ISSN (Online) 2456 -1304

International Journal of Science, Engineering and Management (IJSEM)
Vol 2, Issue 11, November 2017

Transfer Matrix Method for Precise Determination of Thicknesses in A 150- Ply Polyethylene Composite Material and Other Materials

[1] B Jyothi,[2] Vikram vikrant,[3] Shivnandan singh, [4]Shivani singh
[1] Assistant Professor, [2][3][4] B.E 1st year
[1] Department of science and humanities Engineering,[2] Department of Mechanical Engineering,
[3] Department of Electrical and Electronics Engineering [4] Department of Electronics & Communication Engineering,[1][2][3][4] Sri Sai Ram College of Engineering, Anekal, Bengaluru, India.

Abstract:-- **The multilayer structure of an ultra- high molecular weight polyethylene composite material was investigated in the terahertz (THz) spectral range by means of time domain spectroscopy (TDS) technique. Such structures consist of many alternating layers of fibre (~150), each being perpendicular to the other and each having thickness of about 50 micrometer. A transfer matrix method (TMM) and a time domain fitting procedure were used to determine thickness of all layers of the composite material with high accuracy. We apply this technique for various other materials.**

keywords:-- **Time Domain Spectroscopy, Acoustic Waves, Transfer Matrix Method.**

1. INTRODUCTION

Ultra-high molecular weight polyethylene (UHMWPE) consists of extremely long chains of polyethylene with a simple repeated atomic structure. Fibres with diameters of about 17 μm are extruded from the heated gel of UHMWPE using a spinneret and more than 95% of polymer chains have parallel orientation [5]. Next, flexible tapes made of a few about 50-70 μm perpendicular layers (often called plies) consisting of fibres are prepared. These tapes are stacked together and hot pressed to form a multilayer composite material with regular sequences of plies having mutually perpendicular orientations of fibres. Such composites are used in manufacturing bulletproof jackets, helmets and ballistic shields because of their high mechanical resistance. The determination of individual layer thickness as well as investigation of delamination and other defects are crucial for reliable characterization of the manufacturing process and quality control. The THz range is well suited to study these composites, because UHMWPE has a low absorption coefficient varying between 0.3-1.5 cm-1 between 0.1 and 3THz. Its refractive index of 1.53 is almost constant and thus the influence of dispersion is very small.

2. CHARACTERISTIC OF SAMPLE

The considered in this paper HB50 from Dyneema® tape consists of four alternately perpendicular plies of PE fibres (Fig. 1a) with diameters of about 17 μm each [5]. The thickness of a single ply is about 55-70 μm and a pl astic rubber (17% of styrene-isoprene-styrene triblock copolymer (SISTC [5]) is used as a matrix. A long tape reel was cut into smaller sheets which were then layered and hot pressed to form a multilayer composite material. The photograph of HB50 composite samples having the dimensions of 50 x 50 mm2 and thickness 9.45±0.1 mm is shown in Fig. 1b. The exact number of plies is not known to the authors.

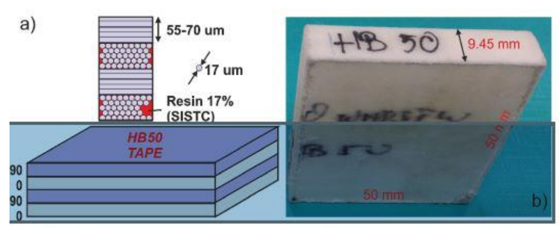

Fig. 1. The scheme of the HB50 tape (a) and the photograph of the manufactured composite.

First we investigated the multilayer HB50 sample having smaller thickness of q=2.95 mm using a standard time domain spectroscopy system Spectra 3000 from Tera View. The complex refractive index N is equal to N=1.521+i·0.002. It was also proved [6] that a single

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layer of UHMWPE fibres exhibits a birefringence (Δn=0.04), which results in the dependence of the value of the refractive index on the orientation of the electric field vector in relation to the direction of fibres. This fact became the basis of the structure analysis in the experiment in the TDS reflection configuration.

