

# Dynamic Response of SDOF System: A Comparative Study Between Newmark's Method and IS1893 Response Spectra

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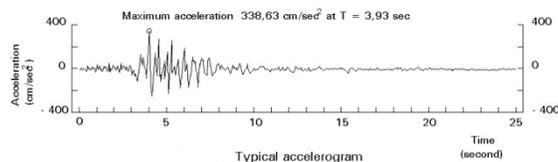
**Abstract:**— There are two basic approaches to numerically evaluate the dynamic response of the structures. The first approach is numerical interpolation of the excitation and the second is numerical integration of the equation of motion. In the present study, the second approach (Newmark's method) is used to find the response of a set of 40 structures idealised as SDOF system. The 40 structures are chosen in such a way that their time period varies from 0.1 to 4 sec. The response of all 40 structures is evaluated for 8 past Indian earthquakes. A program is developed in MATLAB to find the peak response of each structure for each earthquake using Newmark's method. Average peak response of each structure under eight earthquakes is calculated. A response spectrum is developed as average peak response versus time period of 40 structures.

As a second part of the work, the response of 40 structures is evaluated as per response spectra given in IS1893: 2002 based on time period of the structure. A comparative study is made between Newmark's response and the response as per IS1893 response spectra. The study reveals that the responses calculated using IS1893 response spectra and Newmarks method are not similar for all the earthquakes.

**Key word:**-- Response spectrum, Newmarks method, Matlab.

## I. INTRODUCTION

Response spectra provide a very handy tool for engineers to quantify the demands of earthquake ground motion on the capacity of buildings. Data on past earthquake ground motion is generally in the form of time-history recordings obtained from instruments placed at various sites that are activated by sensing the initial ground motion of an earthquake. The amplitudes of motion can be expressed in terms of acceleration, velocity and displacement. The first data reported from an earthquake record is generally the peak ground acceleration (PGA) which expresses the tip of the maximum spike of the acceleration ground motion (Fig 1).



**Fig 1 Typical accelerogram**

Although useful to express the relative intensity of the ground motion (i.e., small, moderate or large), the PGA does not give any information regarding the

frequency (or period) content that influences the amplification of building motion due to the cyclic ground motion. In other words, tall buildings with long fundamental periods of vibration will respond differently than short buildings with short periods of vibration. Response spectra provide these characteristics. Picture a field of lollipop-like structures of various heights and sizes stuck in the ground. The stick represents the stiffness ( $K^*$ ) of the structure and the lump at the top represents the mass ( $M^*$ ). The period of this idealized single-degree-of-freedom (SDOF) system is calculated by the equation:

$$T = 2\pi (M^* / K^*)^{1/2}$$

If the peak acceleration ( $S_a$ ) of each of these SDOF systems, when subjected to an earthquake ground motion, is calculated and plotted with the corresponding period of vibration ( $T$ ), the locus of points will form a response spectrum for the subject ground motion. Thus, if the period of vibration is known, the maximum acceleration can be determined from the plotted curve. When calculating response spectra, a nominal percentage of critical damping is applied to represent viscous damping of a linear-elastic system, typically five-percent.

## II. LITERATURE:

Japanese Society of civil engineers. (2000) stated that Response spectra can also be used in assessing the response of linear systems with multiple modes of oscillation (multi-degree of freedom systems), although they are only accurate for low levels of damping. Modal analysis is performed to identify the modes, and the response in that mode can be picked from the response spectrum. This peak response is then combined to estimate a total response. The result is typically different from that which would be calculated directly from an input, since phase information is lost in the process of generating the response spectrum. The main limitation of response spectra is that they are only universally applicable for linear systems. Response spectra can be generated for non-linear systems, but are only applicable to systems with the same non-linearity, although attempts have been made to develop non-linear seismic design spectra with wider structural application.

Bharat Bhushan Prasad, (2010) stated that PGA, PGV, PGD are the most common and easily recognizable time domain parameters of the strong ground motion. Typically, the ground motion records termed seismograph or time histories have recorded acceleration (these records are termed accelerograms) for many years in analogue form and more recently, digitally. The maximum amplitude of recorded acceleration is termed as peak ground acceleration (PGA) – peak ground velocity (PGV) and peak ground displacement (PGD) are the maximum respective amplitudes of velocity and displacement. The focus is usually on peak horizontal acceleration (PHA) due to its role in determining the lateral inertial forces in structures.

Pankaj Agarwal and Manish shrikande, (2006) stated that the seismic force generated in structures varies according to their dynamic properties even though they stand on the same ground and are subjected to the same seismic motion. The parameters representing the properties of structures and reflected directly in seismic motion, are the natural period and damping constant. The response spectrum thus represents the relation between seismic wave shape and response value as a function of the natural period and damping constant of the structure.

