

Combined Effects of Radiation and Hall Current on MHD Flow past an Impulsively Started Oscillating Vertical Plate with Variable Temperature and Mass Diffusion

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Abstract: The numerical investigation of the effects of radiation and Hall current on an unsteady MHD flow of viscous incompressible electrically conducting fluid past an impulsively started oscillating vertical plate with variable temperature and mass diffusion has been carried out. The fluid is considered a grey, absorbing-emitting radiation but non-scattering medium. The dimensionless governing coupled partial differential equations are solved numerically by using Ritz finite element method. The influence of material parameters such as Prandtl number, Schmidt number, radiation parameter, magnetic parameter, Hall parameter, thermal and mass Grashof numbers and phase angle on the primary and secondary velocity fields, temperature and the concentration distributions are presented with the help of graphs. It has been found that an increase in the radiation parameter there is a fall in the temperature, primary and secondary velocity profiles. An increase in the value of magnetic parameter leads to decrease in the primary velocity and increase the secondary velocity. The primary and secondary velocity fields are increased when the Hall parameter increased. Further, an increase in the thermal and mass Grashof numbers increases the primary and secondary velocity fields.

Key words: Radiation parameter, magnetic parameter, Hall parameter, oscillating vertical plate, partial differential equations.

I. INTRODUCTION

The study of MHD with heat and mass transfer in the presence of radiation have important industrial applications such as glass production and furnace design and space technology applications such as cosmical flight aerodynamics rocket, propulsion systems, plasma physics and aircraft re-entry aerodynamics which operate at higher temperatures radiation effect can be significant. Their applications can be found in the area like Hall accelerators, MHD generators, power engineering, food stuffs processing, MHD pumps and cooling of nuclear reactors. The growing need for chemical reaction in chemical and hydrometallurgical industries require the study of heat and mass transfer with chemical reaction. There are many transport processes that are governed by the combined action of buoyancy forces due to both thermal and mass diffusion in the presence of chemical reaction effect. Muthucumaraswamy and Janakiraman [1] presented MHD and radiation effects on moving isothermal vertical plate with variable mass diffusion. Theoretical study of chemical reaction effects on vertical oscillating plate with variable temperature were reported by Muthucumaraswamy and

Meenakshisundaram [2]. The effects of Hall current on MHD mixed convective flow of a viscous incompressible fluid past a vertical porous plate, immersed in a porous medium with heat source/sink was investigated by Sharma et. al [3]. Radiation effects on MHD flow past an impulsively started infinite isothermal vertical plate were presented by Chandrakala and Raj [4]. Deka [5] analyzed the effects of Hall currents on MHD flow past an accelerated plate. Srihari et. al [6] studied the effects of heat source/sink on MHD free convective flow and mass transfer along an infinite vertical porous plate in the presence of Hall current by finite difference method. Singh and Kumar [7] investigated the combined effects of Hall current and rotation on free convection MHD flow in a porous channel. Thermal radiation effects on unsteady MHD free convection flow past a vertical plate with temperature dependent viscosity was presented by Mohamoud [8]. Rao and Reddy [9] studied heat and mass transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous plate in the presence of radiative heat transfer. Shanaker et. al [10] investigated the effects of radiation and mass transfer on unsteady MHD free convective fluid flow embedded in a porous medium with heat

