

# Numerical Simulation of Electrical Conductivity for Carbon Fiber Reinforced Polymer Composite

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**Abstract**—Carbon fiber composite are generally used in aerospace due its high structural properties. Other than structural properties, Electrical conductivity of composites also play major role such as under lightning strike and health monitoring. Electrical conductivity of carbon fiber is very good. But, Electrical conductivity of carbon composite is as not good as carbon fiber due to use of polymer as matrix. In this paper, electrical conductivity of carbon fiber composite have been numerically simulated using waviness model and curing pressure of composite fabrication as a parameter. This model was successfully implemented unidirectional composites. Conductivity in longitudinal direction conductivity was found to 50 times more than transvers direction.

**Index Terms**—CFRP composite, Electrical conductivity, Fiber waviness, waviness factor

## NOMENCLATURE

$L_c$	Characteristic length of carbon fiber (mm).
$R_a = R_b$	Resistance of carbon fiber having characteristic length ( $\Omega$ ).
$V_f$	Fiber volume fraction (%).
$R_c$	Contact resistance between carbon fiber ( $\Omega$ ).
$I_a$	Current flowing in Resistance $R_a$ (A).
$I_b$	Current flowing in Resistance $R_b$ (A).
$I_c$	Current flowing in Resistance $R_c$ (A).
$\rho$	Intrinsic resistivity of carbon fiber ( $\Omega m$ ).
$d_f$	Diameter of carbon fiber ( $\mu m$ ).
$d_s$	Waviness height (mm).
$\beta$	Fiber waviness factor.
$E$	Elastic modulus of carbon fiber (GPa).
$L$	Length of sample (mm).
$L_u$	Length of waviness (mm)
$B$	Width of sample (mm).
$T$	Thickness of sample (mm).
$R_{n1}$	Equivalent resistance at node 1( $\Omega$ ).
$N_f$	Number of fibers in each load carrying layer.
$N_{cl}$	Average number of a contact point in each fiber.
$N_c$	Number of a node in each fiber layer.
$R_{L1}$	Total resistance of 1st load carrying layer in a longitudinal direction ( $\Omega$ ).
$(R_{com})_L$	Total resistance of all four layers in a longitudinal direction ( $\Omega$ ).
$R_{T1}$	Total resistance of 1st load carrying layer in transverse direction ( $\Omega$ ).
$(R_{com})_T$	Total resistance of all four layers in transverse direction ( $\Omega$ ).
$(R_{com})_T$ thickness	Total resistance of all four layer in a thickness direction ( $\Omega$ ).
$\rho_{Com}$	Resistivity of CFRP composite lamina ( $\Omega m$ ).
$A_c$	Cross-sectional area of composite (mm x mm).
$L_v$	Distance between applied voltage (mm).
$\rho_{(Com)L}$	Resistivity of CFRP composite lamina in longitudinal direction ( $\Omega m$ ).

$\rho_{(Com)T}$	Resistivity of CFRP composite lamina in transverse direction ( $\Omega m$ ).
$\rho_{(Com)Th}$	Resistivity of CFRP composite lamina in thickness direction ( $\Omega m$ ).
$\sigma_{(Com)L}$	Conductivity of CFRP composite lamina in longitudinal direction ( $\Omega m$ ) <sup>-1</sup> .
$\sigma_{(Com)T}$	Conductivity of CFRP composite lamina in longitudinal direction ( $\Omega m$ ) <sup>-1</sup> .
$\sigma_{(Com)Th}$ ickness	Conductivity of CFRP composite lamina in longitudinal direction ( $\Omega m$ ) <sup>-1</sup> .

## 1. INTRODUCTION

Due to its high strength to weight ratio and its excellent corrosion resistance, carbon fiber reinforced polymer (CFRP) have been widely used in aviation, automobile and other industry from the past few years, There are some application which required for electrical property of CFRP composite such as Carbon fiber assisted heating during composite manufacturing[1], self-sensing of damage of composite structure[2], lightning strike protection of composite structure[3].

