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Roman Domination number of some Special graphs

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Abstract:- **Roman domination number was introduced in an article in Scientific American by Ian Stewart. In this paper, we investigate the Roman domination number of some special graphs such as P_n+2K_1, J(m,n),$<K_{1,n}:n>$, AC_n, H_n, $C_n\Theta K_1$, $[P_n:S_m]$ and Fl_n. Also, arrived at an algorithm to find a Roman dominating function of any graph and hence the Roman domination number.**

1. INTRODUCTION

In this paper, we find a variant of the domination number called Roman domination which is suggested by the article in Scientific American by Ian Stewart, entitled "Defend the Roman Empire!" [10] for some special graphs. A Roman dominating function of a graph G = (V,E) is a functionf:V→{0,1, 2}satisfying the condition that every vertex u for which f(u) = 0 isadjacent to at least one vertex v for which f(v) = 2.Stated in other words, a Roman dominating function is a colouring of the vertices of a graph with the colours{0; 1; 2}such that every vertex coloured 0 is adjacent to atleast one vertex coloured 2. The definition of a Roman dominating function is given implicitly in [4] and [5]. The idea is that colours 1 and 2 represent either one or two Roman legions stationed at a given location (vertex v). A nearby location (an adjacentvertex u) is considered to be unsecured if no legions are stationed there (i.e. f(u)=0). An unsecured location (u) can be masde secured by sending a legion to u from an adjacent location (v). But, Emperor Constantine the Great, in the fourth century A.D., disagreed that a legion cannot be sent from a location v if doing so leaves that location unsecured(i.e. if f(v) = 1). Thus, two legions must be stationed at a location (f(v) = 2) before one of the legions can be sent to an adjacent location.

For a graph G = (V; E), let f:V → {0; 1; 2}, and let (V_0, V_1, V_2) be the orderedpartition of V induced by f, where V_i = {v ∈V|f(v) = i}and |Vi| = n_i, for i =0; 1; 2. There exists a 1–1 correspondence between the functions f:V→{0; 1; 2}and the ordered partitions (V_0, V_1, V_2) of V. Thus, we will writef = (V_0, V_1, V_2). A function f =(V_0; V_1; V_2) is a Roman dominating function (RDF) if V_2<V_0,where<means that the set V_2 dominates the set V_0, i.e. V_0⊆N[V_2]. The weight of f is sum of f(v) where v∈V. That is f(v) = $2n_2 + n_1$.The Roman domination number, denoted $\gamma_R(G)$, equals the minimum weight of aRDF of G and we say that a function f=(V_0; V_1; V_2) is a γ_R-function if it is an RDF and f(V) =$\gamma_R(G)$. The following definitions of Special graphs are from [3]. The **join of G_1 and G_2** is the graph G=G_1+G_2 with vertex set V=V_1UV_2 and edge set E=E_1UE_2U{uv;uƐV_1,vƐV_2} The graph P_n+K_1 is called a **fan** and P_n+2K_1 is called the **Double fan**. The **jelly fish** graph J(m,n) is obtained from a 4-cycle, v_1,v_2,v_3,v_4 by joining $v_1$and v_3 with an edge and appending m pendent edges to v_2 and n pendent edges to v_4. The graph which is obtained from $K_{1,n}$ by subdividing all the edges is called the **subdivided star** graph and it is denoted by $<K_{1,n}:n>$. The graph which is obtained by joining a pendant edge to each vertex of C_n by an edge is called the **Armed Crown** graph and it is denoted by AC_n. The graph which is obtained from a wheel W_n by attaching a pendant edge to each rim vertex is called the **Helm** graph.It is denoted by H_n. The graph which is obtained by joining a pendant edge to each vertex of C_n is called the **crowngraph** and it is denoted by $C_n\Theta K_1$. $[P_n:S_m]$ is the graph obtained from a path P_n by joining every vertex of a path to a root of a star S_m by an edge. The graph which is obtained from a Helm graph H_n by joining each pendant vertex to the apex of the helm is called the **flower graph** and it is denoted by Fl_n In this paper, we investigate the Roman domination number of some special graphs such as P_n+2K_1, J(m,n),$<K_{1,n}:n>$, AC_n, H_n, $C_n\Theta K_1$, $[P_n:S_m]$ and Fl_n. Also, arrived at an algorithm to find a Roman dominating function of any graph and hence the Roman domination number. For Graph Theoretic terminology refer Harary[1].

