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Particle Swarm Based Crack detection in Structure using One Dimensional PZT Patch Model

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***Abstract:*— The concept of damage identification scheme for single/multiple crack in beam is presented, based on one dimensional Piezo Electric ZirconiumTtitanium (PZT) patch with beam model. Hybrid element constituted of one dimensional beam element and a PZT sensor is used with reduced material properties. The Finite Element Model (FEM) model of hybrid element constituted of beam and a PZT sensor is formulated based on One Dimensional approach. The reduced piezoelectric stiffness, coupling and dielectric properties are used for the PZT patch sensor. The Crack Identification is posed as an inverse problem whereby unknown crack parameters(depth and location) are identified by minimizing Objective function of the mean square of the deviation between predicted and measured voltage response obtained from the PZT patches. A non-classical heuristic Particle Swarm Optimization (PSO) is used to minimize this objective function. Using fracture mechanics concept, crack is modeled as a hinge which provides an additional flexibility to the element and cracked stiffness matrices are formulated based on FEM procedure. In the proposed method, PZT patches are attached to either end of the concerned beam. This model is convenient and simple for identification of beams. The signals are polluted with 5% Gaussian noise to simulate experimental data. The feasibility of the proposed approach is proved by numerical studies on a beam with single and double crack per element.**

Index Terms - **Crack Identification, PZT Patches, Particle swarm Optimization, Voltage matching**

I. INTRODUCTION

Structural health monitoring is the process in which directly assessing the state of structural health using nondestructive approach. Damage occurs during service because of the operational cyclic loading, aging, mechanical vibration, changing of ambient conditions, shocks and chemical attack. Hence, early detection of structural stiffness degradation and damage is very important in current scenario. System Identification (SI) or structural identification of structures is typically an inverse problem is an alternate technique whereby structural parameters such as stiffness and cracks are identified from the input excitation as well as output responses. Doebling et al.[1]reviewed the literature on detection, location and characterization of structural damage with techniques that determine the change in structural vibration response. Yang and Wang [2] developed a new damage detection method based on the concept of natural frequency vector (NFV) and natural frequency vector assurance criterion (NFVAC) and verified by both numerically and experimental examples. Viola et al [3] is developed interpolation functions of a cracked Timenshenko beam element based on Hamilton principle, with crack sections represented as elastic springs. A nondestructive evaluation procedure for identification of the magnitude and location of the structure based on experimentally measured frequency and mode shape was formulated. Douka et al [4] investigated the effect of two transverse cracks on the mechanical impedance of a double-cracked cantilever beam by analytically and experimentally. It was found there was significant change of anti-resonance frequency as well as natural frequency and that information was used for crack identification. Sinha et al [5] developed a multi crack model in Euler-Bernoulli beam based on small modification of the local flexibility in the vicinity of crack. There, crack models were incorporated in to the finite element model of the structure and estimated the crack location and size using model updating from the experimentally measured modal data. Philips et al [6] identified the damage in beam using combined measurement of displacement modes from accelerometer and with fiber Bragg grating (FBG) sensors. Cibu Varghese and Shankar [7] was applied the combined instantaneous power flow balance and conventional acceleration matching concept for the sub structural identification of multiple crack parameters of beam.

The high sensitivity, reliability and electro mechanical coupling property of PZT has gained significant attention for potential application as sensors for structural health monitoring. Bendary and Raid [8] developed a one dimensional integrated beam element using hermite cubic and Lagrangian interpolation function,and carried out static and dynamic analysis. Zemcik and Sadilek [9] developed one dimensional hybrid PZT element based on the Euler Bernoulli beam using bilinear Lagrangian interpolation polynomial for electric potential. In the present proposed method, integrated beam structure with PZT sensor which is introduced into the finite element model of the structure has been used for the

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Vol 2, Issue 3, March 2017

direct identification of single/multiple damage at various location in structure.

