

Optimized Modeling of Vehicle Induced Vibrations

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Abstract:— Vibrations induced in bridges due to vehicles have a considerable impact on it. To study this effect profoundly the more accurate formulation to a sprung mass model of a vehicle running on bridge is stated in this paper. Numerical solution for finding responses of vehicle modeled as sprung mass model considering fundamental mode of vibration, using Newmark-beta method is explained. Dynamic responses, displacement, velocity and acceleration, obtained from this model are compared to the responses from moving load and moving mass model. Contribution of first few modes is analogized for different models. Vibration in bridge due to vehicle is function velocity of vehicle. Effects of this parameter on responses are evaluated on an old bridge.

Index Terms :- Fundamental Mode, Moving Load Model, Moving Mass Model, Sprung Mass Model.

I. INTRODUCTION

The vehicle traveling through a roadway bridges, creates the dynamic effect on the bridge. These effects can be appreciably larger than the static effects considered due to moving load. The dynamic responses due to moving vehicles on bridges have been the topic of research from past few years. For obtaining responses vehicles can be modeled as Moving Load, Moving Mass and Sprung Mass. Moving load model is the simplest one, less time consuming but the least accurate amongst the three. The most sophisticated model is sprung mass and gives more accurate responses.

Chopra [2] has given the solution for the moving load problem, which is the simplest one, using modal equations. Biggs [1] have extended this method for moving load and sprung mass model which includes the inertial masses and dynamic properties of vehicles and are more accurate. Yang and Yau [6] has given numerical method (Newmark-beta) to solve the bridge-vehicle coupled equation. Further, Yang and Lin [5] has compared the results for the mentioned three types of model.

The research on sprung mass model was done using some assumptions. One of them is finding the responses without considering the effective mass as shown in equation (6). Researchers generally consider the mass M_n as given in equation (5) instead of M_{eff} . Roadway bridges generally are designed for life period more than 50 yrs. As years goes by, speed of vehicles increases due to technological advancement. Because of this increased velocity of moving loads dynamic effects also increases significantly. Research on such effects of increased speed

on old bridges is inadequate. For this research paper, design of old bridge is taken from IRC:SP-13(1998). Responses is computed for various speeds and compared with each other.

II. METHODOLOGY

The vehicle is modeled as sprung mass. All the dynamic responses for the model are computed with and without considering effective moving mass. Bridge-vehicle coupled Equations obtained for both the cases were solved using Newmark-Beta Method. Comparative study of the dynamic responses for different velocities has been done in this paper. Comparison of contribution of different modes is also done for different velocities of the vehicle

III. MODELING OF BEAM

The bridge is modeled as Euler Bernoulli beam with its end as simply supported. It's mass per unit length being m_b , stiffness k_b , modulus of elasticity E , moment of inertia I , length L , damping C_b , first modal frequency ω_{b1} . The vehicle running on bridge is modeled as sprung mass with mass m_v , running with the velocity v , and its stiffness and damping are K_v and C_v . Fig.1 shows the bridge vehicle system to be analyzed. The modal interaction equation for n th mode considering damping is given by

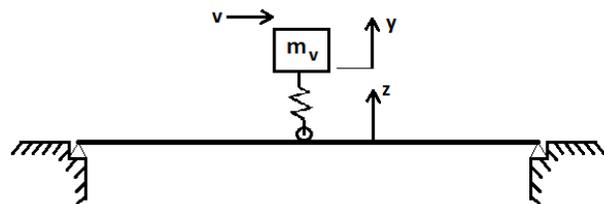


Fig.1 Sprung Mass Model

$$M_n \ddot{q}_n(t) + C_n \dot{q}_n(t) + K_n q_n(t) = P_0 \sin \frac{n\pi vt}{L} \quad (1)$$

Where P_0 is load due to vehicle. For sprung mass model considering inertial effect load P_0 can be written as

$$P_0 = m_v(\ddot{g} - \ddot{y}) + C_v(\dot{y} - \dot{z}) + k_v(y - z) \quad (2)$$

$$y = q \sin \frac{n\pi vt}{L}, \dot{y} = \dot{q} \sin \frac{n\pi vt}{L}, \ddot{y} = \ddot{q} \sin \frac{n\pi vt}{L} \quad (3)$$

Value of y and \ddot{y} is given considering only first mode.