1.1 Theorem:[2] If a graph G is of order n which contains a vertex of degree n-1,then $\gamma_R(G)$=2.

II. ROMAN DOMINATION NUMBER OF SOME SPECIAL GRAPHS

2.1 Theorem: $\gamma_R(G)$= 2 if and only if G has a full degree vertex (or) G is K_2(or) G is $\overline{K}_2$

$\gamma_R(G)=2$ => f (v)=2

=> $2n_2+n_1 = 2$

=>$n_2=1$ & $n_1=0$ (or) $n_2=0$ & $n_1=2$.

Now, $n_2=1$ &$n_1=0$ implies G has a full degree vertex (or) G is K_2 and $n_2=0$&$n_1=2$ implies G is$\overline{K}_2$

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Conversely,Suppose G = K_2 (or) $\overline{K}_2$

Then assigning 1 to both the vertices of G

$n_2=0=n_0$ & $n_1=2$

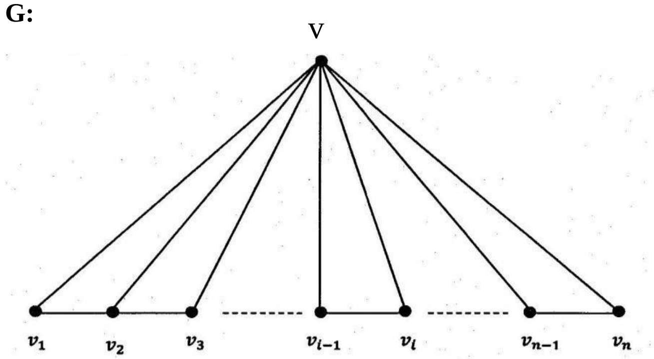
f(v)=2*0+2 = 2

$\gamma_R(G)=2$

The case G has a full degree vertex is proved in Theorem 1.1.

2.2 Remark:

G:



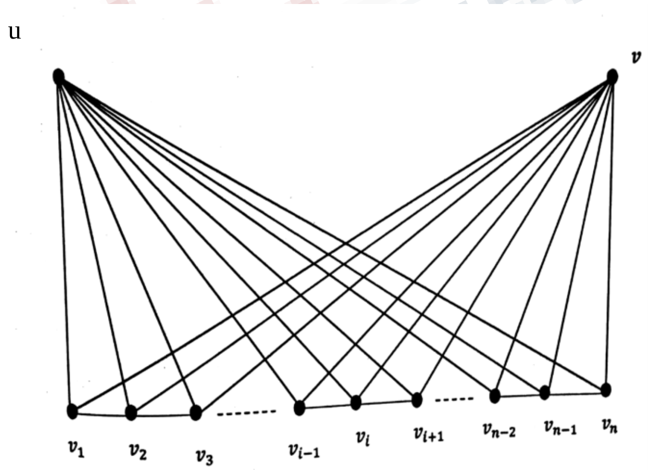
Here,G has a full degree vertex.

By theorem 1.1, $\gamma_R(G)=2$.

2.3Theorem: $\gamma_R(P_n+2K_1)=3$

Let G $=P_n+2K_1$

G:



Here ,V(G)={u,v,v_i; $1 \leq i< n$} and E(G)={uv_i,vv_i; $1\leq i \leq n$}U{v_iv_{i+1}; $1\leq i \leq n-1$}

Let V_0 = {v_i\ $1\leq i \leq n$}; V_1={u} ;V_2 ={v}

Clearly, The set V_2 dominates the set V_0. (i.e)$V_0 \subseteq N(V_2)$

f = (V_0,V_1,V_2) is a Roman domination function.

Now, f (v)=$2n_2+n_1$= 2(1)+1=3

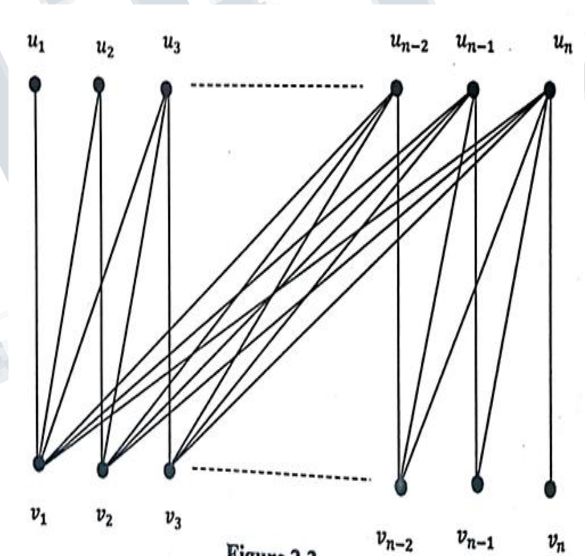
Obviously, there is no function f such that f(v) <3.