II. CONSTITUTIVE EQUATIONS OF PZT

Piezo Electric Materials are used to transform the mechanical displacement into an electrical field (voltage potential) in which case the piezoelectric material acts as a sensor(direct effect), and its converse effect acts as an actuator. The constitutive equations for the transversely isotropic piezo electric medium which define the interaction between the stress(σ)strain(ε) and electric displacement(D), and electric field € in the form

$$\sigma_j = C_{jk}\,\varepsilon_k - e_{jm}\,E_m \qquad (1)$$

$$D_l = e_{lj}\,\varepsilon_k + \in_{lm}\,E_m \qquad (2)$$

Where Cjk , Єlm and elj(j,k=1,…,3) are elastic, dielectric and piezo electric coupling coefficient respectively. The transverselypoled piezoelectric material is bonded/embedded in host structure.The Euler-Bernoulli beam theory is applied for one dimensional beam with piezo electric patch, which neglects the shear effect. The model is assumed to be plane stress and width in the y direction is stress free. The polarization axis z is aligned with the thickness direction of the beam. Apply these condition into three dimensional constitutive equation of piezo electric material and it is reduced in the one dimensional form.

$$\begin{bmatrix}\sigma_x\\ D_x\end{bmatrix}=\begin{bmatrix}C & -e\\ e & \in\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ E_z\end{bmatrix} \qquad (3)$$

Here ,C, e and Є are reduced elastic, dielectric and piezo electric coupling coefficient respectively. This reduced property is used for further numerical study of one dimensional beam with PZT patch in MATLAB.

III. ANALYTICAL AND FEM FORMULATION

The beam element is based on the Euler-Bernoulli theory and element has two nodes. The two independent polynomial are used for interpolation of mechanical and electrical field variable. First hermit cubic polynomial is used for the interpolation of mechanical quantities of vertical displacement (w) and rotation (θ) as shown in the Figure 1.

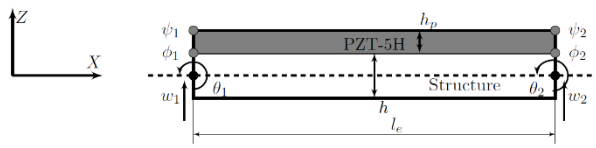

Figure 1. One dimensional beam element:PZT and Supporting Structure sharing Common Nodes

Let the vertical displacement (w) is approximated across the length as

$$w(x) = a_0 + a_1x + a_2x^2 + a_3x^3 = N_w w^e w(x,z) = B_w w^e \qquad (4)$$

where *Nw* are the shape interpolation functions and *Bw* is the strain displacement matrix consisting of derivatives of shape functions.

The electric potential is ϕ considered as a function of the thickness and the length of the beam. Hence, let Langrangian bi-linear function be estimated for the interpolation as

$$\emptyset_{(x,z)} = a_4 + a_5x + a_6z + a_7xz = \phi_{\emptyset}\emptyset^e$$

$$E(x,z) = B_{\emptyset}\emptyset^e \qquad (5)$$

Where Bϕ is the Electrical field-potential matrix consisting of derivative shape functions.

IV. EVALUATION OF ELEMENTAL MATRICES.

$$M_{ww}^e = \int N_w^T \rho N_w\, dv$$

$$K_{ww}^e = \int B_w^T C B_w\, dv \qquad (6)$$

$$K_{w\emptyset}^e = \int B_w^T e B_{\emptyset}\, dv$$

$$K_{\emptyset\emptyset}^e = \int B_{\emptyset}^T \in B_{\emptyset}\, \mathrm{dv}$$

$$\begin{bmatrix}M_{ww}^e & 0\\ 0 & 0\end{bmatrix}\begin{Bmatrix}\ddot{w}^e\\ \ddot{\emptyset}^e\end{Bmatrix} + \begin{bmatrix}C_{ww}^e & 0\\ 0 & 0\end{bmatrix}\begin{Bmatrix}\dot{w}^e\\ \dot{\emptyset}^e\end{Bmatrix} + \begin{bmatrix}K_{ww}^e & K_{w\emptyset}^e\\ K_{w\emptyset}^T & K_{\emptyset\emptyset}^e\end{bmatrix}\begin{Bmatrix}w^e\\ \emptyset^e\end{Bmatrix} = \begin{Bmatrix}F^e\\ Q^e\end{Bmatrix} \qquad (7)$$

Where M_{ww}^e is the mass matrix K_{ww}^e is the stiffness matrix corresponding to the mechanical degree of freedom $K_{w\emptyset}^e$ is the stiffness matrix dueto electromechanical coupling , $K_{\emptyset\emptyset}^e$ is the stiffness matrix due to the electrical degrees of freedom alone, F^e mechanical load vector and Q^e electrical charge load vector. The matrices of equations are then assembled to obtain the global dynamic system equation and solve this equationto get corresponding electrical (voltage)and mechanicalquantities(acceleration,velocityand displacement).

