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Fuzzy Fourier series using Hexagonal, Reverse order Pentagonal Fuzzy Number

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Abstract: **— In this paper we have used Hexagonal fuzzy number in Fuzzy Fourier series for the period of $[-\pi,\pi]$ .We have observed that Reverse order pentagonal fuzzy number also satisfies the periodic function for the fuzzy Fourier series.**

Keywords:- **Fuzzy Numbers, Pentagonal Fuzzy number, Hexagonal Fuzzy number, Fourier series, Fourier Coefficients.**

I. INTRODUCTION

Lotfi A. Zadeh was introduced the fuzzy set theory in 1965. Fuzzy mathematics is used in many applications such as decision making, control system, signal processing, etc. [3]. Fuzzy numbers are used as an one of the significant tools in fuzzy application found by Dubois, Prade and Yager (1978) [1]. Triangular fuzzy number, Trapezoidal fuzzy number and Pentagonal fuzzy numbers are most common forms of fuzzy numbers used in various applications [5]. The fundamental idea of L-R representation of a fuzzy number is to split the membership function of a fuzzy number into two curves left (L) and right(R) fuzzy number was proposed by Michael Hans in 2004 [7]. The Pentagonal fuzzy number and the reverse order pentagonal fuzzy number were introduced by T. Pathinathan and Ponnivalavan [6]. Jean Baptiste Joseph Fourier introduced the idea that any periodic function can be represented by a series of sines and cosines which are harmonically related. An infinite series of trigonometric functions which represents an expansion or approximation of a periodic function is used in Fourier analysis [3]. In this paper we have verified that hexagonal fuzzy number satisfies the fuzzy Fourier coefficients in the real line. Fourier coefficients are non-linear coefficients of a special family of crisp subsets. We also verified that reverse order pentagonal fuzzy number satisfies the periodic fuzzy Fourier series in the interval.

II. PRELIMINARY NOTIONS

A *fuzzy number* is a fuzzy set $\underset{\sim}{A}$ on the real axis, whose membership function $\mu_{\underset{\sim}{A}}$ satisfies the following conditions.

(i) $\mu_{\underset{\sim}{A}}$ is normal; there exists an $x_0 \in \mathbb{R}$ such that $\mu_{\underset{\sim}{A}}(x_0)=1$.

(ii) $\mu_{\underset{\sim}{A}}$ is = convex;

$$\mu_{\underset{\sim}{A}}\left[\lambda x+(1-\lambda)y\right]\geq \mu_{\underset{\sim}{A}}(x)\wedge\mu_{\underset{\sim}{A}}(y)\ \forall x,y\in\mathbb{R} \text{ and } \forall\lambda\in[0,1].$$

(iii) $\mu_{\underset{\sim}{A}}$ is piece-wise continuous,

$$\forall \in>0, \textit{there exists } \delta>0 \textit{ such that } \mu_{\underset{\sim}{A}}(x)-\mu_{\underset{\sim}{A}}(y)<\in$$

whenever $|x-x_0|<\delta$.

III. FOURIER SERIES FOR FUZZY VALUED FUNCTIONS OF PERIOD 2π

A. Definition: Let f^{t} be a 2π -periodic fuzzy valued function on a set A. The Fourier series of fuzzy valued function f^{t} of period 2π is defined as follows:

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$$f^{t}(x)=\frac{a_0}{2}\oplus\sum_{n=1}^{\infty}\left(a_n\cos nx\oplus b_n\sin nx\right)$$

with respect to the fuzzy coefficients a_0, a_n and b_n which converges uniformly in $\lambda\in[0,1]$ for all $n\in N$

B. Definition: Pentagonal fuzzy number

A pentagonal Fuzzy number (PFN), $\mu_{\tilde{A}}(x)$ has a piecewise continuous function consisting of five points in its domain, forming a pentagonal shape. As chosen, the points in the domain have the ordering $a\le b\le c\le d\le e;a,b,c,d,e\in\square$ of a fuzzy set and its membership function is given by,

$$\mu_{\tilde{u}}(x)=\begin{cases}0 & for\quad x<a\\ \frac{(x-a)}{(b-a)} & for\quad a\le x\le b\\ \frac{(x-b)}{(c-b)} & for\quad b\le x\le c\\ 1 & x=c\\ \frac{(d-x)}{(d-c)} & for\quad c\le x\le d\\ \frac{(e-x)}{(e-d)} & for\quad d\le x\le e\\ 0 & for\ x>e\end{cases}\qquad(1)$$