3.EXPERIMENTAL SETUP AND RESULTS

A THz TDS setup in reflection configuration with normal incidence was used for the investigation (Fig. 2a). The TDS setup uses a Ti: Sapphire laser, an InAs surface emitter, a GaAs photoconductive detector and a mechanical chopper. Figure 2b shows a THz pulse with FWHM=0.3 ps and side lobes. The 105-mm focal length resulting in a spot size at the surface of the sample of about 2 mm @1⇒ THz

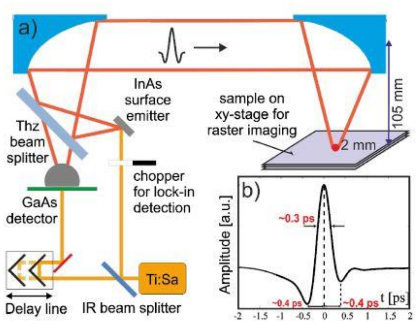

Fig. 2. The experimental setup (a) and the reference signal (b).

A bandwidth of the system is up to 3 THz with a dynamic range of about 40 dB. The setup is situated in a box purged with dry air, which provides low humidity (< 1%). The sample was placed on a rotating platform and the THz radiation was precisely focused on their front surfaces using a z-stage. The sample was also carefully aligned with respect to the parallel orientation of the electric field vector in relation to the direction of fibres of the front layer. Figure 3 plots a typical waveform reflected from the sample with duration of about 140 ps with 13441 measuring points with spacing 0.0104 ps. In reflection geometry the propagating THz pulses are partly reflected from the front surface and from the interfaces between the media having different refractive indices (like layers/plies). Because of the multilayered structure of composite materials a sequence of pulses shifted in time are observed; each pulse coming from a reflection from a particular interface. In principle, knowing the refractive index of the single layer we can determine its thickness on the basis of the time difference between two pulses (their peaks) reflected from the front and back surfaces of this layer. However, for thin layers (~ 60 µm), pulses reflected from adjacent layers overlap in the time domain and cause errors in determining thicknesses of consecutive layers.

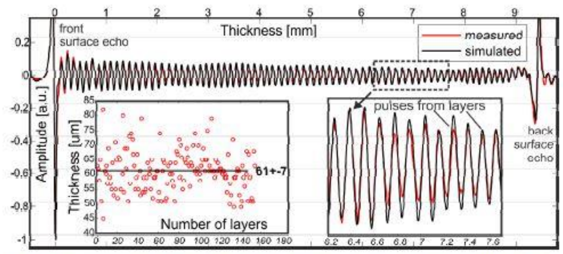

Fig. 3. Waveform reflected from the sample – measured and simulated. The inset on the left illustrates calculated thicknesses of particular layers.

4.SIMULATION RESULTS

The transfer matrix method and a time-domain fitting procedure [7] were used to the UHMWPE sample composed of about 150 layers with a quasi-binary profile of the refractive index, which means that values of the refractive index for subsequent layers are: $n+\Delta n1$, $n-\Delta n2$, $n+\Delta n3$, …, respectively; $n >> \Delta ni$ and $\Delta n1 \approx \Delta n2 \approx \Delta n3$… . The optimization of both parameters, i.e. the refractive index and the thickness for each layer independently would be computationally demanding. Therefore, we can assume that $\Delta n1 = \Delta n2 = \Delta n3 = \Delta n$ and Δn was optimized for the whole sample, while calculations were limited to the optimization of thicknesses of layers. Due to the large number of layers, we significantly increased the optimization efficiency and accuracy by application of a hybrid fitting approach, which is based first on a brute force method for coarse searching followed by the accurate direct optimization.

