and for all $\gamma_1, \gamma_2, \gamma$ in Γ which contradicts the assumption (4). Hence e is left semicentral.

COROLLARY: III.13

For an idempotent e of an Γ-near ring M the following conditions are equivalent:

(i) $e \in M$ is right semicentral

(ii) $e\gamma_1 a = e\gamma_1 a\gamma_2 e$ for all units $a \in M$, $\gamma_1, \gamma_2 \in \Gamma$

(iii) $e\gamma_1 a = e\gamma_1 a\gamma_2 e$ for all nilpotents $a \in M$, $\gamma_1, \gamma_2 \in \Gamma$

(iv) $e\gamma f$ is an idempotent for all idempotents $f \in M$ & γ in Γ

(v) $e\gamma_1 f = e\gamma_1 f\gamma_2 e$ for all idempotents $f \in M$ & $\gamma_1, \gamma_2 \in \Gamma$ which are isomorphic to e

(vi) $(e\gamma_1 f)^n = (e\gamma_1 f\gamma_2 e)^n$ for all idempotents $f \in M$ & $\gamma_1, \gamma_2 \in \Gamma$ which are isomorphic to e where n is some positive integer.

Proof:

It follows as similar to proposition: III.12 and by proposition: III.7

COROLLARY: III.14

For an idempotent e of an Γ-near ring M the following conditions are equivalent:

(i) $e \in M$ is central

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(ii) $a\gamma_2 e = e\gamma_1 a\gamma_2 e$ *for all units* $a \in M$, $\gamma_1, \gamma_2 \in \Gamma$

(iii) $a\gamma_2 e = e\gamma_1 a\gamma_2 e$ *for all nilpotents* $a \in M$, $\gamma_1, \gamma_2 \in \Gamma$

(iv) $f\gamma e$ *is an idempotent for all idempotents* $f \in M$ *&* γ *in* Γ

(v) $f\gamma_2 e = e\gamma_1 f\gamma_2 e$ *for all idempotents* $f \in M$ *&* $\gamma_1, \gamma_2 \in \Gamma$ *which are isomorphic to e*

(vi) $(f\gamma_2 e)^n = (e\gamma_1 f\gamma_2 e)^n$ *for all idempotents* $f \in M$ *&* $\gamma_1, \gamma_2 \in \Gamma$ *which are isomorphic to e where n is some positive integer.*

Proof:

It follows form proposition: III.12 and corollary: III.13

COROLLARY: III.15

For a gamma near ring M an idempotent $e \in$ M is left semicentral iff $1 - e$ is right semicentral.

Proof:

Let e be an left semicentral idempotent of M. Then $f\gamma_1 e = e\gamma f\gamma_1 e$ for all idempotents $f \in M$ & $\gamma_1, \gamma \in \Gamma$ by proposition: III.12. Therefore, $(1-e)\gamma f\gamma_1(1-e) = (f - e\gamma f)\gamma_1(1-e) = 1 - f\gamma_1 e - e\gamma f + e\gamma f\gamma_1 e = f - e\gamma f = f(1-e)$. This implies that $1-e$ is a right semicentral idempotent of M by corollary: III.13. By retracing the steps the converse follows.

CONCLUSION:

In this paper, it is proved that a zero-symmetric reduced gamma near ring with unity is left semicentral, right semicentral and central gamma near rings is proved. It is proved that zero-symmetric left (right) regular unital Γ-near ring with zero divisor is right (left) semicentral Γ-near ring. Also, some characterizations of left (right) semicentral and central gamma near ring is proved. And it is proved that an idempotent e is left semicentral iff $1-e$ is right semicentral.

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