Although electrical conductivity of carbon fiber and polymer matrix in CFRP differs by several order of magnitude. Carbon fiber is having good electrical conductivity in the range of  $10^5$  ( $\Omega m$ )<sup>-1</sup>[4]. In opposite, the polymer matrix treated as good insulator having electrical conductivity rage of  $10^{-10}$  ( $\Omega m$ )<sup>-1</sup> to  $10^{-20}$  ( $\Omega m$ )<sup>-1</sup>[5]. Conduction mechanism of electrical conductivity along the fiber direction is administrated by continuous conduction while as in transverse and through-thickness direction is governed by the shortest conduction path. Thermal

conduction and mechanical behavior study have been performed by microstructure based modeling for CFRP[6]. An existing modeling approach, it is assumed that fibers are thin (order of  $10^{-6}$ - $10^{-9}$ ) long cylinder and placed parallel to each other throughout the width and thickness direction and microstructure is represented by fiber spacing information from the cross-section of the composite.

Gutowski et al [7] relaxed the assumption of straight fiber, and introduced a new model parameter for multiple connecting points along its length with neighboring fiber due to fiber waviness which result in continuous conduction path in transverse and thickness direction.

In this paper, a 2D micro-mechanism model of electrical conductivity is presented. An equivalent electrical resistance model is built by describing the microstructure relationship between waviness and their contact point. This model predicts electrical resistivity of CFRP as a function of fiber volume fraction, intrinsic carbon fiber property, fiber waviness, and processing pressure. For numerical simulation, network resistor model is solve using Kirchhoff's current law.

**2. MODELING**

In existing electrical conductivity model fiber are assumed to be straight and placed throughout in volume. Percolation theory helps to explain electrical conductivity in CFRP composite. In percolation theory critical loading called percolation threshold, the conductivity enhance to several order of magnitude In CFRP composite, due to fibers waviness, It is assumed that there is the number of contact point along the fiber direction with neighbor fiber. Hence this makes CFRP structure as a larger electrical network. Existing microstructure model helps to generate a resistor network model.

Cross-sectional fiber arrangement perpendicular to fiber length is generated first so that 2D fiber arrangement builds. After that this 2D fiber network extended along the length with fiber waviness information to describe the full 3D micro-structure. Fiber arrangement and fiber waviness parameter describe numerically and experimentally. It is assumed that square, hexagonal or random packing order helped in generation of 2D fiber arrangement of the composite micrograph. Waviness parameter can be obtained experimentally from compression behavior model developed by Gutowski[7] or from a numerical simulation. Each carbon fiber of CFRP composite is divided into small section with fiber contact point throughout length of the fiber. Contact point in carbon fiber considered as a node in the electrical resistor network model. Carbon fiber divided in the number of section in which each section modeled as a resistor whose value can be determined by the length of carbon fiber segment, carbon fiber diameter and intrinsic

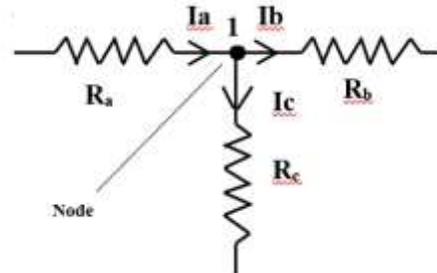
resistivity of carbon fiber. A contact resistor added in electrical resistor network model which represent contact point as in 3D micro- structure. It means that there are two types of resistor in the electrical resistor network model

- a) Resistance which representing a length of carbon fiber.
- b) Resistance which representing contact resistance between carbon fiber.

Kirchhoff's law helps to solve resistor of fiber and contact resistance of carbon fiber. According to **Kirchhoff's current law** which is a state that-“For a parallel path the total charge(current ) entering in a circuit junction is equal to the total current leaving the same junction this is because it has no other place to go or no charge is lost.”

Mathematically-

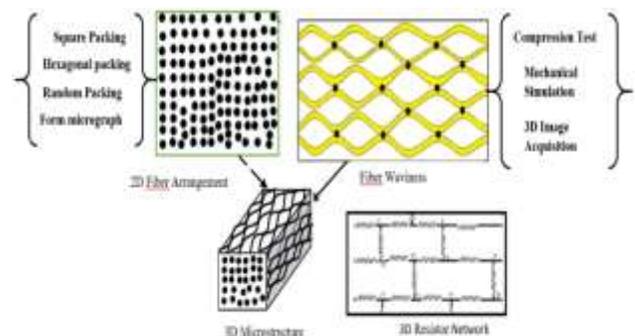
$$\Sigma I_{in} = \Sigma I_{out} \dots\dots(1)$$



**Fig. 1. Fiber and contact resistance arrangement for at node1**

In fig. 1 shown, when potential difference applied across the terminal, First current passed through fiber resistor ( $R_a$ ) but when current reach to node point 1, it divided into two parts one is fiber resistor and second is contact resistor. So according to Kirchhoff's current law algebraic sum of current entering at node point 1 is equal to current leaving the node point. i.e.

$$I_a = I_b + I_c \dots\dots(2)$$



**Fig. 2. Resistor network model formation from 2 D fiber arrangement and fiber waviness parameter**

When electrical potential applied, Motion of electron started and motion of electron in carbon fiber depends of resistivity of fiber. Higher resistivity of carbon, lesser will conductivity.