ISSN (Online) 2456-1290
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Vol 2, Issue 3, March 2017

V. PARTICLE SWARM OPTIMIZATION (PSO) ALGORITHM

A heuristic optimization technique referred to as PSO is used here which mimicsthe social behavior of swarms. It was first proposed by James and Ebehart[9].Heuristic methods are preferred over calculus based methods due to their robustnessand ability to attain the global optima. It imitates the social behaviour of aswarm of birds. Each bird tends to follow the general swarm direction in searchof the target (food), but it has a component of its own intelligence and memory(i.e. local search) which influences its action. Each bird is visualized as a `particle'which approaches the target (i.e the global optima) with a `velocity'. Thenumber of particles (i.e. population) and their initial random positions are specified. As the particles progress to the global optima through many generations,their current position is updated using two parameters Gbest which represent thehistorically best co-ordinate of all the particles in the population and Pbest; thehistorically best co-ordinate of the i^{th} particle. The equations giving the velocityv and position x for the i^{th} particle in the k + 1 generation are given by

$$v_i(k+1) = \varphi(k)v_i(k) + \propto_1 [\gamma_{1i}(P_{best,i} - x_i(k)] + \alpha_2[\gamma_{2i}[(G_{best} - x_{i(k)})]$$
$$x_i(k+1) = x_i(k) + v_i(k+1) \qquad (8)$$

where *i* is the particle index, *k* the discrete time index, v the velocity of thei^{th} particle,*x* the position of the i^{th} particle/present solution. Here $\gamma 1$and $\gamma 2$represents two random numbers between 0 and 1, φ is an inertia term uniformlydecreasing from 0.9 to 0.4 with generations $\propto_1$and and α_2 are two accelerationconstants set to two[10]. Several studies have pointed out the superiority ofPSO algorithm over the more conventional heuristic algorithms such as GeneticAlgorithm (GA) for inverse problem applications [11, 12].

VI. FEM FORMULATION OF CRACKED BEAM STRUCTURE

A Beam element with single crack

The Finite Element Formulation of single cracked model is presented here. Here it is combined with One-dimensional PZT patch element for inverse problem. The FEM model is shown in Figure 2

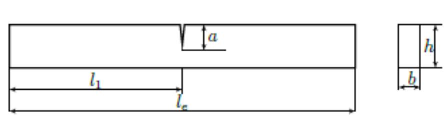

Figure 2. Beam element with two cracks

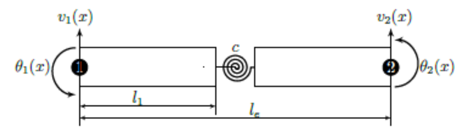

Figure 3. Equivalent model of cracked beam element

Let 'l_e 'be the length of element, 'l_1 'be the location of the crack from the left node, 'a' be the crack depth measured from the top of the crack section. The element has two nodesand with two degrees of freedom (transverse displacement and rotation) in eachnode. Since it is discontinued at the cracked plane, two different polynomials are

$$v_1(x) = a_1 + a_2x + a_3x^2 + a_4x^3; 0 \le x \le l_1$$
$$v_2(x) = a_5 + a_6x + a_7x^2 + a_8x^3; l_1 \le x \le l_e \qquad \textbf{(9)}$$

The boundary conditions are applied at cracked sections where the elastic hinge is located as follows

$$v_1(0) = Y_1;\ \theta_1(0) = \Theta_{1;} v_2(l_e) = Y_2; \theta_2(l_e) = \Theta_{2;}$$
$$v_1(l_1) = v_2(l_1);\ v_2''(l_1) = \frac{1}{K_c}((\theta_2(l_1) - \theta_1(l_1))$$
$$v_1''(l_1) = v_2''(l_1);\ v_1'''(l_1) = v_2'''(l_1)$$

Where $K_c$is the flexibility coefficient, EI is the flexural stiffness and 'c' torsional flexibility. From FEMprocedure, the stiffness and mass matrix of the cracked element can be obtained.

A Beam element with two cracks

The finite element formulation of double cracked element model is an extension f above single cracked model. In the current study, itusedmulti crack identification using one dimensional PZT patch model.

