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Investigation of In-Plane and Out-of-Plane Buckling of Tied Arch

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***Abstract:*— This paper conducts the study to investigate the in-plane and out-of-plane buckling of tied-arch bridge. Several analytical models of tied arch with varying span to height ratio and hanger numbers are studied. It has been shown from results that the recommended values of European standards (Eurocode 3: 1993-2) for predicting the in-plane and out-of-plane buckling length factor of tied arch bridges is not rational and often leads to unsafe design. The in-plane buckling length factor not only depends on the stiffness of main arch girders but also depends on the stiffness of deck and vertical ties. Furthermore, it is difficult to predict the first mode of buckling, It largely depends on the effectiveness of a bracing system. To predict the actual critical buckling force it is of paramount importance to know the first buckling mode. In last it is concluded that tied arches are complex structures owing to its various parameters which are difficult to formulate in terms of codal procedures, Therefore more emphasis should be given in actual modeling**

***Key word:*-- Buckling length factor, In-plane buckling, Out-of-plane buckling, Tied-arch bridges.**

I. INTRODUCTION

The arches have widespread use in infrastructure projects owing to its effective load carrying mechanism. While designing the arch the primary step is to calculate its buckling load due to high compression forces. The subject of arch buckling has been extensively dealt by many researchers. But there is few research in the field of tied-arch buckling with flexible hangers, Though tied arches are used extensively in bridges.

Most of the international codes do not give guidelines to predict elastic buckling of tied arch. The one which provides is Eurocode-3 in form of graphs of in-plane and out-of-plane buckling and the buckling length factors are not rational sometimes even leading to over-predicting of critical buckling normal force.

Ju (2003) gave the effective lengths for arches through statistical analysis. Arie and Charalampos (2008) in their paper came out with a conclusion that buckling length factor depends upon arch in-plane stiffness which leads to unsafe design. Palkowski (2012) gave buckling length factors which are in close agreement with Eurocode 3. But none of the papers discuss the bending stiffness of deck and nature of buckling mode. Even after the effective cross-bracing of arches, the global out-of-plane buckling mode can be the first mode

II. THEORY AND EUROCODE PROVISIONS

A. Eigenvalue buckling analysis

To carry out the buckling analysis an eigenvalue problem is
formulated and solved. Eq. 1 is an eigenvalue problem [1].

$$([K]+\lambda_{cr}[K_\sigma]_{ref})\{\delta D\}=\{0\} \qquad (1)$$

Where [K] is the conventional stiffness matrix, Matrix [kσ] is a function of element geometry and displacement. The smallest root λcr defines the smallest level of external load for which there is buckling which is to be multiplied by applied load P. The eigenvector {δD} associated with λcr is the
buckling mode.

$$\{P\}_{cr}=\lambda_{cr}\{P\}_{ref} \qquad (2)$$

Where {P}cr is the critical buckling load. From buckling analysis the critical buckling load obtained is applied to the model which gives the critical buckling normal force Ncr at support in the arch.

$$N_{cr}=\frac{\pi^2 EI_y}{(\beta S)^2} \qquad (3)$$

Where, E = young's modulus of elasticity, Iy = the second moment of area of cross-section of arch rib, S = half arch length and β is the buckling length factor for an arch which is obtained by

$$\beta=\left(\frac{\pi}{S}\right)\sqrt{\frac{EI_y}{N_{cr}}} \qquad (4)$$

B. Eurocode provisons

The buckling of tied-arch is dealt in Annex D.3, part 2. For in-plane buckling a graph is given as shown in Figure

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Vol 2, Issue 3, March 2017

1. The buckling length value β can be obtained for a given arch rise and a number of hangers. The critical buckling normal force Ncr at support is given by Eq. 3. Where m is the number of hangers, f is the arch rise, l is the length, p is the spacing and q is load intensity.