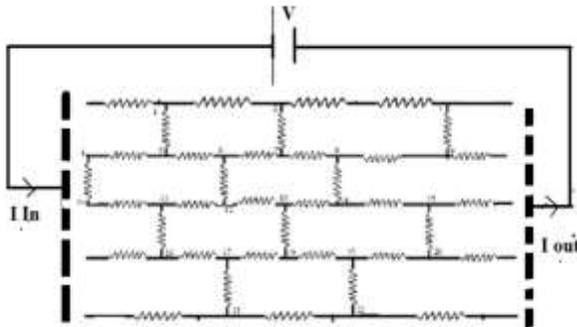
**a) Conduction in longitudinal direction**

The Polymer matrix can be treated an insulator because its conductivity are 10-20 order smaller than that of carbon fiber. Highly anisotropic behavior of CFRP leads to different

conduction mechanism in a different direction. Along the fiber direction, an overall length of carbon fiber is divided into the number of a small segment which has equal length. Length of this segmental portion is known as Characteristic length ( $L_c$ ) and resistance of segmental portion is known as fiber resistance ( $R_a$  or  $R_b$ ) shown in fig.1.

When potential difference applied across the length of the sample, First electrical current is passed through resistor  $R_a$  having electrical current  $I_a$  as shown in fig.1. At node 1, Current  $I_a$  divided in to two parts that are  $I_b$  and  $I_c$  having resistance  $R_b$  and  $R_c$  respectively. Hence,  $R_b$  and  $R_c$  are parallel to each other and series with  $R_a$ .

To solve the network resistor model shown in fig. 3 when current entered along length direction, It passed in all the fiber simultaneously. To determine the overall resistance along the longitudinal direction solution can be made on one node in the network model explained above and multiple by number of node in the layer.



**Fig. 3. A Resistor network model for longitudinal direction having four layer**

The Resistance ( $R_a$  or  $R_b$ ) of characteristic length of carbon fiber is calculated from the intrinsic carbon resistivity, carbon fiber diameter and characteristic length ( $L_c$ ) as-

$$R_a = \rho \frac{4 \cdot L_c \cdot V_f}{\pi \cdot d_f^2} \dots\dots\dots(3)$$

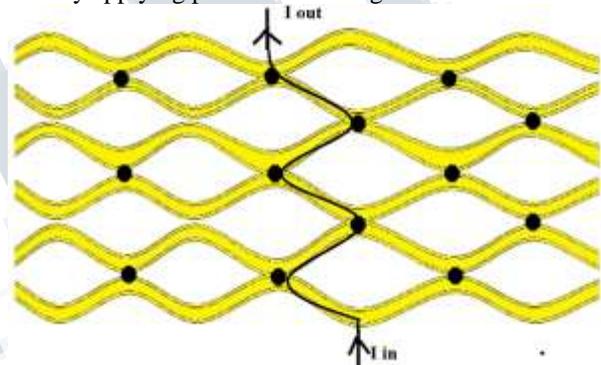
And contact resistance ( $R_c$ ) can be calculated with the

help of Hertz contact theory as-

$$R_c = \frac{\rho \cdot V_f}{\sqrt[3]{3P \cdot (d_f + d_s) \cdot d_s \cdot \beta \cdot d_f}} \dots\dots\dots(4)$$

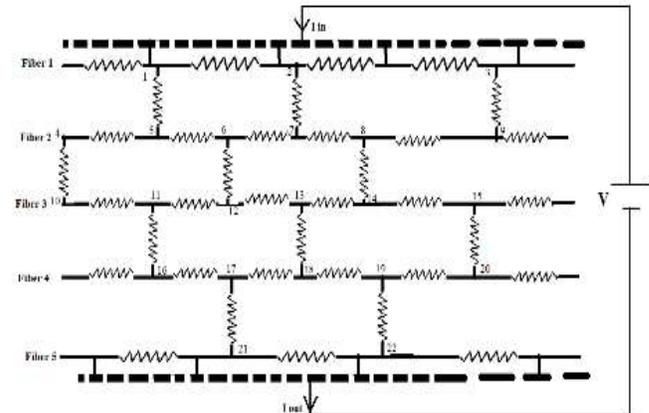
**b) Conduction in transverse direction**

Park et el [28 ] explained random fiber to fiber contact that contributes transverse electrical conductivity and anisotropy of conductivity. Overall electrical conductivity can be contributed by fiber to fiber contact point, which provided a continuous conduction path to flow electron from one side to another side of CFRP composite. Waviness is created in CFRP by applying pressure in through-thickness direction.