generation/absorption. Narahari and Nayan [11] studied free convection flow past an impulsively started infinite vertical plate with Newtonian heating in the presence of thermal radiation and mass diffusion. Ahmed and Sarma [12] analyzed the effects of Hall current on MHD transient flow past an impulsively started infinite horizontal porous plate in the presence of rotation. The effects of radiation and thermal diffusion on an unsteady MHD free convection mass transfer flow past an infinite vertical porous plate with Hall current and heat source was presented by Reddy and Rao [13]. Raju et. al [14] analyzed the effects of Hall current on unsteady MHD flow between stretching sheet and an oscillating porous upper parallel plate with constant suction. Sarkar et. al [15] investigated the effects of Hall current on MHD free convective couette flow in a rotating system in the presence of radiation. The effects of rotation and Hall current on mixed convection MHD flow through a porous medium filled in a vertical channel in presence of thermal radiation were presented by Singh and Pathak [16]. Satya Narayana et. al [17] analyzed the effects of Hall current and radiation absorption on MHD micro-polar fluid in a rotating system. Rajput and Kumar [18] presented the effects of radiation on MHD flow through a porous media past an impulsively started vertical with variable heat and mass transfer. Recently, MHD flow past a vertical plate with variable temperature and mass diffusion in the presence of Hall current were reported by Rajput and Kanaujia [19]. Muthucumaraswamy and Siva Kumar [20] studied MHD past a parabolic flow past an infinite isothermal vertical plate in the presence of thermal radiation and chemical reaction. The aim of this paper is to analyze the effects of radiation on unsteady MHD flow past an impulsively started oscillating vertical plate with variable temperature and mass diffusion in the presence of Hall current. The Ritz finite element method has been adopted to solve the governing system of coupled boundary layer equations under the boundary conditions. The effects of the material parameters on the primary and secondary velocity fields, temperature and concentration distributions have been presented through graphs and then discussed.

II. BASIC EQUATIONS

A two dimensional unsteady MHD viscous incompressible electrically conducting fluid past an impulsively started oscillating vertical plate is considered. The fluid is considered a gray, absorbing-emitting radiation but non-scattering medium. The flow is assumed to be in x' -direction, which is taken along the plate in the upward direction and y' -axes is taken to be normal to the direction of the plate. A uniform magnetic field B is assumed to be applied normal to the flow. The magnetic Reynolds number of the flow is taken to be small enough, so that the induced

magnetic field can be neglected. Initially, at time $t' \leq 0$, the temperature of the fluid and the plate is T_∞' and the concentration of the fluid is C_∞' . At time $t' > 0$, the plate starts oscillating in its own plane with frequency ω' , the temperature of the plate and the concentration of the fluid, respectively are raised to T_w' and C_w' . Further, we assume that the plate is non-conducting. Using the relation $\nabla \cdot B = 0$ for the magnetic field $\bar{B} = (B_{x'}, B_{y'}, B_{z'})$, we obtain $B_{y'}$ (say B_0) constant, i.e. $B = (0, B_0, 0)$, where B_0 is externally applied transverse magnetic field. Let $V(u', v', w')$ be the velocity and u', v', w' are the velocity components along x', y' and z' -directions respectively. The governing continuity equation is

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} + \frac{\partial w'}{\partial z'} = 0$$

Since there is no variation of flow in the y' -direction, therefore $v' = 0$. The generalized Ohm's law including the effect of Hall current is given as:

$$\bar{J} + \frac{\omega_e \tau_e}{B_0} (\bar{J} \times \bar{B}) = \sigma (E + \bar{V} \times \bar{B})$$

The external electric field $E = 0$, since the polarization of charges is negligible. Let $(j_{x'}, j_{y'}, j_{z'})$ be the components of current density J in x', y' and z' directions, respectively. Using above assumption, one obtain

$$J_{x'} = \frac{\sigma B_0^2}{1+m^2} (u' + mw') \quad \text{and} \quad J_{z'} = \frac{\sigma B_0^2}{1+m^2} (mu' - w')$$

Under the usual Boussinesq's and boundary layer approximations with above assumptions the governing equations of the flow are:

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T_\infty') + g\beta^*(C' - C_\infty')$$

$$-\frac{\sigma B_0^2}{\rho(1+m^2)} (u' + mw') \tag{1}$$

$$\frac{\partial w'}{\partial t'} = \nu \frac{\partial^2 w'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho(1+m^2)} (w' - mu') \tag{2}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y'} \tag{3}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} \tag{4}$$