Hence $\gamma_R(P_n+2K_1)=3$

2.4 Theorem: $\gamma_R(H_{n,n})=4$.

Let G= ($H_{n,n}$)

G:



Let V_0 ={v_i,u_j/ i=1 to n-1 and j=2 to n};V_1= ϕ ; V_2={v_1,u_n}

Clearly, The set V_2 dominates the set V_0. (ie) $V_0 \subseteq N(V_2)$

f = (V_0,V_1,V_2) is a Roman domination function.

Now,f(v)=$2n_2+n_1$=2(2)+0=4

Obviously,there is no function f such that f(v)<4.

Hence $\gamma_R(H_{n,n})=4$

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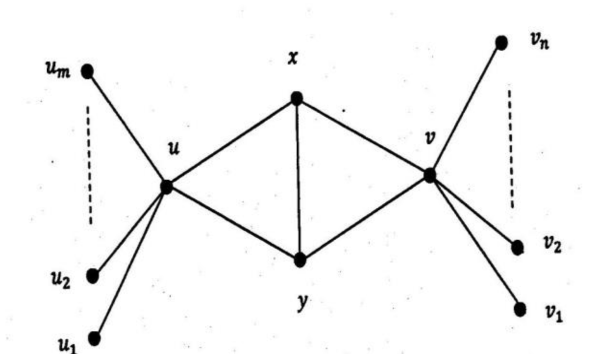
2.5 Theorem: $\Upsilon_R(J_{(m,n)})=4$.

Let $G = J_{(m,n)}$

G:



Here $V(G)= \{u,v,x,y,u_i,v_j;1\leq i\leq m\ ;\ 1\leq j\leq n\}$ and $E(G)= \{uu_i,u_y,u_x,xy,xv,yv\ ,vv_j;1\leq i\leq n\}$

Let $V_0=\{u_i,v_i,x,y/1\leq i\leq n\};V_1= \phi\ ;V_2=\{u,v\}$

Clearly, The set V_2 dominates the set V_0. (ie) $V_0\subseteq N\ (V_2)$

$f = (V_0,V_1,V_2)$ is a Roman domination function.

Now, $f(v)=2n_2+n_1=2(2)+0=4$

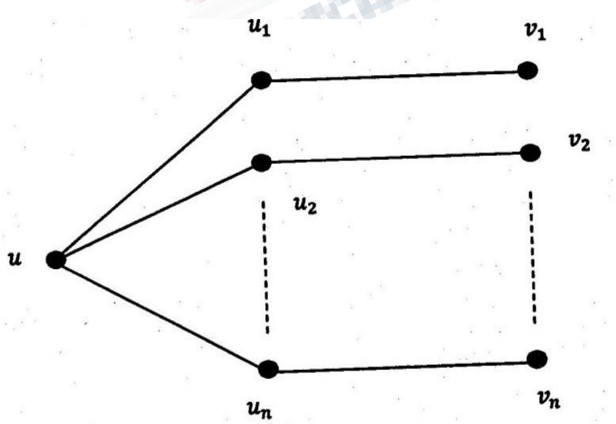
Obviously,there is no function f such that $f(v)<4$

Hence $\Upsilon_R(J_{(m.n)})=4$

***2.6 Theorem*:**$\Upsilon_R(<K_{1,n};n>)=n+2$

Let G $<K_{1,n};n>$

G:



Here$V(G)=\{u,u_i,v_i;1\leq i\leq n\}$

and $E(G)=\{uu_i,u_iv_i;1\leq i\leq n\}$

Let $V_0=\{u_i/1\leq i\leq n\};V_1=\{v_i/1\leq i\leq n\};V_2=\{u\}$

Clearly, The set V_2 dominates the set V_0. (ie) $V_0\subseteq N\ (V_2)$

$f = (V_0,V_1,V_2)$ is a Roman domination function.