**Fig. 4. Direction of electron flowing in transverse direction**

Waviness model is useful to explained the conduction mechanism in a transverse direction as shown in fig. 5. In common model it is assumed that fibers are straight and there is no contact between neighbor fibers. Fibers are separated by a matrix which assumed as an insulator. In waviness model current flow in shortest path to travel from one end to another end. A solution of transverse resistor model is same as for longitudinal direction.



**Fig. 5. An electrical resistor network model for transverse direction**

But here is some difference such a direction of current entering in CFRP composite. When applied a potential difference along the transverse direction, current ( $I_c$ ) first entered in resistance  $R_c$ . When this current reached at a node point, it divided in two part one is  $I_a$  which is flowing in resistance  $R_a$  and one is  $I_b$  which is flowing in a resistance  $R_b$ . These two resistance  $R_a$  and  $R_b$  are parallel to each other and series with  $R_c$ .

To calculate overall conductivity in transverse direction of Fig. 5, First calculate resistance about one node and multiply it by the number of nodes available in a layer.

**c) Conduction on thickness direction**

In common model, It was assumed that fibers are straight cylinder and parallel to length of a composite. In such model parallel arrangement of carbon fiber provided good conductivity in longitudinal direction due to an intrinsic property of carbon fiber are responsible for conduction in a longitudinal direction. But in a thickness direction, conductivity is very low due to lack of contact point in a thickness direction. This is reason that resistivity in thickness direction is  $10^{20}$  order higher than in longitudinal direction.

But in case of waviness model, It is assumed that fiber has waviness nature in the thickness direction, so model applicable in thickness direction is same as the transverse direction and procedure to determine resistance is same in a transverse direction which explained above.

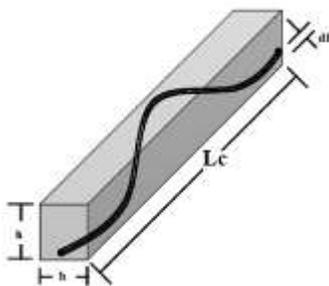
The advantage with waviness model is that resistance in transverse direction is equal to thickness direction and which improve conductivity in transverse direction also.

**3. NUMERICAL SIMULATION**

Consider a ply model with stacking sequence of [0/0/0/0] i.e. four layers having  $0^\circ$  orientation parallel with a length of the sample.

**Assumption-**To evaluate resistance in each direction (Longitudinal, Transverse and Thickness). It followed by certain assumption-

- Fibers are not straight they have waviness on entire sample.

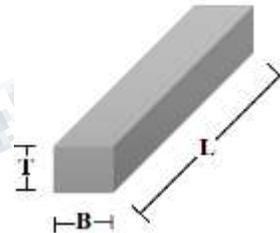


**Fig. 6. Representation of fiber waviness**

$$\text{Fiber waviness } (\beta) = \frac{L_c}{d_s} = \frac{L_c}{h-d_f}$$

- $d_s = 2 \times d_f$
- Distribution of fiber is the same throughout in sample.
- Contact between the fiber is uniform and perfect(Direct contact of carbon fiber) in a composite sample.
- CFRP composite has orthotropic and carbon fibers property is anisotropic.
- Matrix is insulative in nature and strongly holds carbon fibers.

With considered above assumption and model parameter shown in table 1, we have a dimension of lamina are-  
 Length of a sample (L) = 10.0 mm  
 Width of a sample (B) = 1.0 mm  
 Thickness of a sample (T) = 1.0 mm



**Fig. 7. Sample model of CFRP composite**

To investigate electrical conductivity of CFRP composite, consider following property of carbon fiber (AS-4 carbon fiber) and dimension[8].