Equation for vehicle can be written as

$$m_v \ddot{z} + C_v(\dot{y} - \dot{z}) + k_v(y - z) = 0 \quad (4)$$

Substituting equation (2) and (3) in equation (1) and (4) and arranging these equation in matrix form, a coupled equation as in (6) is obtained for the system. This equation is to be solved using Newmark-Beta method to obtain dynamic responses of both bridge and vehicle. Modal parameters used in these equations can be given as in Chopra (2003)

$$M_n = \frac{m_b L}{2}, \quad K_b = \frac{n^4 \pi^4 EI}{2L^3} \quad (5)$$

$$M_{eff} = \frac{m_b L}{2} + m_v \sin^2 \frac{\pi vt}{L} \quad (6)$$

$$\begin{bmatrix} M_{eff} & 0 \\ 0 & m_v \end{bmatrix} \begin{Bmatrix} \ddot{q} \\ \ddot{z} \end{Bmatrix} + \begin{bmatrix} C_b + C_v \sin^2 \frac{\pi vt}{L} & -C_v \sin \frac{\pi vt}{L} \\ -C_v \sin \frac{\pi vt}{L} & C_v \end{bmatrix} \begin{Bmatrix} \dot{q} \\ \dot{z} \end{Bmatrix} + \begin{bmatrix} K_b + K_v \sin^2 \frac{\pi vt}{L} & -K_v \sin \frac{\pi vt}{L} \\ -K_v \sin \frac{\pi vt}{L} & K_v \end{bmatrix} \begin{Bmatrix} q \\ z \end{Bmatrix} = \begin{Bmatrix} m_v \sin \frac{\pi vt}{L} \\ 0 \end{Bmatrix} \quad (7)$$

IV. NUMERICAL VERIFICATION

IRC:SP-13(1998) have given design for small bridges. Investigation of the above method is done using the design of simply supported bridge with span 8m . Elastic modulus $E = 25000 \text{ N/mm}^2$, moment of inertia $I = 0.0176 \text{ m}^4$, mass per unit length $L = 1985 \text{ kg/m}$, maximum mass of vehicle as stated in IRC6 $m_v = 11630 \text{ kg}$ (11.4 tons), $K_v = 500000 \text{ N/m}$. Dynamic responses of midpoint of beam is calculated for both the cases using Newmark-Beta. For velocity 25 m/s Fig. 1, Fig. 2, Fig 3 shows the comparison of maximum displacement, velocity and acceleration response with and without considering effective mass and differences of response in percentage due to effective mass.

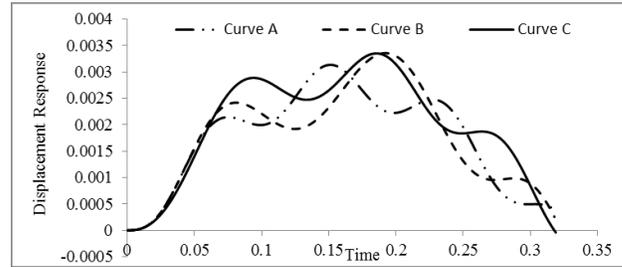


Fig. 2 Displacement Response for v=25m/s

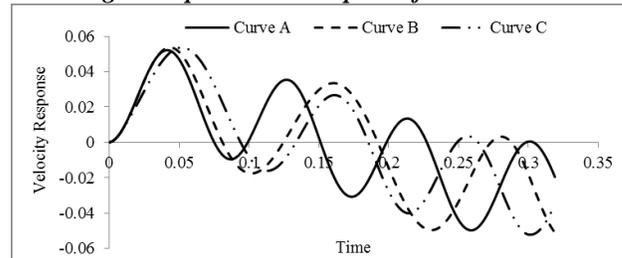


Fig. 2 Velocity Response for v=25m/s

Curve A is of responses obtained not considering M_{eff} , Curve B obtained considering M_{eff} , and Curve C obtained considering contribution of all modes.

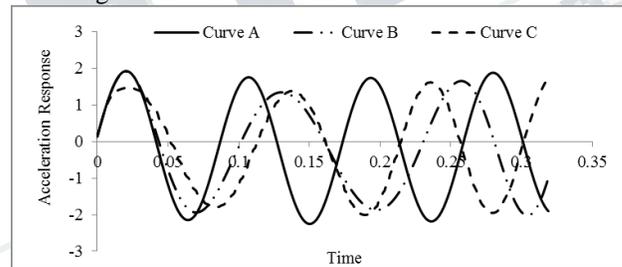


Fig. 2 Acceleration Response for v=25m/s

From the graph it can be observed that there is much differences in the responses calculated from both types of equation. There is appreciable difference in maximum values of displacement response. Both the equations are giving different number of peaks for the same type of response. Further investigation has been carried out by comparing maximum values of all types of dynamic responses for different values of velocities as shown in Table 1, Table 2 and in Table 3. Percentage difference of displacement response from both the equation for velocity 20 m/s was computed to be approximately 15%.