Now $f(v)=2n_2+n=2(1)+n=n+2$

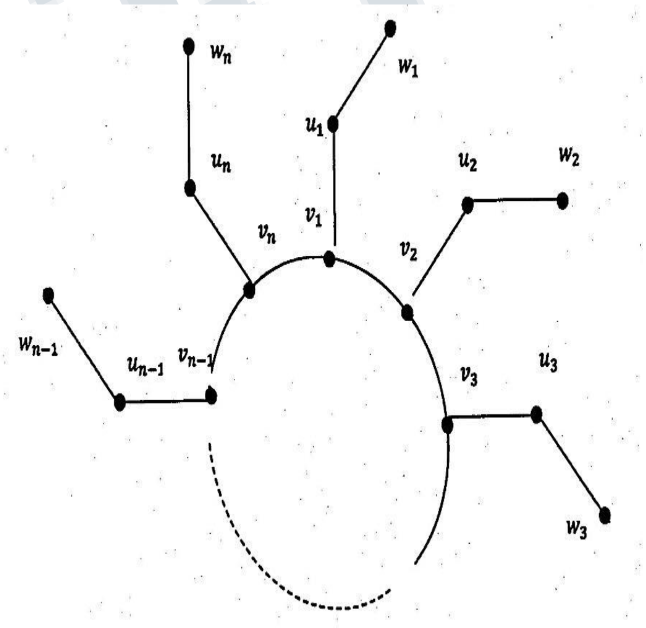
Obviously there is no f function such that $f(v)<n+2$

Hence $\Upsilon_R(<K_1,n;>)=n+2$

2.7 Theorem: $\Upsilon_R(AC_n) = 2n$

Let $G = AC_n$

G:



Here $V(G)=\{u_i,v_i,w_i\ :1\ \leq I\ \leq n\}$and $E(G)=\{v_i\ v_i+1;\ 1\leq i\leq n-1\}U\{u_iv_i,u_iw_i,1\leq i<n\}u\{v_1v_n\}$

Let $V_0 = \{w_i,v_i/1\leq i\leq n\};\ V_1= \phi\ ;V_2=\{u_i\ /1\leq i\ \leq n\}$

Clearly, The set V_2 dominates the set V_0.

(ie) $V_0\subseteq N\ (V_2)$

$f = (V_0,V_1,V_2)$ is a Roman domination function.

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Now, $f(v)=2n_2+n=2(n)+0=2n$

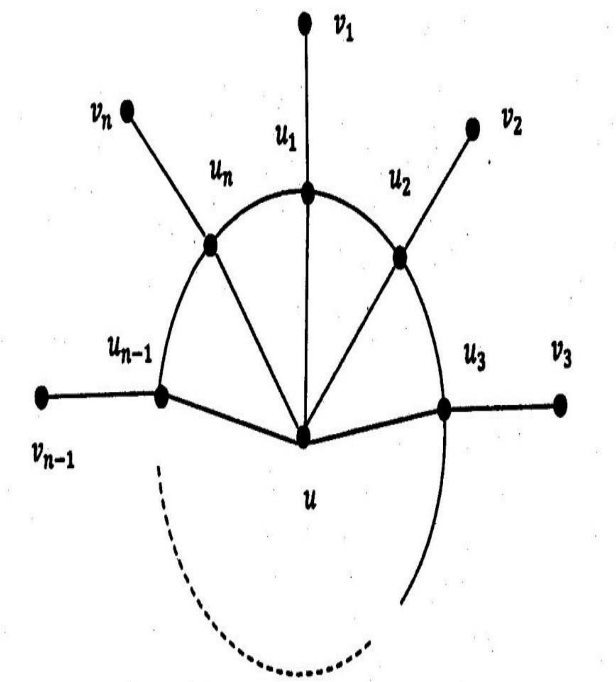
Obviously, there is no f function such that $f(v) <2n$

Hence, $\Upsilon_R(AC_n)=2n$

2.8Theorem: $\Upsilon_R(H_n)=n+2$

Let $G = H_n$

G:



Here $V(G) = \{u,u_iv_i;1\leq i\leq n\}$and $E(G) = \{uu_i,u_iv_i:1\leq i \leq n\}U\{u_iu_{i+1};1\leq i \leq n-1\}U\{u_1u_n\}$

Let $V_0 = \{u_i \backslash 1\leq i\leq n\};V_1= \{v_i\backslash 1\leq i \leq n\}; V_2 = \{u\}$

Clearly, The set V_2 dominates the set V_0. (ie) $V_0\subseteq N (V_2)$

$f = (V_0,V_1,V_2)$ is a Roman domination function.