The appropriate initial and boundary conditions are:

$$t' \leq 0; \quad u' = 0, w' = 0, T' = T_\infty', C' = C_\infty' \quad \text{for all } y'$$

$$t' > 0;$$

$$u' = u_0 \cos \omega' t', w' = 0, T' = T_\infty' + (T_w' - T_\infty'), C' = C_w' \quad \text{at } y' = 0$$

$$u' \rightarrow 0, w' \rightarrow 0, T' \rightarrow T_\infty', C' \rightarrow C_\infty' \quad \text{as } y' \rightarrow \infty \quad (5)$$

where u' is the velocity of the fluid in x' -direction, w' is the velocity of the fluid in z' -direction, $m (= \omega_e \tau_e)$ is the Hall parameter, g is the acceleration due gravity, β is the volumetric coefficient of thermal expansion, β^* is the volumetric coefficient of concentration expansion, t' is the time, C_∞' is the concentration in the fluid far away from the plate, C' is the species concentration in the fluid, C_w' is the species concentration at the plate, D is the mass diffusion, T_∞' is the temperature of the fluid near the plate, T_w' is the temperature of the plate, T' is the temperature of the fluid, k is the thermal conductivity, ν is the kinematic viscosity, ρ is the fluid density, σ is the electrical conductivity, C_p is the specific heat at constant pressure, μ is the magnetic permeability, ω_e is the cyclotron frequency of electrons and τ_e is the electron collision time. The radiation flux on the basis of the Rosseland diffusion model for radiation heat transfer is expressed as:

$$q_r = -\frac{4\sigma^* \partial T'^4}{3k^* \partial y'} \quad (6)$$

where σ^* and k^* are Stefan-Boltzmann constant and the spectral mean absorption coefficient of the medium. It is assumed that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as linear function of the temperature. It can be established by expanding T'^4 in a Taylor series about T_∞' and neglecting higher order terms, we obtain

$$T'^4 = 4T_\infty'^3 T' - 3T_\infty'^4 \quad (7)$$

Using Eqs. (6) and (7) in Eq. (3), we arrive at the modified energy equation:

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma^* T_\infty'^3}{3k^*} \frac{\partial^2 T'}{\partial y'^2} \quad (8)$$

Let us introduce the following non-dimensional quantities to make Eqs. (1), (2), (4), (5) and (8) dimensionless:

$$u = \frac{u'}{u_0}, y = \frac{y' u_0}{\nu}, t = \frac{t' u_0^2}{\nu}, w = \frac{w'}{u_0}, \omega = \frac{\omega' \nu}{u_0^2}, S_c = \frac{\nu}{D},$$

$$P_r = \frac{\mu C_p}{k}, N = \frac{k k^*}{4\sigma^* T_\infty'^3}, F^* = \frac{3N}{3N+4}, M = \frac{\sigma B_0^2 \nu}{\rho u_0^2},$$

$$\theta = \frac{(T' - T_\infty')}{(T_w' - T_\infty')}, C = \frac{(C' - C_\infty')}{(C_w' - C_\infty')}, G_r = \frac{g \beta \nu (T_w' - T_\infty')}{u_0^3},$$

$$G_m = \frac{g \beta \nu (C_w' - C_\infty')}{u_0^3} \quad (9)$$

The governing systems of equations are transformed into the following one-dimensional form:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r \theta + G_m C - \frac{M}{(1+m^2)} (u + mw) \quad (10)$$

$$\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial y^2} - \frac{M}{(1+m^2)} (w - mu) \quad (11)$$

$$F^* \frac{\partial \theta}{\partial t} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial y^2} \quad (12)$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2} \quad (13)$$

where u is the dimensionless velocity of the fluid in x -direction, w is the dimensionless velocity of the fluid in z -direction, θ is the dimensionless temperature, C is the dimensionless concentration, G_r is the Grashof number for heat transfer, G_m is the Grashof number for mass transfer, M is the magnetic parameter, m is the Hall parameter, P_r is the Prandtl number, N is the radiation parameter and S_c is the Schmidt number.