From the tables it can be noted that for some velocities such as 20 and 32 m/s there is significant difference in displacement response which cannot be neglected. However, for other two types of response this difference is lower. Responses with considering all modes have pronounced effect for some velocity of vehicle For further investigation of contribution of modes all the

Table 2 Comparison of Maximum Displacement Response at Midpoint

Velocity m/s	Maximum Displacement		
	Considering M_{eff}	Not Considering M_{eff}	Considering M_{eff} and all modes
5	0.0029	0.0028	0.0029
10	0.003	0.0029	0.0032
16	0.0031	0.003	0.0031
20	0.0034	0.0029	0.0033
25	0.0033	0.0031	0.0034
32	0.0036	0.0032	0.0034

Table 2 Comparison of Maximum Velocity Response at Midpoint

Velocity m/s	Maximum Velocity		
	Considering M_{eff}	Not Considering M_{eff}	Considering M_{eff} and all modes
5	0.011	0.011	0.012
10	0.022	0.021	0.028
16	0.035	0.034	0.036
20	0.045	0.042	0.044
25	0.056	0.052	0.054
32	0.071	0.066	0.066

Table 3 Comparison of Maximum Acceleration Response at Midpoint

Velocity m/s	Maximum Acceleration		
	Considering M_{eff}	Not Considering M_{eff}	Considering M_{eff} and all modes
5	0.4	0.4	0.45
10	0.76	0.79	1.18
16	1.25	1.26	1.2
20	1.5	1.55	1.42
25	1.8	1.93	1.73
32	2.24	2.45	2.3

responses were plotted for velocity 10 m/s. Fig. 5, Fig. 6, Fig. 7 shows displacement, velocity and acceleration response respectively. From these figures it can clearly be marked that for velocity 10 m/s contribution of modes other than fundamental are noteworthy.

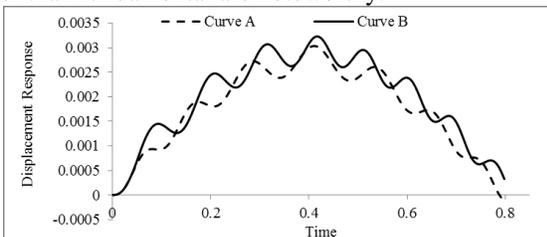


Fig. 5 Displacement Response for v=10m/s

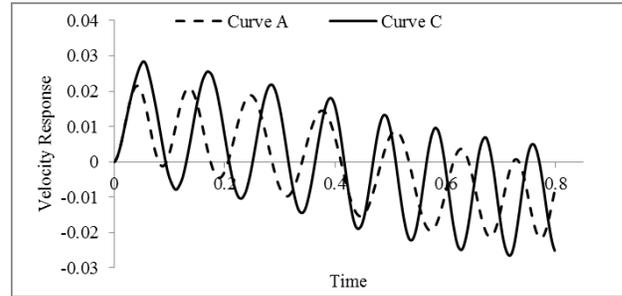


Fig. 2 Velocity Response for v=10m/s

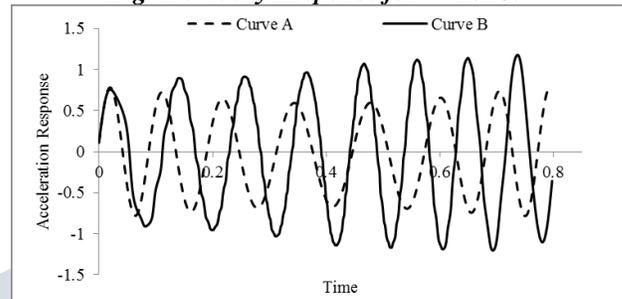


Fig. 2 Acceleration Response for v=10m/s

Curve A represents response computed considering M_{eff} , and curve B represents Response obtained considering contribution of all modes.

CONCLUSION

For small bridges all the dynamic responses have significant variance with change in velocity. Responses increases as speed increases. Effects of velocities, contribution of all modes and effective mass is shown through table for maximum midpoint responses and through graph of responses for certain velocities. From the comparison of responses it can be concluded that for some velocities negligence of effective mass will lead to under estimation of response. Contribution of modes also is sensitive to velocity. For some velocities their contribution is negligible and for some velocities it is considerable.

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