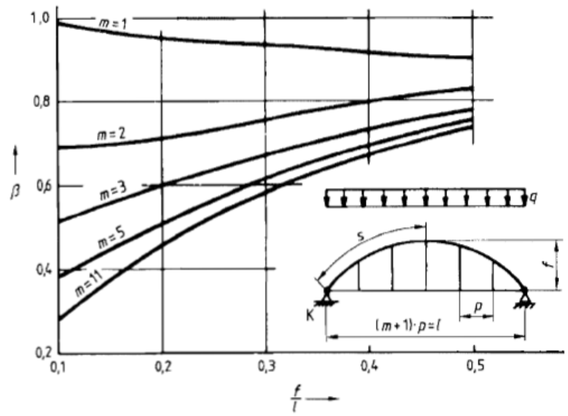

Figure 1[2]. Buckling factor β.

The out-of-plane buckling is verified by a stability check of the end portal. The buckling factor β is obtained from Figure 3 by using the geometry shown in Figure 2.

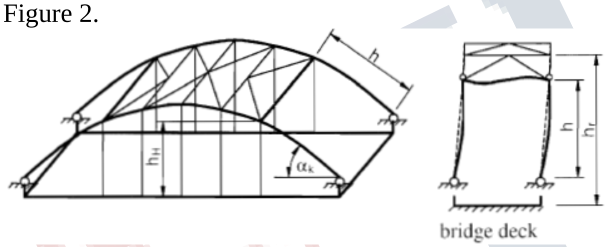

Figure 2[2]. Buckling of portal for arches.

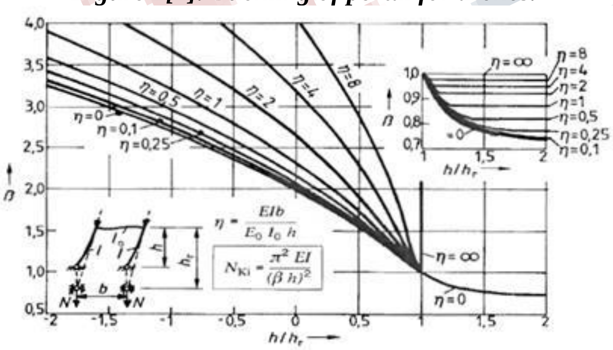

Figure 3[2]. Buckling length factors β for end portal.

Where h is the minimum distance between support and first cross bracing between arch ribs, αk is the angle at support between arch rib and deck, hr is the mean of all lengths hH of the hangers multiplied by 1 / sin αk , Io is the second moment of area of arch cross bracing and I is the second moment of inertia of arch rib along minor axis.

III. MATHEMATICAL MODELLING

SAP 2000v14 is adopted to conduct Elastic buckling and static analysis for tied arch bridges. A total of 25 arch models are analyzed for different arch rise ratio f / L (0.1, 0.2, 0.3, 0.4, 0.5) where f is the arch rise and L is the length of the arch. Each arch rise has five different cable configuration, m =1,2,3,5 and 11 cables (For modeling the same cross-section details from [3] are used as shown in Figure 4).

- The main arch rib is the rectangular box section with a height of 4000 mm, 3000 mm width and thickness of 60 mm.
- Tie member is the rectangular box section with a height of 2300 mm, 3000mm width and thickness of 25 mm.
- Crossbeam is also a rectangular cross-section with height of 2300m, a width of 1400 mm and thickness of 20mm.
- Arch cross beam is hollow circular cross-section with 2000 mm diameter and a thickness of 20 mm.
- Figure 4[3]. Different cross-se

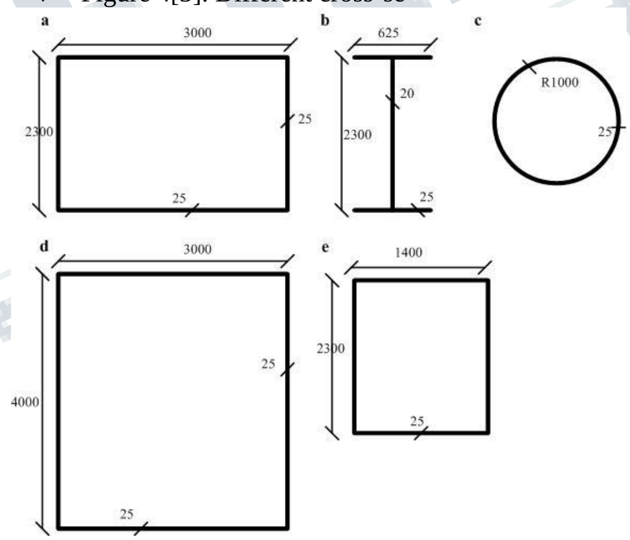

Figure 4[3]. Different cross-sections used : a) Tie; b) Stringer; c) Arch; d) Arch bracing; e) Cross girder.