The corresponding initial and boundary conditions in non-dimensional form are:

$$t \leq 0; \quad u = 0, w = 0, \theta = 0, C = 0 \quad \text{for all } y$$

$$t > 0; \quad u = \cos \omega t, w = 0, \theta = t, C = 1 \quad \text{at } y = 0$$

$$u \rightarrow 0, w \rightarrow 0, \theta \rightarrow 0, C \rightarrow 0 \quad \text{as } y \rightarrow \infty \quad (14)$$

III. SOLUTIONS OF THE EQUATIONS

Eqs. (10), (11), (12) and (13) are solved numerically subject to the physically realistic boundary conditions given in Eq. (14) by applying the Ritz finite element method. The

algorithm for the Ritz finite element method can be summarized by the following steps.

Division of the whole domain into smaller elements of finite dimensions called “finite elements”.

Generation of the element equations using variational formulations.

Assembly of element equations as obtained in step (2).

Imposition of boundary conditions to the equations obtained in step (3).

Solution of the assembled algebraic equations.

The assembled equations can be solved by any of the numerical technique viz. Gauss-Seidal iteration Numerical solutions for the primary velocity u , secondary velocity w , temperature θ and concentration C are computed by using C - program. To judge the convergence and stability of the Ritz FEM, computations are carried out by making small changes in time and space directions, no significant change was observed in the values of u, w, θ and C . Hence, we conclude that the Ritz finite element method is convergent and stable.

IV. NUMERICAL RESULTS AND DISCUSSION

The velocity profiles for x and z -directions commonly, known as primary and secondary velocity profiles. The numerical calculations have been carried out to study the effects material parameters encountered in the problem under the investigation on the primary and secondary velocity fields, temperature and concentration distributions. The obtained numerical results have been presented through graphs. During the numerical computations of the values of the Prandtl number are chosen $P_r = 0.71, 1.00$ and 7.00 , which corresponds air, electrolytic solution and water at $20^\circ C$ and one atmosphere pressure and the values of the Schmidt number are taken $S_c = 0.22, 0.60$ and 0.78 which corresponds to hydrogen, water-vapour and ammonia respectively. The other physical parameters are chosen arbitrarily.

Temperature distribution: Fig. 1 displays the effects of the Prandtl number (P_r) on the temperature distribution. It is observed that there is a decrease in the temperature and temperature boundary layer as the Prandtl number increased. Fig. 2 illustrates the effects of the radiation parameter (N) on the temperature distribution. It is clearly observed from this figure that increasing values of the radiation parameter decreases the temperature and temperature boundary layer.

Concentration distribution: The effects of the Schmidt number (S_c) on the concentration distribution are shown in

Fig. 3. It is seen that the increasing values of Schmidt number leads to fall in the concentration distribution.

Physically, increase of S_c means decrease of molecular diffusivity D , this results in a decrease of concentration boundary layer. Hence, the concentration of the species is higher for small values of S_c and lowers for large values of S_c .

Primary and secondary velocity profiles: Figs. 4 and 5 illustrate the effect of the Prandtl number (P_r) on the primary and secondary velocity fields. From these figures it is seen that an increase in the value of the Prandtl number results a decrease in both primary and secondary velocities.

Figs. 6 and 7 show the effect of the Schmidt number (S_c) on the primary and secondary velocity fields. It is observed that an increase in the Schmidt number leads to decrease in both primary and secondary velocity fields. The effects of the radiation parameter N on the primary and secondary velocity fields are presented in Figs. 8 and 9. It can be clearly seen that an increase in the radiation parameter N decreases both the primary and secondary velocity fields. Figs. 10 and 11 illustrate the effects of the magnetic parameter M on the primary and secondary velocity fields. It is obvious that an increase in the magnetic parameter decreases the primary velocity and the reverse effect is observed on the secondary velocity. That is retarded under the effect of transverse magnetic field. The effects of the

