

# The Sun-Jupiter Lagrange Points and the Observation of the Captured Trojan Asteroids

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**Abstract**— In this paper Jupiter’s Lagrange points are elucidated in detail and the calculation of each point is given, the asymmetric nature of the L4 and L5 points are noted and their characteristics are presented. Furthermore, the trajectory of the first mission to the Trojan asteroids, Lucy, following its earth gravity assists are discussed and mentioned.

**Index terms:** Jupiter’s Lagrange points and calculation of each points.

## I. INTRODUCTION

Icarus don’t fly too close to the sun” -a Greek mythology quote, was greatly denied by the Aditya L1 launched by ISRO on September 2nd 2023, after it’s earth-bound maneuvers, it has escaped the sphere of (earth’s) influence (SOI) and is on its way to the earth-sun Lagrange point L1.

## II. SUN-EARTH AND EARTH-MOON SYSTEMS

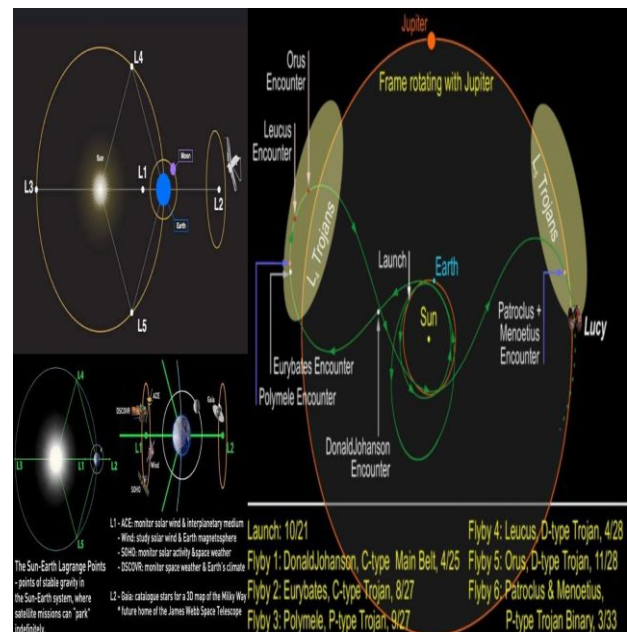
Aditya L1 is a name that everyone is familiar with, but what is “L1”? Lagrange points or Lagrangian points are produced due to the gravitational forces of two massive bodies (such as the earth and sun or the earth and moon) equaling at one point in space. These points were discovered by Joseph Louis Lagrange. L1 is the point where both the gravitational forces travel along the lines joining the bodies. L2 is the point along the opposite direction of the bodies near the smaller body. L3 is the point in the direction near the larger body. There are a ton of satellites placed in the L1 and L2 points in the sun-earth and earth-moon Lagrange systems.

There are various satellites present in the sun-earth Lagrange points. In L1: The Solar and Heliospheric observatory (SOHO), The Advanced composition explorer (ACE), WIND, The Deep Space Climate Observatory (DSCOVER), Aditya-L1. In L2: The ESA Gaia probe, The joint Russian-German high-energy astrophysics observatory Spektr-RG, The joint NASA, ESA and CSA James Webb Space Telescope (JWST), The ESA Euclid mission. In L3 there are no orbiters as of now. In L4: STEREO, OSIRIS-REx, along with asteroids 2010 TK7 and 2020 XL5. In L5: Asteroid 2010 SO, STEREO B, The Spitzer Space Telescope and Hayabusa2 have passed through.

The satellites in the Earth-moon system comprises of THEMIS, Chang’e 5-71, Queqiao relay satellite, EQUULEUS nanosat at L2 and kordylewski clouds are dusts that exist at L4 and L5. Though we are aware of the sun-earth or the earth-moon systems, Jupiter has captured multitudes of asteroids in its Lagrange points.

## III. LAGRANGE POINTS OF JUPITER

In this paper, the Lagrange points of our solar system’s largest planet, Jupiter will be discussed. Jupiter, named after the Roman king of Gods, has around 80 to 95 moons, which is an infinitesimal amount, compared to the number of Trojan asteroids captured by Jupiter’s Lagrange points (approximately over 10,000 asteroids have been detected as of now). There are 5 Lagrange points: L1, L2, and L3 (founded by mathematician Leonhard Euler) are not as stable as L4 and L5 Lagrange points in Jupiter (with a resonance of 1:1) . The L4 and L5 points are calculated by making two equilateral triangles with the Sun and Jupiter on two ends of the vertex and the third vertex at an angle of 60 degrees lies the L4 Lagrange point and mirroring this is L5 at the opposite side. As Jupiter has greater mass, the Lagrange points of Jupiter have shown greater stability, which has been able to capture the trojan asteroids which are the trailing asteroids at L5 and the Greek asteroids which are the leading asteroids at L4 named after the Trojan war.