The longitudinal girders are I-section cross-section having top and a bottom flange width of 650 mm, a flange thickness of 25mm, a web thickness of 20mm and total depth of I section is 1800mm.

The cross-section as shown and discussed are modeled as beam elements. The cables are modeled as cable element in SAP 2000 with a diameter of 120mm. The concrete deck above longitudinal girder is 450mm thick which is modelled as a composite beam with I-section longitudinal girder. The total length of a bridge is considered

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Vol 2, Issue 3, March 2017

as 300 m which remain constant for every model and cross section width is 25m.

For modelling two types of steel material and a concrete material have been defined. Members other than cable and slab, steel is used with yield strength fy = 345 Mpa, Elastic modulus E = 200GPA, poison ratio υ = 0.3, for steel cables fy = 975Mpa, E = 200 GPA, υ = 0.3 and a concrete of fck = 35 MPa , E = 29580, υ = 0.3 is used for deck slab.

Appropriate boundary conditions are applied. To model support, four end nodes are selected. Two nodes at one of the end have been restrained from translations, the in-plane rotation is allowed. The remaining nodes at other end are allowed both in longitudinal translation and in-plane rotation. A schematic diagram is shown in Figure 5.

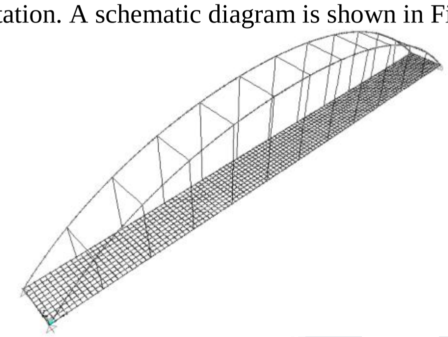

Figure 5. A schematic diagram of Tied-arch bridge

IV. RESULTS

A. In-plane buckling

As mentioned earlier a total of 25 arches with five different cable configurations for five different arch rise ratio have been analyzed. At first, the deck slab has been kept as 450 mm thick then the whole analysis is repeated for 250 mm thick deck slab. The first in-plane buckling mode is considered as shown in Figure 6 to calculate normal critical buckling force at support and then the β value is obtained from Eq. 4

The results are summarized and shown in Table 1, βEN is the buckling length from Eurocode 3, βM1 is the buckling length factor obtained from mathematical model for 450 mm thick deck slab, βM2 is the buckling length factor obtained from mathematical model for 250 mm thick deck slab, δ1 and δ2 are the percentage change in βM1 and βM2 with respect to βEN.

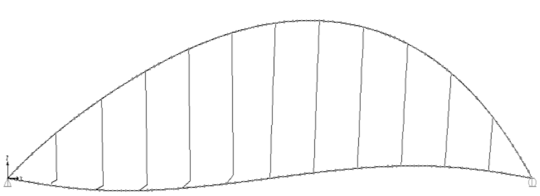

Figure 6. First in-plane buckling mode

Table 1. Buckling length factors from analysis.