Hall parameter (m) on the primary and secondary velocity fields are shown in Figs. 12 and 13. It is seen that an increase in the Hall parameter increases both primary and secondary velocity fields. This situation supports the fact that the Hall parameter induces a cross-flow in the boundary layer. Figs. 14 and 15 demonstrate the effects of the thermal

Grashof number (G_r) on the primary and secondary velocity fields. It is noticed that both primary and secondary fields increase with an increase value of the thermal Grashof number. Here, the positive values of G_r correspond to externally cooling of the plate. Figs. 16 and 17 show the

effects of the mass Grashof number (G_m) on the primary and secondary velocity fields. We found that an increase in the value of mass Grashof number increases both primary and secondary velocity fields. Also, it is observed that the peak values of the velocity increases rapidly near the plate due to cooling of the plate and after attaining a maximum value, it decreases as y increases. The effects of the phase

angle (wt) on the primary and secondary velocity fields are presented in Figs. 18 and 19. It is clear that both primary and secondary velocity fields decreases with increasing values of the phase angle.

FIGURES

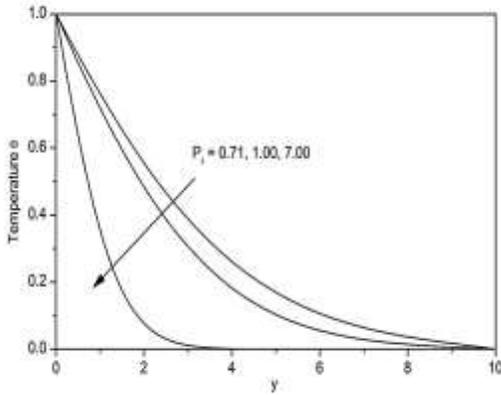


Fig. 1: Effect of P_r on the temperature distribution.

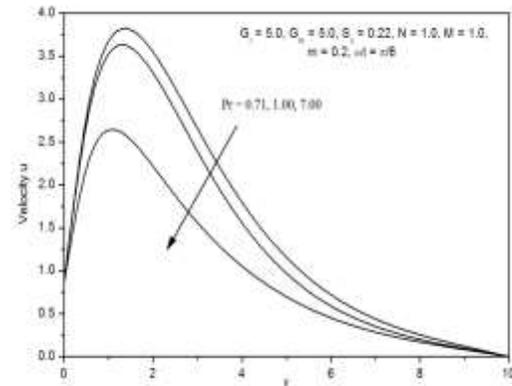


Fig. 4: Effect of P_r on the primary velocity u .

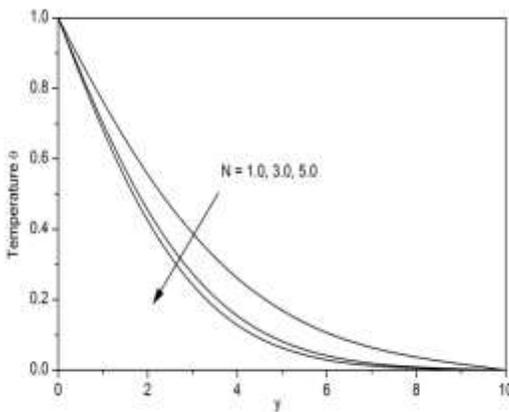


Fig. 2: Effect of N on the temperature distribution.

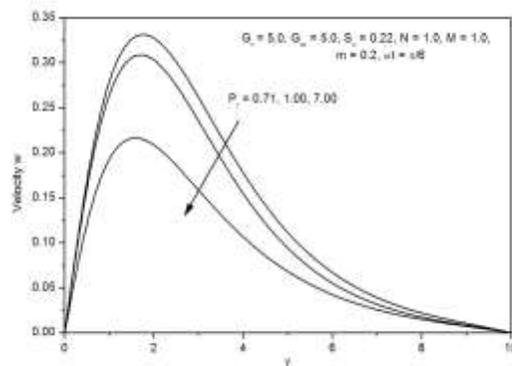


Fig. 5: Effect of P_r on the secondary velocity w .