**1. Model Parameter**

Description	Parameter	Typical value (A S4 Carbon Fiber)
Fiber Volume Fraction.	VF	50%
Fiber Packing Order.	Pack order	Hexagonal, Square, Random,
Sample Length.	Spac_Len	10.0 mm.
Sample Width.	Spac_Wid	1.0 mm.
Sample Thickness.	Spac_Th	1.0 mm.
Processing Pressure.	Pressure	8 bar (Autoclave)
Fiber Waviness Term.	Beta	~300.
Average Fiber Diameter.	Fib_Dia	7 $\mu$ m.
Elastic Modulus of Fiber.	E_Mod	231GPa.
Electrical Resistivity of Fiber.	Fiber_Rho	$1.7 \times 10^{-5} \Omega$ m.

**a) For longitudinal direction**

When potential difference applied across the length of composite shown in fig. 3, Resistance of carbon fiber characteristic length ( $L_c$ ) is given from equation 3 using model parameter in table 1-

$$R_a = \rho \frac{4 * L_c * V_f}{\pi * d_f^2}$$

**$R_a = 927.64 \Omega$ .**

Contact resistance ( $R_c$ ) between neighbor carbon fiber due to waviness from equation (4) using model parameter in table 1-

$$R_c = \frac{\rho * V_f}{\sqrt{\frac{3P * (d_f + d_s) * d_s * \beta * d_f}{2E}}}$$

**$R_c = 5.76 \Omega$ .**

From fig. 1, it assumed that when potential difference applied across its length, current entered from resistance  $R_a$  (fiber resistance) and then passed through node it divided in  $R_b$  (fiber resistance) and  $R_c$  (contact resistance).

**$R_a = R_b = 927.64 \Omega$ .**

and

**$R_c = 5.76 \Omega$ .**

Hence, According Kirchoff's law from fig. 1 for equivalent resistance at node 1-

$$R_{n1} = R_a + \frac{1}{\frac{1}{R_b} + \frac{1}{R_c}} \dots\dots(5)$$

**$R_{n1} = 933.36 \Omega$ .**

Total number of fibers in each load carrying layer ( $N_f$ ) is given as-

$$N_f = \frac{B}{d_f + d_s} \dots\dots(6)$$

**$N_f = 47.619 \sim 48$**

The average number of contact point( $N_{cl}$ ) in each fiber is given by-

$$N_{cl} = \frac{L}{d_s * \beta} \dots\dots(7)$$

**$N_{cl} = 2.3 \sim 3$**

Number of a node in each fiber layer( $N_c$ ) is given as-

$$N_c = N_{cl} * N_f \dots\dots(8)$$

**$N_c = 144$**

All the node in one layer is connected in parallel resistor

network system because when current is pass from 1<sup>st</sup> node, then it will go 2<sup>nd</sup>, 3<sup>rd</sup> and so on.

Hence total resistance of 1<sup>st</sup> load carrying layer ( $R_{L1}$ ) in longitudinal direction is given as –

$$R_{L1} = \frac{\text{Equivalent resistance at a node } 1(R_{n1})}{\text{Number of node in each fiber layer } (N_c)}$$

**$R_{L1} = 6.48 \Omega$ .**

There are four load carrying layer ply arrangement which are parallel to each other, Hence

$$\text{Total resistance } (R_{Com})_L \text{ of CFRP composite} = \frac{R_{L1}}{4}$$

**$(R_{Com})_L = 1.62 \Omega$ .**

Resistance of the composite,  $(R_{Com})_L$ , in terms of  $\rho_{Com}$ , composite resistivity, area of the composite,  $A_c$ , and distance between the electrodes (length of sample for this case)  $L$ .

$$R_{(Com)} = \frac{\rho_{Com} * L_v}{A_c} \dots\dots(9)$$

Here

$$A_c = 10^{-6} \text{ m}^2, L_v = 10^{-2} \text{ m.}$$

**$\rho_{(Com)} = 1.62 \times 10^{-4} \Omega \text{m}$ .**

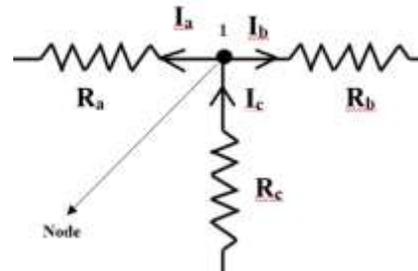
Relationship in between conductivity and resistivity of material is given as-

$$\sigma_{(Com)} = \frac{1}{\rho_{(Com)}} \dots\dots(10)$$

**$\sigma_{(Com)} = 6.17 \times 10^3 (\Omega \text{m})^{-1}$ .**

**b) For transverse direction**

When potential difference applied across its width direction as shown in fig. 5, A current is first passed through contact resistance( $I_c$ ) between the fiber and when it reached to node point 1 it divided in two part ( $I_a$  and  $I_b$ ) as shown in fig. 8.