Model	β_{EN}	β_{M1}	β_{M2}	δ_1 (%)	δ_2 (%)
0.1_1	0.993	1.289	1.380	29.81	38.92
0.1_2	0.692	0.949	1.019	37.07	47.28
0.1_3	0.515	0.755	0.812	46.67	57.75
0.1_5	0.384	0.534	0.574	38.89	49.24
0.1_11	0.283	0.398	0.502	40.82	77.42
0.2 1	0.953	1.197	1.282	25.54	34.43
0.2_2	0.712	0.856	0.920	20.22	29.23
0.2_3	0.600	0.673	0.723	12.11	20.54
0.2_5	0.509	0.522	0.527	2.56	3.68
0.2_11	0.456	0.476	0.484	4.39	6.01
0.3_1	0.936	1.083	1.159	15.78	23.90
0.3_2	0.756	0.749	0.805	-0.94	6.38
0.3_3	0.672	0.654	0.662	-2.73	-1.43
0.3_5	0.619	0.618	0.627	-0.27	1.20
0.3_11	0.579	0.588	0.598	1.62	3.28
0.4_1	0.762	0.934	1.001	22.60	31.39
0.4_2	0.915	0.777	0.790	-15.00	-13.66
0.4_3	0.800	0.742	0.755	-7.23	-5.60
0.4_5	0.734	0.719	0.732	-2.02	-0.17
0.4_11	0.669	0.700	0.712	4.58	6.40
0.5_1	0.738	0.958	1.018	29.85	37.90
0.5_2	0.695	0.892	0.915	28.45	31.77
0.5_3	0.907	0.877	0.899	-3.33	-0.90
0.5_4	0.831	0.862	0.882	3.74	6.20
0.5_11	0.780	0.748	0.861	-4.14	10.34

Following are the observations from above results.

i. It can be observed that for an arch rise ratio of 0.1 Eurocode overestimates the Ncr which is increasing as we reduce the number of cables.

ii. For an arch rise ratio of 0.2 and above with a number of cables 5 or more the Eurocode values are best applicable as δ1 is within 5%. Most of the tied arch bridges have cable numbers more than 5.

iii. By comparing β1 and β2 it is observed that with a decrease in lateral and transverse stiffness of deck the buckling length factor increases or Ncr decreases, that concludes buckling factor not only depends upon arch ribs in-plane stiffness [4] but also upon the stiffness of deck. Since the Eurocode β values do not depends upon deck stiffness it may lead to unsafe design depending upon the deck stiffness.

B. Out-of-plane buckling

First out-of-plane buckling mode is considered as shown in Figure 7 and its corresponding Eigenvalue is used to calculate normal critical out-of-plane buckling force. The results are summarized and shown in Table 2. Ncro is the normal critical buckling force at support as per clause D.3.4 of EN:1993 part 2. Ncro,M1 and Ncro,M2 are the critical out-of-plane buckling force at support from a mathematical model for deck slab of thickness 250 mm and 450 mm.

Figure 7. First out of plane buckling mode.

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Vol 2, Issue 3, March 2017

Table 2. Out-of-plane normal critical buckling force at support.

Model	N_{cro} (KN)	$N_{cro,M1}$ (KN)	$N_{cro,M2}$ (KN)	δ_1 (%)	δ_2 (%)
0.1_1	305694	222505	222014	27.21	27.37
0.1_2	345100	225408	225106	34.68	34.77
0.1 3	406070	229308	228994	43.53	43.61
0.1_5	379879	232700	232379	38.74	38.83
0.1_11	392647	235508	235341	40.02	40.06
0.2_1	268299	180722	180013	32.64	32.91
0.2_2	287122	189167	188629	34.12	34.30
0.2_3	357203	199160	198555	44.24	44.41
0.2_5	331234	209226	208526	36.83	37.05
0.2_11	319298	218408	217406	31.60	31.91
0.3_1	178498	133295	132539	25.32	25.75
0.3_2	199688	144243	143570	27.77	28.10
0.3_3	238312	155450	154724	34.77	35.07
0.3_5	215983	168899	168026	21.80	22.20
0.3_11	234357	183347	182289	21.77	22.22
0.4_1	133297	93089	92267	30.16	30.78
0.4_2	141995	103240	102372	27.29	27.90
0.4_3	168704	111652	110764	33.82	34.34
0.4_5	153019	122586	121607	19.89	20.53
0.4_11	168704	136992	135769	18.80	19.52
0.5_1	88341	52588	51174	40.47	42.07
0.5_2	86556	61972	60393	28.40	30.23
0.5_3	116234	67075	65536	42.29	43.62
0.5_4	99761	72251	70839	27.58	28.99
0.5_11	117614	78070	76874	33.62	34.64

Following observations are made from Table 2

i. The codal procedure given in Eurocode part-3 over-estimates the out-of-plane critical buckling force compared to mathematical model which may lead to unsafe design.

ii. Unlike in-plane buckling, the change in deck stiffness does not have a pronounce effect over out-of-plane normal critical buckling force.