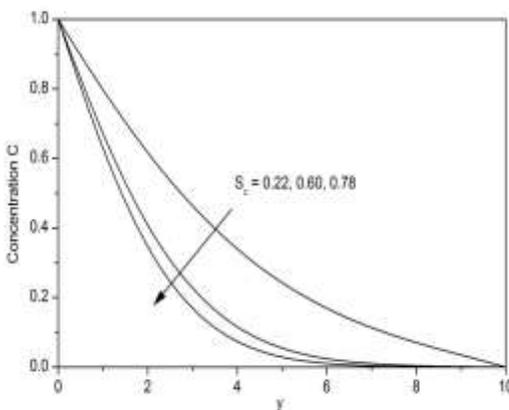


Fig. 3: Effect of S_c on the concentration distribution.

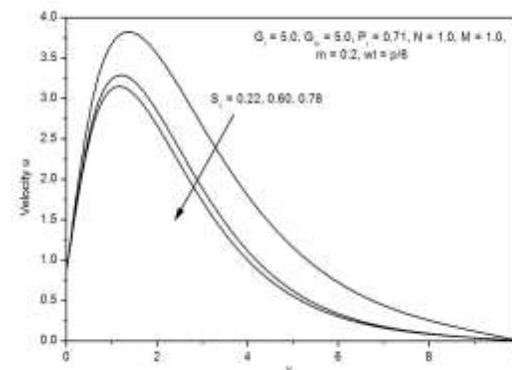


Fig. 6: Effect of S_c on the primary velocity u .

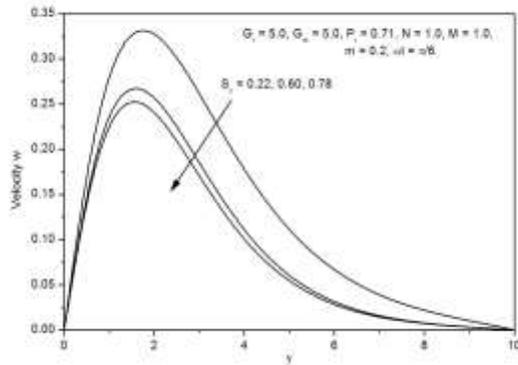


Fig. 7: Effect of S_c on the secondary velocity w

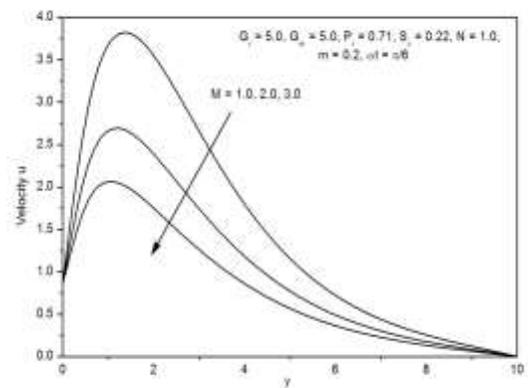


Fig. 10: Effect of M on the primary velocity u .

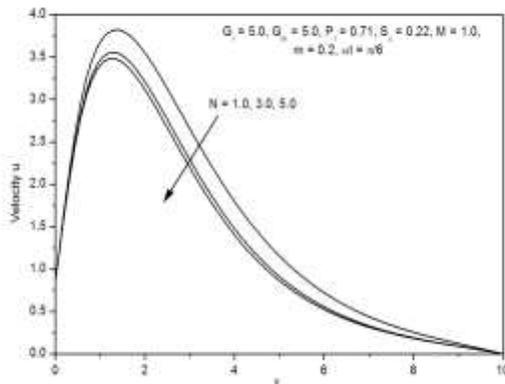


Fig. 8: Effect of N on the primary velocity u .