**Fig. 8. Direction of flow of current for transverse potential difference**

In such arrangement of resistance shown in fig. 7 resistance  $R_a$  and  $R_b$  are parallel to each other and resultant of  $R_a$  and  $R_b$  is in series.

From assumption, In case of transverse direction, Fiber resistance and contact resistance are same as in longitudinal direction calculated above. An average number of a contact point ( $N_{cl}$ ) in each layer and total number of fiber in each load carrying layer ( $N_f$ ) are also same which is calculated in case of longitudinal direction resistance.

Hence, On applying Kirchoff's law in fig. 7 for equivalent resistance at node 1

$$R_{n1} = R_c + \frac{1}{\frac{1}{R_a} + \frac{1}{R_b}} \dots\dots(11)$$

$$R_{n1} = 469.58 \Omega.$$

All node in one layer connected in parallel, Hence total resistance of 1<sup>st</sup> load carrying layer ( $R_{T1}$ ) in transverse direction is given as –

$$R_{T1} = \frac{\text{Equivalent resistance at a node 1}(R_{n1})}{\text{Number of node in each fiber layer } (N_c)}$$

$$R_{T1} = 3.2 \Omega.$$

There are four load carrying layer ply arrangement which are parallel to each other, Hence

$$\text{Total resistance}(R_T) \text{ of CFRP composite} = \frac{R_{T1}}{4}$$

$$(R_{Com})_T = 0.815 \Omega.$$

Using equation 9, resistivity in transvers direction given as-  
Here,  $A_c = 10^{-5} \text{ m}^2$ ,  $L_v = 10^{-3} \text{ m}$ .

$$\rho_{(Com)T} = 8.15 \times 10^{-3} \Omega \text{m}.$$

From equation 10, conductivity is given as-

$$\sigma_{(Com)T} = 122.69 (\Omega \text{m})^{-1}.$$

**c) For thickness direction**

In case of fiber waviness model, it has an advantage over common model is that it provided symmetry between transverse direction and thickness direction model. The only difference in between transverse and thickness direction model is that number of layer in transverse direction model is equal to total number of fiber in thickness direction model and number of fiber in transverse direction model is equal to number of later in thickness model.

From assumption, In case of transverse direction, Fiber

resistance and contact resistance are same as in longitudinal direction calculated above. Average number of contact point ( $N_{cl}$ ) in each layer and total number of fiber in each load carrying layer ( $N_f$ ) are also same which is calculated in case of longitudinal direction resistance, Hence

Total resistance in thickness direction of CFRP composite –

$$(R_{Com})_{\text{Thickness}} = 0.815 \Omega \text{ and}$$

$$\sigma_{(Com)\text{Thickness}} = 122.69 (\Omega \text{m})^{-1}.$$

**4. RESULT AND DISCUSSION**

Calculating conductivity on the basis of model parameter given in table 1. Conductivity in longitudinal direction is high as compared to transverse and through thickness direction because in longitudinal direction intrinsic property of carbon fiber are responsible for conductivity of CFRP composite. Result for model ply in shown in table 2.

**2. Calculated electrical conductivity for model parameter.**

Longitudinal Conductivity ( $\Omega \text{m})^{-1}$	Transverse Conductivity ( $\Omega \text{m})^{-1}$	Through Thickness Conductivity ( $\Omega \text{m})^{-1}$
$6.17 \times 10^3$	122.69	122.69

Model considered in this paper is successfully simulate the conductivity in all direction for model parameter shown in table 1. Conductivity in longitudinal direction is 50 times higher than transvers and thickness direction.

**CONCLUSION**

Numerical simulation using waviness model for four layer CFRP composite with  $[0^0/0^0/0^0/0^0]$  stacking sequence was simulated waviness model and taking curing pressure as a parameter. Following conclusions were made:

1. Waviness model has successfully simulated the conductivity in all three directions of composites. Conductivity was very high in longitudinal direction. But, it was very in transverse and in through thickness direction.

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**International Journal of Engineering Research in Mechanical and Civil Engineering  
(IJERMCE)**

**Vol 4, Issue 5, May 2019**

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