The Trojan asteroids follow heliocentric orbits with almost the same semi-major axis of Jupiter, at 5.2 AU (astronomical unit). The calculation of the Lagrange points can be obtained by using Kepler's law or using the centripetal force and gravitational force,

$$\omega^2 = G(M_1 + M_2) / |r_1 - r_2|^3 = GM_2 / |r_1 - r_2|^2 |r_1|$$

Where G is Newton's gravitational constant

M1 is the mass of Sun and M2 is the mass of Jupiter along with r1 and r2 being the distance respectively.

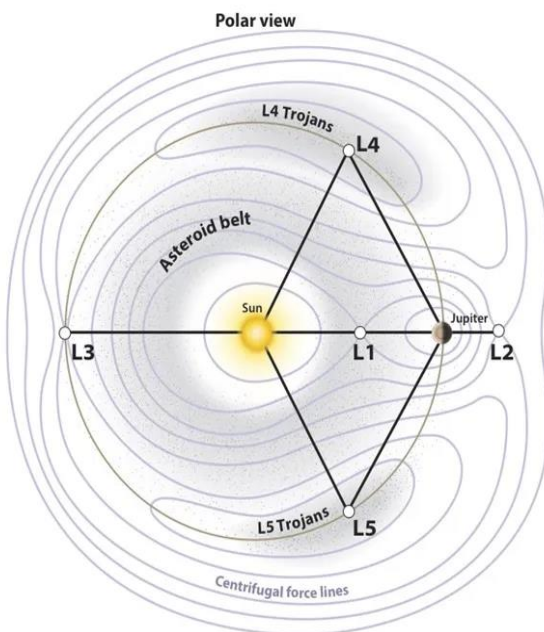
As this is a non inertial frame, two other forces are centrifugal force and the Coriolis force. The forces Fm on mass m with angular velocity ω is given by

$$F_m = - (GM_1 m (r-r_1) / |r - r_1|^3) - (GM_2 m (r-r_2) / |r - r_2|^3) + (m\omega^2 r) - 2m\omega \times r$$

The table below gives the location of each point

**LOCATION OF LAGRANGE POINTS FROM THE SUN:**

Point	Location
L1	7.25861726660483e+11
L2	7.80357861875457e+11
L3	7.78656009093795e+11
L4	< 3.88258289037167e+11, 6.73767764144293e+11, 0 >
L5	< 3.88258289037167e+11, 6.73767764144293e+11, 0 >



Though the L4 and L5 points are observed to be at almost the same distance, the asymmetric nature of the two points is evident. The L4 having more mass than the L5 point which results in the ratio of N4/N5 (number of trojans) ~ 1.6 (~ golden ratio). This asymmetric nature is still a mystery to us

though many theories have been given, to state a few, it could be the result of Jupiter's inward migration through the planetary discs during its accretion or a part of Jupiter's embryo serving as Trojans, but the asteroids have dark photometric colours, which are resemblant to the Trans-Neptunian objects (TNO). Predominantly dark red P-type and D-type asteroids are found which also resemble those asteroids in the icy bodies of Kuiper Belt. C-type Trojans are minimal (7%) and are found between the asteroid belt of Mars and Jupiter.

Though many satellites have been launched to Jupiter like the pioneer 10, pioneer 11, voyager 1 and 2, Juno etc; Lucy was the first satellite sent to the Trojan asteroids which was launched on October 16, 2021 on an atlas V 401 rocket making it's flyby across the asteroid belt and make it's way to the trojan asteroids. Lucy's trajectory uses Evolutionary Mission Trajectory Generator(EMTG). This is used for Multiple Gravity Assist with n Deep Space Maneuvers using Shooting (MGAnDSMs). Gravity Assist is the process of using Earth's gravitational force to gain momentum. Lucy will be using a total of 3 EGAs.

Lucy's first mission is to study the L4 swarm containing Eurybates and its satellite, Queta, Polymele, Leucus and Orus. Lucy will then return back to earth for another gravity assist which will then head to the L5 swarm as their orbital motion would have moved around the sun, where it will flyby Patroclus and it's companion Menoetius

**IV. CONCLUSION**

The lagrange points of Jupiter serves to be a heap of not just asteroids, but also mysteries. After the successful mission to Mar's orbit (MoM) by the ISRO, they plan on exploring the gas planet Jupiter, which could take up to 26 months to travel to the Jovian orbit.

The future for the exploration of the Trojan asteroids seems promising just like the victory of the Trojan war.

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