Then, $\left[\mu_{\tilde{A}}\right]_{\lambda}=\left[\mu_{\tilde{A}}^{-}(\lambda),\mu_{\tilde{A}}^{+}(\lambda)\right]_{=}\left[(c-a)\lambda+a+b,(c-e)\lambda+d+e\right]$

C. Definition: Hexagonal Fuzzy Number

A hexagonal fuzzy number whose membership function is characterized by six parameters $\{a,b,c,d,e,f\}$ where $a\le b\le c\le d\le e\le f;a,b,c,d,e,f\in\square$.

$$\mu_{\tilde{A}}(x)=\begin{cases}0 & for\quad x<a\\ \frac{1}{2}\left(\frac{x-a}{b-a}\right) & for\quad a\le x\le b\\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a}{c-b}\right) & for\quad b\le x\le c\\ 1 & for\quad c\le x\le d\\ 1-\frac{1}{2}\left(\frac{x-d}{e-d}\right) & for\quad d\le x\le e\\ \frac{1}{2}\left(\frac{f-x}{f-e}\right) & for\quad e\le x\le f\\ 0 & for\ x>f\end{cases}\qquad(2)$$

$$\left[\mu_{\tilde{A}}\right]_{\lambda}=\left[\mu_{\tilde{A}}^{-}(\lambda),\mu_{\tilde{A}}^{+}(\lambda)\right]_{=}$$
$$\left[a(1-2\lambda)+c(2\lambda-1)+2b,d(2\lambda-1)+f(1-2\lambda)+2e\right]$$

IV. CASE: 1

Consider the periodic fuzzy hexagonal function shown below,

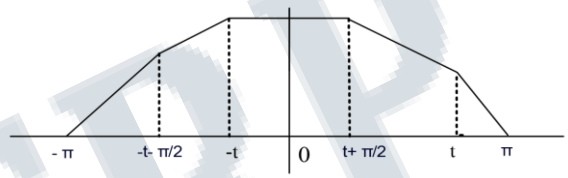

Figure 1

Let f^{t} be 2π -periodic fuzzy valued function on the Fuzzy integral on $[-\pi,\pi]$ with hexagonal form defined by

$$f^{t}(x)=\begin{cases}0 & for\quad x<-\pi\\ \frac{1}{2}\left(\frac{x+\pi}{-t+\pi/2}\right) & for\quad -\pi\le x\le-\frac{\pi}{2}-t\\ \frac{1}{2}+\frac{1}{2}\left(\frac{x+t+\pi/2}{\pi/2}\right) & for\quad -\frac{\pi}{2}-t\le x\le -t\\ 1 & for\quad -t\le x\le t\\ 1-\frac{1}{2}\left(\frac{x-t}{\pi/2}\right) & for\quad t\le x\le t+\frac{\pi}{2}\\ \frac{1}{2}\left(\frac{\pi-x}{\pi/2-t}\right) & for\quad t+\frac{\pi}{2}\le x\le\pi\\ 0 & for\ x>\pi\end{cases}\qquad(3)$$

which is fuzzy integral on $[-\pi,\pi]$ for each $x,t\in[a,f]$ and $\lambda\in[0,1]$. By using Definition. III.C, the level set $\left[f^{t}\right]_{\lambda}$ of the membership function of f^{t} can be defined as,

International Journal of Science, Engineering and Management (IJSEM)
Vol 3, Issue 5, May 2018

$$\left[f^{t}\right]_{\lambda}=\left[f_{\lambda}^{-}(t), f_{\lambda}^{+}(t)\right]=\left[2\lambda(\pi-t)-2\pi-t, 2\lambda(t-\pi)+2\pi+t\right] \quad (4)$$