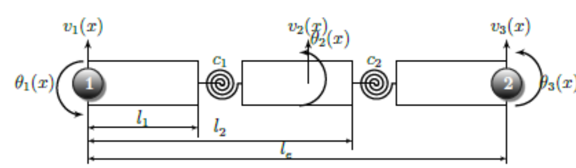

Figure 4. Equivalent model of double cracked beam element

Three different polynomials are assumed for the field variables of this element since it has three different segments. Here, it is discontinued at the two cracked plane, therefore three different polynomials are used. Similar to the previous section with different boundary conditions and FEM procedure, stiffness and mass matrix are derived for double crack per element.

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Vol 2, Issue 3, March 2017

VII. CRACK IDENTIFICATION USING PZT PATCH

In the proposed method, a patch is attached to either end of the beam memberwhose crack parameters have to be identified. The time domain based approach is used and voltage history of the patch is used as the main response quantity in the identification of structural stiffness and crack parameters. A few experimentallymeasured voltage potential responses are measured from PZT patches. Theestimated voltage potential is obtained from mathematical model and for exact identification, ϕ_e has to match with experimentally measuredresponses, ϕ_m.In this method, experimental responses are simulatedfrom a known numerical model and polluted with Gaussian noise of zero meanand a certain standard deviation. Using Particle Swarm Optimization (PSO)algorithm the following fitness (objective) function is minimized, which is thesum of squares of deviations between the measured and estimated voltage.

$$f = \frac{\sum_{i=1}^{M}\sum_{j=1}^{L}(\emptyset^{m}(i,j)-\emptyset^{e}(i,j))^{2}}{ML} \qquad (10)$$

The superscripts m and e denote measured and estimated responses for fitness evaluation, *M* is the number of measurement sensors used and *L* is the number of time steps. Ideally, it must be minimized to zero, but usually it approaches avalue close to zero.

VIII. NUMERICAL EXAMPLES AND RESULTS

A Single Crack Identification (Depth and Location) using PZT Patch

In this case, damage detection by the proposed method of voltage matching is applied to determine the magnitude (Depth) and location simultaneously for a single crack in the

Beam structure. The fixed-fixed beam is divided into five Euler Finite elements with an open edge crack is at a distance of 275.6 mm from the left fixed end as shown in Figure 5 . The absolute normalized crack location measured from the left end of the beam is *(λ (l/L)* =0.53. Inthe finite element model of the fixed-fixed beam, the crack is located in element 3. In this study four different cases of normalized crack depth(η_n=*a/h*) areconsidered viz;0.03, 0.05,0.1 and 0.5 respectively. The normalized crack location is measured from theleft node of the element 3 is 0.6. In this study, as the first case two PZT patches with size PZT:5% i.e.(26x50x1 mm^3PZT) is bonded on either side of the structure as shown in Figure

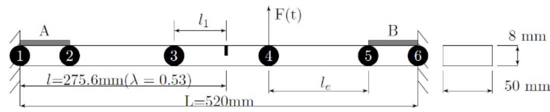

Figure 5. Finite element Model of single cracked beam element with PZT Patches

First, the displacement, velocity and acceleration time history data arecalculated for each nodal point using Newmarks method with constant time step of0.002 Sec in .01 Sec using the impulse force of 5 N in the vertical (upward) direction at node 4. Using the displacement response history, voltage responses are measuredthrough two PZT patches.The numerically calculated voltages are polluted by artificially adding Gaussian white noise with zero mean and standard deviation of 5% to simulate experimental errors.It is assumed that the mass and flexural rigidity (EI) are known. Crack magnitude and location are set as the unknown variable for each element and thus there are tenunknown variables in this problem. Thus the optimizations variables to be identified arenormalized crack depth (η_n=*a/h*) and location in 5 elements. The experimentallymeasured voltage response of PZT patches is required for the fitness evaluation. The meansquare error(MSE) between measured and predicted voltage response at PZT patchesare minimized by particle swarm optimization.The lower and upper bound for PSOoptimization to identify crack magnitude are set at zero and one. Similarly the lower andupper bound to identify normalized crack location are set as zero and le respectively.
In this PSO parameters are to 100 particles(swarm size) and 500 generations as perprevious literature. The identified crack parameters (depth and location) using PZT:5% are shown in the Table:1. The convergence plot of different patch length is also shown in the Figure 6. From the plot, it can be seen that convergence is minimum for the PZT:20% , but PZT:5% is sufficient for the sufficient accurate identification .