C. Effect of different types of bracing over buckling strength

Three different types of bracing have been considered as shown in Figure 8 are defined as a,b and c having the same cross-section as defined earlier for arch bracing. In this present study, the effect of different bracings over buckling strength have been studied. Five different arches with f / L = 0.1, 0.2, 0.3, 0.4 and 0.5 for 11 cables in one side each are analyzed. The results are summarized in Table 3 and 4.

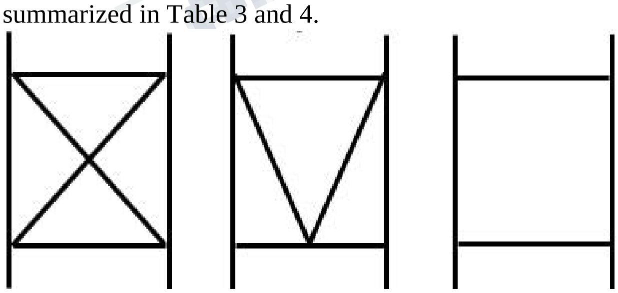

Figure 8. Bracing type a,b and c.

Table 3. First buckling mode type for bracing type a,b and c.

Model	1st buckling mode		
	a	b	c
0.1_11	Out-of-plane	In-plane	In-plane
0.2_11	Out-of-plane	In-plane	In-plane
0.3_11	Out-of-plane	In-plane	In-plane
0.4_11	Out-of-plane	In-plane	In-plane
0.5_11	Out-of-plane	In-plane	In-plane

Table 4. Normal critical buckling force Ncr for 1st mode of buckling for bracing type a,b and c.

Model	N_{cr} (KN)		
	a	b	c
0.1_11	222505	1103637	1104741
0.2_11	180722	637733	638664
0.3_11	133295	342678	341661
0.4_11	93089	191309	190301
0.5_11	52588	100098	99574

It can be observed from Table 3 the governing mode is the out-of-plane buckling mode for type a and in-plane buckling mode for type b and c. From Table 4 which shows the corresponding normal critical buckling force, it can be said that in-plane buckling mode is the preferred mode of buckling because of higher normal critical buckling force than out-of-plane buckling. Therefore type b and c should be preferred over type a, owing to the high stiffness of b and c against out-of-buckling. The increase in stiffness is due to the fact that the number of members are increased from 11 in type a to 31 in type b and c. Even if the number of bracings are increased for type a the out-of-plane buckling could have been resisted However the codal provisions of Eurocode part 3 are silent over different types of bracing and their effect over buckling strength. This may lead to an over conservative and uneconomical design. Because of large difference in normal critical buckling force for in-plane and out-of-plane buckling.

CONCLUSION

From the study following observations are derived

i. The in-plane buckling length factor also depends upon the longitudinal stiffness of deck apart from arch rise ratio and

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Vol 2, Issue 3, March 2017

cable numbers. This fact is ignored in Eurocode part 3 designing through it may lead to deficient design.

ii. The out-of-plane normal critical buckling force by considering the end portal as given in Eurocode part 3 may predict results on the higher side as compared to the mathematical model.

iii. The bracing has a significant effect over buckling strength that can change the fundamental mode of buckling. Which is unfortunately not present in Eurocode part 3 provision. Depending upon the aesthetics or any other reason designer may choose any particular type of bracing and can design without knowing the actual mode of failure, which may produce a very conservative and uneconomical design.

iv. From results, it is observed codal provisions are not rational and it is difficult to predict actual buckling load. The tied-arch bridges are complex structures as it depends on many factors like in-plane and out-of-plane stiffness of arch, the stiffness of cross bracing, The longitudinal and transverse stiffness of deck, cable numbers and many others which is difficult to formulate in terms of codal procedure. Therefore more emphasize should be given in modeling which can predict result with sufficient accuracy.

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