We could find the fuzzy Fourier coefficients $a_0, a_n, and\ b_n$.

$$a_0=\frac{1}{2\pi}\left[\int_{-\pi}^{\pi}\left[2\lambda(\pi-t)-2\pi-t\right]dt, \int_{-\pi}^{\pi}\left[2\lambda(t-\pi)+2\pi+t\right]dt\right]$$
$$=\frac{2}{\pi}\left[-6\pi^2, 6\pi^2\right]=-12\pi[-1,1]$$

$$a_n=\frac{1}{2\pi}\left[\int_{-\pi}^{\pi}\left[2\lambda(\pi-t)-2\pi-t\right]\cos nt\,dt, \int_{-\pi}^{\pi}\left[2\lambda(t-\pi)+2\pi+t\right]\cos nt\,dt\right]$$
$$=\frac{2}{\pi}[0,0]=0$$

$$b_n=\frac{1}{2\pi}\left[\int_{-\pi}^{\pi}\left[2\lambda(\pi-t)-2\pi-t\right]\sin nt\,dt, \int_{-\pi}^{\pi}\left[2\lambda(t-\pi)+2\pi+t\right]\sin nt\,dt\right]$$
$$=\frac{2}{\pi}\left[\frac{2\pi}{n}(2\lambda+1), -\left(\frac{2\pi}{n}(2\lambda+1)\right)\right]=\frac{4}{n}\left[(2\lambda+1), -(2\lambda+1)\right]$$

By using the above coefficients $a_0, a_n, and\ b_n$ in equation (2), we have

$$f^{t}(x)=-12\pi(-1,1)\oplus\sum_{n=1}^{\infty}\left(\left[\frac{4(2\lambda+1)}{n}, \frac{-4(2\lambda+1)}{n}\right]\sin nt\right)$$

$$=\left[\pi(\lambda-2), \pi(2-\lambda)\right]\oplus\sum_{n=1}^{\infty}\left(\left[\frac{2}{n}, \frac{-2}{n}\right]\sin nt\right)$$

$$f^{t}(x)=12\pi+(2\lambda+1)\left[4\sin t+2\sin 2t+\frac{4}{3}\sin 3t+......\right], -12\pi-(2\lambda+1)\left[4\sin t+2\sin 2t+\frac{4}{3}\sin 3t......\right] \quad (5)$$

Example: 1

Let the membership value λ be 0.6 and $t=30°$ in equation (5). For the hexagonal fuzzy number of six parameters we can obtain the Fourier fuzzy valued function is a symmetric periodic function.

$$\left[f^{30°}\right]_{0.6}=[51.61, -51.61]$$

V. DEFINITION: REVERSE ORDER PENTAGONAL FUZZY NUMBER

A pentagonal Fuzzy number (PFN), $\mu_{\tilde{A}}(x)$ has a piecewise continuous function consisting of five points in its domain, forming a pentagonal shape. As chosen, the points in the domain have the ordering $a \le b \le c \le d \le e; a,b,c,d,e \in \mathbb{R}$. of a fuzzy set and its membership function is given by,

$$\mu_{\tilde{A}}(x)=\begin{cases}1 & for \quad x<a \\ -\frac{1}{a}x & for \quad -a \le x \le -b \\ -\frac{1}{b}x & for \quad -b \le x \le c \\ 0 & x=c \\ \frac{1}{d}x & for \quad c \le x \le d \\ \frac{1}{e}x & for \quad d \le x \le e \\ 1 & for\ x>e\end{cases} \quad (6)$$

Then, we have $\left[\mu_{\tilde{A}}(x)\right]_{\lambda}=\left[\mu_{\tilde{A}}^{-}(\lambda), \mu_{\tilde{A}}^{+}(\lambda)\right]=$ $\left[-(a+b)\lambda, \lambda(d+e)\right]$

VI. Case: 2

Consider the periodic reverse order pentagonal fuzzy Fourier function shown below,