Table 1:Identified values of normalized damage magnitude and location using PZT:5%

Normalized Crack Depth	NormalizedCrack Depth(identified)		Normalized location(identified)	Crack
	Noise Free(error)	5%Noise(error)	Noise Free(error)	5%Noise (error)
0.03	0.031(-3.33)	0.0327(-9)	0.532(-0.377)	0.5421(-2.2)
0.05	0.0489(2.2)	0.053(-6)	0.5307(-0.14)	0.5405(-1.98)
0.1	0.0996(0.4)	0.104(-4)	0.5295(0.094)	0.5375(-1.41)
0.5	0.4999(.02)	0.499(0.2)	0.5249(0.019)	0.535(-0.94)
	Mean absolute error 1.48975	4.8	Mean absolute error 0.1575	1.6325

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Vol 2, Issue 3, March 2017

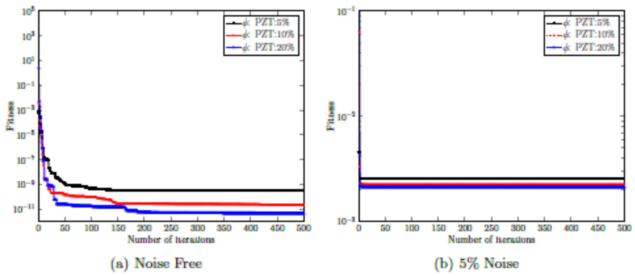

Figure 6. Convergence plot for η_n=0.03 BMultiple crack Identification (double crack per element)using PZT Patch

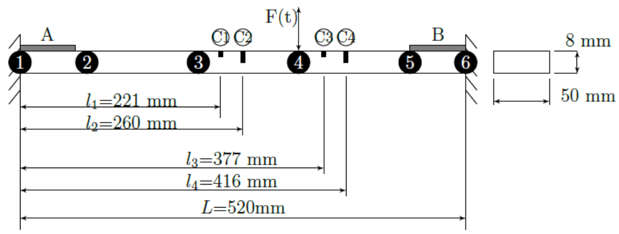

Figure 7. Finite element Model of doublecracked beam element with PZT Patches

The same beam in example 1 is considered for the crack identification with doublecracked beam element. Here, four open edge crack of depths 0.4 mm, 4 mm,0.8 mm and 2 mm are assumed to be located at a distances of 221 mm,260 mm, 377 mm and 416 mm respectively from the fixed end. In this study, the beamis divided into five elements; two open edge cracks C1, C2 are placed in element3 and C3, C4 are placed in element 4 as shown in Figure 5. Here PZT:5% is used based on the previous numerical results. As the first case two PZT patches for the size PZT:5% are bonded on either side of the structure as shown in Figure 5. The identified structural parameters (crack and location) as shown in Figure7.

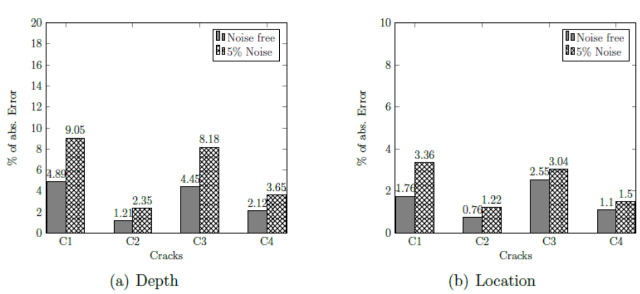

Figure 8. Absolute Error in identified parametrs in Fixed-Fixed beam with PZT:5%

IX. CONCLUSIONS:

This study presents a multiple crack detection scheme in beam structures by minimization of measured and estimated voltage responses of PZT patches.A one dimensional hybrid beam element with PZT sensor bonded to beam represented by reduced material properties is used.Numerical examples show that, the smallest patch length PZT:5% (5% of the length of beam) under study is even sufficient for effective and accurate crack parameter identification. The proposed method estimated the crack depth error in the range of 0.66% to 9% and location error in the range of 0.11% to 4% (noise free and noisy case).

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