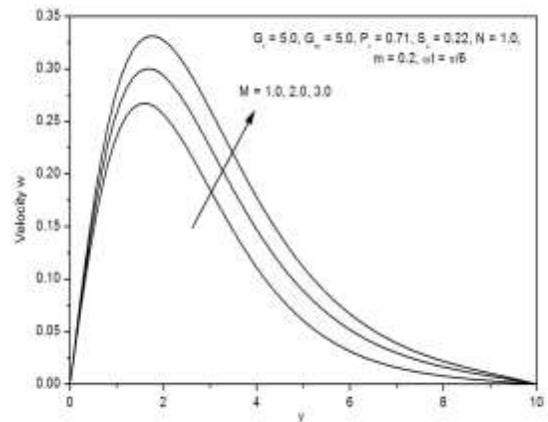


Fig. 11: Effect of M on the secondary velocity w .

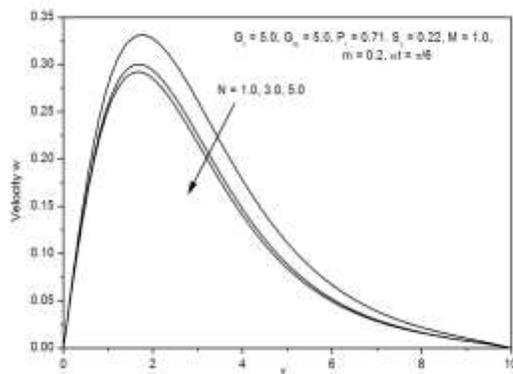


Fig. 9: Effect of N on the secondary velocity w .

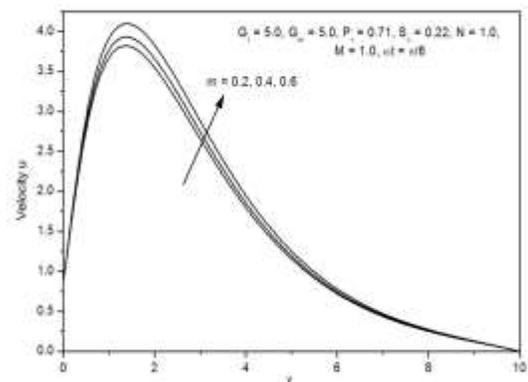


Fig. 12: Effect of m on the primary velocity u .

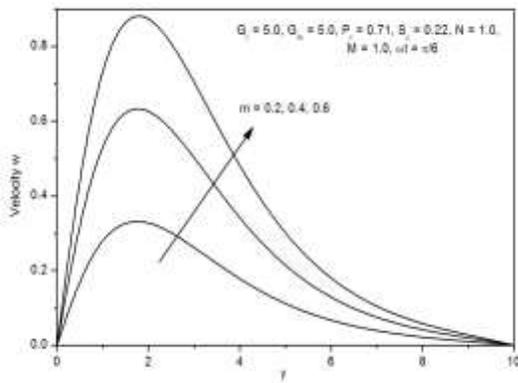


Fig. 13: Effect of m on the secondary velocity w .

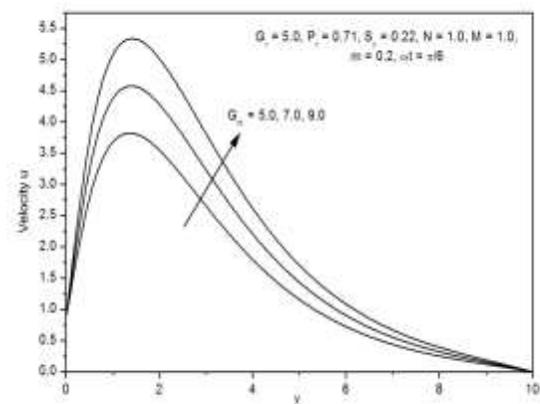


Fig. 16: Effect of G_m on the primary velocity u .