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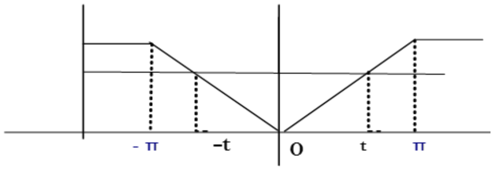

Figure 2

Let f^t be 2π -periodic fuzzy valued function on the Fuzzy integral on $[-\pi,\pi]$ with reverse order pentagonal form defined by

$$\mu_{\tilde{A}}(x)=\begin{cases}1 & for \quad x<-\pi \\ -\frac{1}{\pi}x & for \quad -\pi\le x\le -t \\ -\frac{1}{t}x & for \quad -t\le x\le 0 \\ 0 & x=0 \\ \frac{1}{t}x & for \quad 0\le x\le t \\ \frac{1}{\pi}x & for \quad t\le x\le \pi \\ 1 & for\ x>\pi \end{cases} \qquad (7)$$

which is fuzzy integral on $[-\pi,\pi]$ for each $x,t\in[-\pi,\pi]$ and $\lambda\in[0,1]$. By using Definition V, the level set $\left[f^t\right]_\lambda$ of the membership function of f^t can be defined as,

$$\left[f^t\right]_\lambda=\left[f^-_\lambda(t), f^+_\lambda(t)\right]=[-\pi\lambda-\lambda t, \lambda\pi+\lambda t]$$

.

We could find the fuzzy Fourier coefficients $a_0, a_n, and\ b_n$.

$$a_0=\frac{1}{2\pi}\left[\int_{-\pi}^{\pi}(-\pi\lambda-\lambda t)\,dt, \int_{-\pi}^{\pi}(\lambda\pi+\lambda t)\,dt\right]$$

$$=\frac{2}{\pi}\left[-4\pi^2\lambda, 4\pi^2\lambda\right]=8\pi\lambda[-1,1]$$

$$a_n=\frac{1}{2\pi}\left[\left[\int_{-\pi}^{\pi}(-\pi\lambda-\lambda t)\cos nt\,dt, \int_{-\pi}^{\pi}(\lambda\pi+\lambda t)\cos nt\,dt\right]\right]$$

$$=\frac{2}{\pi}[0,0]=0$$

$$b_n=\frac{1}{2\pi}\left[\int_{-\pi}^{\pi}\left[-(\pi\lambda+\lambda t)\right]\sin nt\,dt, \int_{-\pi}^{\pi}(\pi\lambda+\lambda t)\sin nt\,dt\right]$$

$$=\frac{2}{\pi}\left[\frac{2\pi\lambda}{n}, -\left(\frac{2\pi\lambda}{n}\right)\right]=\frac{4\lambda}{n}[1,-1]$$

By using the above coefficients $a_0, a_n, and\ b_n$ in equation (2), we have

$$f^t(x)=8\pi\lambda(-1,1)\oplus\sum_{n=1}^{\infty}\left(\left[\frac{4\lambda}{n}, \frac{-4\lambda}{n}\right]\sin nt\right)$$

$$f^t(x)=-8\pi\lambda+\lambda\left[4\sin t+2\sin 2t+\frac{4}{3}\sin 3t+........\right], 8\pi\lambda-\lambda\left[4\sin t+2\sin 2t+\frac{4}{3}\sin 3t......\right] \qquad (8)$$

Example: 2

Let the membership value λ be 0.6 and $t=30^\circ$ in equation (8). For the reverse order pentagonal fuzzy number of five parameters we can obtain the Fourier fuzzy valued function is a symmetric periodic function.

$$\left[f^{30^\circ}\right]_{0.6}=[-11.28, 11.28]$$

Conclusion:

In this paper we conclude that, Fuzzy Fourier series can be verified for the Hexagonal fuzzy Fourier series which has six even parameter domain. The Reverse order pentagonal fuzzy number has five odd parameter domains. Both have a value in fuzzy Fourier coefficients for a_0 and b_n in terms of sine series. The cosine terms become zero. The pentagonal fuzzy number and the reverse order pentagonal fuzzy number have different value in the fuzzy Fourier coefficients a_0 and b_n but they have similar sine series.

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