Now, $f (v)= 2n_2+n= 2(1)+n=n+2$

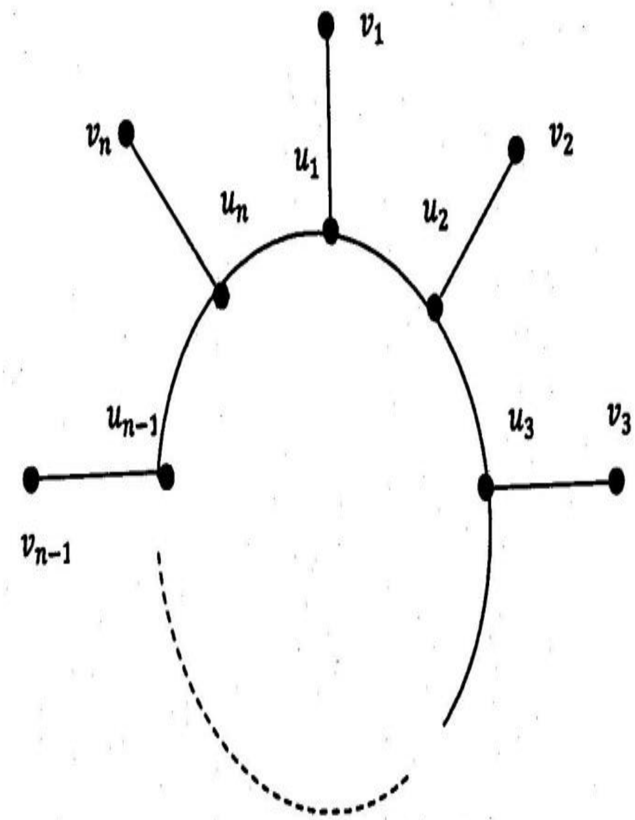
Obviously, there is no function f such that $f (v)<n+2$

Hence,$\Upsilon_R (H_n)=n+2$

2.9 Theorem: $\Upsilon_R (C_n\Theta K_1)= 2n$

Let $G =(C_n\Theta K_1)$

G:



Here $V(G) = \{u_i,v_i;1\leq i\leq n\}$

and$E(G)= \{u_iui+1;1\leq 1\leq n-1\}U\{u_1u_n\}U\{u_iv_i;1\leq i\leq n\}$

Let $V_0=\{v_i/1\leq i\leq n\}$; $V_1= \phi$;$V_2=\{u_i/1\leq i\leq n\}$

Clearly, The set V_2 dominates the set V_0. (ie) $V_0\subseteq N (V_2)$

$f = (V_0,V_1,V_2)$ is a Roman domination function.

Now, $f (v) = 2n_2+n_1= 2(n) + 0=2n$

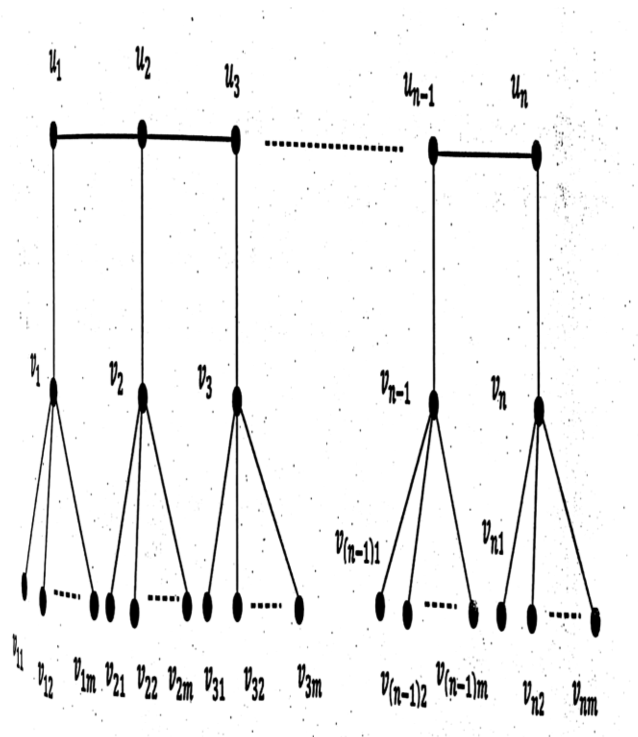
Obviously, there is no function f such that $f (v) < 2n$

Hence,$\Upsilon_R (c_n\Theta k_1) = 2n$

2.10 Theorem: γ_R ([P_n: S_m]) = 2n

Let G = [P_n:S_m]