Due to the complexity resulting from this partial superposition of pulses, the waveform reflected from the multilayer structure with the refractive index of i-th layer $Ni=ni+i\kappa$, (i=1...k) can be simulated using the TMM [7]. The structure is
surrounded from both sides by air with the complex refraction index $N0=Nk+1=1$. In TMM each layer is modeled with the following two matrices:
- Dij describing the behaviour of the terahertz pulse at the interface between i-th and j-th ($j=i+1$) layer containing the corresponding Fresnel coefficients:

$$D_{i,j} = \frac{1}{t_{ij}} \begin{bmatrix} 1 & r_{ij} \\ r_{ij} & 1 \end{bmatrix}, \qquad (1)$$

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- *Pi* describing the propagation of the pulse through the *i*-th layer:

$$P_i(\omega) = \begin{bmatrix} \exp\left(\frac{i\omega N_i d_i}{c}\right) & 0 \\ 0 & \exp\left(-\frac{i\omega N_i d_i}{c}\right) \end{bmatrix}, \quad (2)$$

where ω is the angular frequency and c is the speed of light in vacuum. The multilayer structure can be described as the multiplication of all layers' matrices:

$$M_{total}(\omega) = \prod_{i=0}^{k} P_i(\omega) \cdot D_{i,i+1} = \begin{bmatrix} M_{11}(\omega) & M_{12}(\omega) \\ M_{21}(\omega) & M_{22}(\omega) \end{bmatrix} \quad (3)$$

The transfer function for the reflection geometry (*R(ω)*) can be calculated from the equation:

$$R(\omega) = \frac{M_{21}(\omega)}{M_{11}(\omega)}. \quad (4)$$

Finally, the simulated signal (*Er(t)*) reflected from the multilayer sample can be calculated using the inverse numerical Fourier transform (F^{-1}) according to:

$$E_r(t) = F^{-1}\left[R(\omega) \cdot F\left[E_0(t)\right]\right], \quad (5)$$

where *F*[*E*0 (*t*)] is the numerical Fourier transform of the incident THz pulse. The time-domain fitting algorithm seeks for minima of:

$$QERR(d_1 \ldots d_k) = \sum_t \left| sign_{meas}(t) - sign_{sim}(d_1 \ldots d_k, t) \right| \quad (6)$$

QERR(*d*1 ... *d*k) describes the mean square deviation between the measurement and simulation. Thicknesses of individual layers are frequently varied until an optimal correlation between the measured and the simulated pulse is achieved. Figure 3 (inset) shows optimal thicknesses calculated according to the proposed algorithm. We determined 154 plies with an average thickness of 61±7 µm. Almost 90% of calculated thickness values are within a range of 50-70 µm, what is in good agreement with other results. The waveform simulated basing on the calculated thicknesses agrees very well with the measurement with a correlation coefficient 0.97. As a result, we were able to determine the thickness of all layers of multilayer 154 piles structure by means of the reflection TDS technology with high accuracy.

The transfer-matrix method is a method used in optics and acoustics to analyze the propagation of electromagnetic or acoustic waves through a stratified medium.[8] This is for example relevant for the design of anti-reflective coatings and dielectric mirrors.

The reflection of light from a single interface between two media is described by the Fresnel equations. However, when there are multiple interfaces, such as in the figure, the reflections themselves are also partially transmitted and then partially reflected. Depending on the exact path length, these reflections can interfere destructively or constructively. The overall reflection of a layer structure is the sum of an infinite number of reflections.

The transfer-matrix method is based on the fact that, according to Maxwell's equations, there are simple continuity conditions for the electric field across boundaries from one medium to the next. If the field is known at the beginning of a layer, the field at the end of the layer can be derived from a simple matrix operation. A stack of layers can then be represented as a system matrix, which is the product of the individual layer matrices. The final step of the method involves converting the system matrix back into reflection and transmission coefficients

CONCLUSION:

Transfer matrix is used in optics and acoustics to analyse the propagation of electromagnetic or acoustic wave through a stratified medium.

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ISSN (Online) 2456 -1304

International Journal of Science, Engineering and Management (IJSEM)
Vol 2, Issue 11, November 2017

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