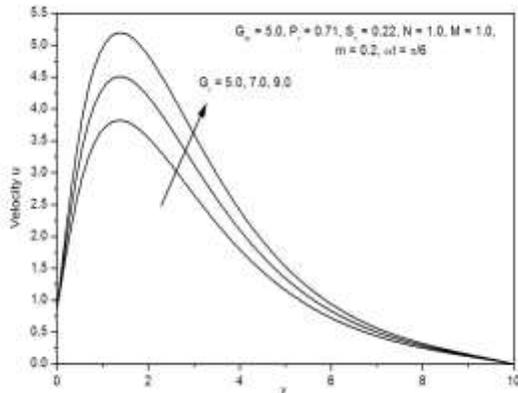


Fig. 14: Effect of G_r on the primary velocity u .

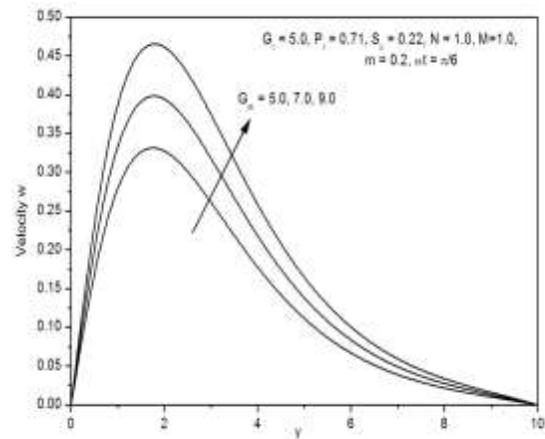


Fig. 17: Effect of G_m on the secondary velocity w .

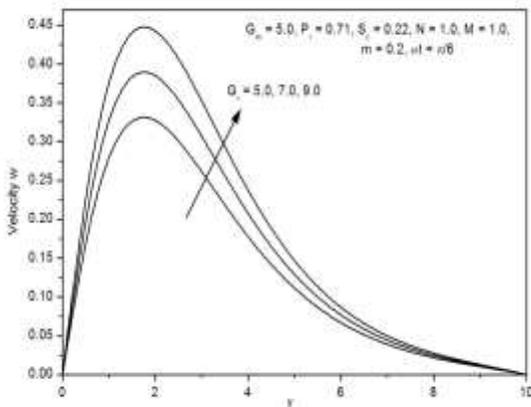


Fig. 15: Effect of G_r on the secondary velocity w .

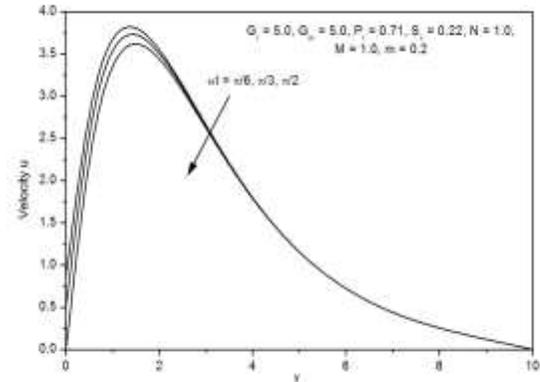


Fig. 18: Effect of ωt on the primary velocity u .

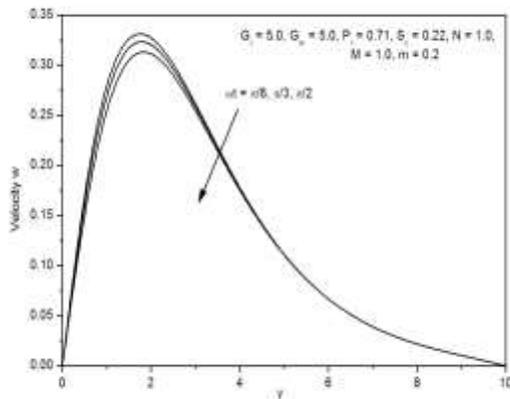


Fig. 19: Effect of w^t on the secondary velocity w^w .

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