G:



Here,

V(G)={u_iv_i;1≤i≤n}U{v_{ij};1≤ i ≤ n;1≤j≤m}

and

E(G)={u_iv_i;1≤i≤n}U{u_iu_i+1;1≤ n-1}U{v_iv_{ij};1 ≤ i ≤n;1≤ j≤ m}

Let V_0 = {u_i,v_{im}| 1 ≤i≤ n} ;V_1 = ϕ ; V_2 = {v_i|1≤i≤n}

Clearly, The set V_2 dominates the set V_0. (ie) V_0⊆N (V_2)

f = (V_0,V_1,V_2) is a Roman domination function.

Now, f (v) =2n_2+n_1= 2(n) + 0= 2n.

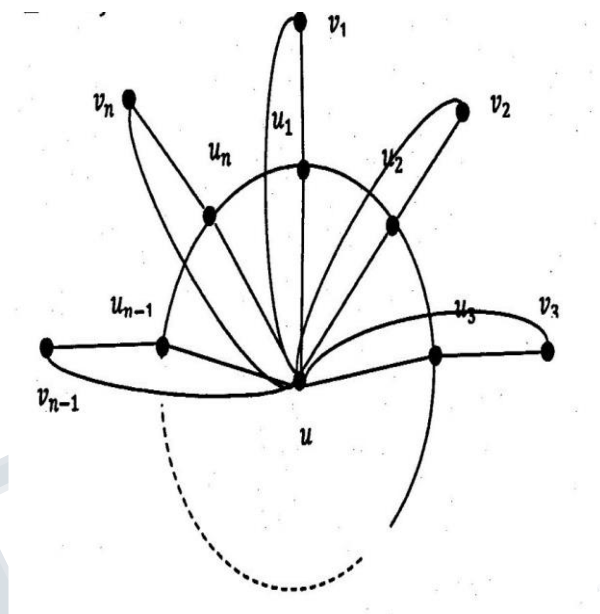
Obviously, there is no function f such that f (v)<2n.

Hence,γ_R ([P_n:S_m]) = 2n

2.11 Theorem*:* γ_R (Fl_n)=2

Let G = (Fl_n)

G:



Here V(G) ={u,u_i,v_i: 1≤ i≤n}and E(G) ={U_i,U_i=1 ; 1≤ i ≤ n-1} U {u,u_n} U{uu_i,uv_i,u_iv_i;1≤i≤n}

Let V_0={u_i, vi \ 1≤i≤n};V_1=ϕ;V_2={u}

Clearly, the set V_2 dominates the set V_0. (ie) V_0⊆N (V_2)

f = (V_0,V_1,V_2) is a Roman domination function.

Now, f (v) =2n_2+n_1= 2(1) + 0= 2.

Obviously, there is no function f such that f (v) < 2.

Hence,γ_R (Fl_n) =2

2.12 Remark:

For the above graph assigning 1 to all the vertices also gives a Roman domination function.

III. ALGORITHMTO GENERATE A ROMAN DOMINATION FUNCTION

Let V ={v_1 ,v_2,…v_n} such that d_i = d(v_i) and δ=d_1≤d_2≤d_3≤….≤d_n=Δ

Step 1:Let f(v_n)=2.

Rename v_n with w_1

Step 2: Put $f(v_i)=0$ for all $v_i \in N(w_1)$

***Step 3**:* Find $| N[v] \setminus N[w_1] |$ for all $v \neq w_1$

***Step 4**:* Find max $| N[v] \setminus N[w_1] |$

Suppose maximum is obtained for some v_i

Step 5: Rename v_i with w_2

Step 6: Label w_2 with 2. That is, $f(w_2)=2$

***Step 7**:* Label $f(v_i)=0$ for all $v_i \in N(w_2) \setminus N[w_1]$.

***Step 8**:* Find $| N[v] \setminus (N[w_1] \cup N[w_2]) |$ for all $v \neq w_1, w_2$

Repeat this above procedure until the maximum is 0.

***Step 9**:* m=i where i is the suffix of w_i at this stage.

Label the vertices of $V - \{N[w_1] \cup N[w_2] \ldots\ldots\ldots \cup N[w_m]\}$ as 1.

Obviously, the resulting function is a Roman dominating function giving the Roman domination number.

IV. CONCLUSION:

We manually verified that the above algorithm works nice to find the Roman dominating function for any graph.

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