## III. METHODOLOGY

In 1959, N.M. Newmark developed a family of time-stepping methods based on the following equations:

$$\dot{u}_{i+1} = \dot{u}_i + [ (1 - \gamma) \Delta t ] \ddot{u}_i + ( \gamma \Delta t ) \ddot{u}_{i+1} \quad (1a)$$

$$u_{i+1} = u_i + ( \Delta t ) \dot{u}_i + [ (0.5 - \beta) (\Delta t)^2 ] \ddot{u}_i + [ \beta (\Delta t)^2 ] \ddot{u}_{i+1} \quad (1b)$$

Equation 1 can be reformulated to avoid iteration and to use incremental quantities:

$$\Delta u_i = u_{i+1} - u_i \quad \Delta \dot{u}_i = \dot{u}_{i+1} - \dot{u}_i \quad \Delta \ddot{u}_i = \ddot{u}_{i+1} - \ddot{u}_i \quad (2)$$

$$\Delta p_i = p_{i+1} - p_i$$

While the incremental form is not necessary for analysis of linear systems, Equation 1 can be rewritten as

$$\Delta \dot{u}_i = (\Delta t) \ddot{u}_i + (\gamma \Delta t) \Delta \ddot{u}_i$$

$$\Delta u_i = (\Delta t) \dot{u}_i + \frac{(\Delta t)^2}{2} \ddot{u}_i + \beta (\Delta t)^2 \Delta \ddot{u}_i \quad (3)$$

The second of these equations can be solved for

$$\Delta \ddot{u}_i = \frac{1}{\beta(\Delta t)^2} \Delta u_i - \frac{1}{\beta \Delta t} \dot{u}_i - \frac{1}{2\beta} \ddot{u}_i \quad (4)$$

Substituting Equations (4) into Equation (3a) gives

$$\Delta \dot{u}_i = \frac{\gamma}{\beta \Delta t} \Delta u_i - \frac{\gamma}{\beta} \dot{u}_i + \Delta t (1 - \frac{\gamma}{2\beta}) \ddot{u}_i \quad (5)$$

Next, Equations (4) and (5) are substituted into the incremental equation of motion:

$$m \Delta \ddot{u}_i + c \Delta \dot{u}_i + k \Delta u_i = \Delta p_i \quad (6)$$

Obtained by subtracting equation (  $m \ddot{u}_i + c \dot{u}_i + (fs)_i = p_i$  ) from (  $m \ddot{u}_{i+1} + c \dot{u}_{i+1} + (fs)_{i+1} = p_{i+1}$  ) both specialized to linear systems with  $(fs)_i = ku_i$  and  $(fs)_{i+1} = ku_{i+1}$ . This substitution gives

$$k_{cap} \Delta u_i = \Delta p_{capi} \quad (7)$$

$$\text{Where, } k_{cap} = k + \frac{\gamma}{\beta \Delta t} c + \frac{1}{\beta (\Delta t)^2} m \quad (8)$$

And,

$$\Delta p_{capi} = \Delta p_i + (\frac{1}{\beta \Delta t} m + \frac{\gamma}{\beta} c) \dot{u}_i + [\frac{1}{2\beta} m + \Delta t (\frac{\gamma}{2\beta} - 1) c] \ddot{u}_i \quad (9)$$

With  $k_{cap}$  and  $\Delta p_{capi}$  known from the system properties  $m$ ,  $k$ , and  $c$ , algorithm parameters  $\gamma$  and  $\beta$ , and the  $\dot{u}_i$  and  $\ddot{u}_i$  at the beginning of the time step, the incremental displacement is computed from

$$\Delta u_i = \frac{\Delta p_{capi}}{k_{cap}} \quad (10)$$

Once  $\Delta u_i$  is known,  $\Delta \dot{u}_i$  and  $\Delta \ddot{u}_i$  can be computed from Equations (5) and (4), respectively, and  $u_{i+1}$ ,  $\dot{u}_{i+1}$ , and  $\ddot{u}_{i+1}$  from equation (2).

The acceleration can also be obtained from the equation of motion at time  $i+1$ ,

$$\ddot{u}_{i+1} = \frac{p(i+1) - c \dot{u}(i+1) - ku(i+1)}{m} \quad (11)$$

Below steps summarizes the time-stepping solution using Newmark's method as it might be implemented on the computer.

1. Average acceleration method ( $\gamma = 1/2$ ,  $\beta = 1/4$ )

Initial calculations

- $\ddot{u}_0 = \frac{p_0 - c \dot{u}_0 - k u_0}{m}$ .
- select  $\Delta t$ .
- $k_{cap} = k + \frac{\gamma}{\beta \Delta t} c + \frac{1}{\beta (\Delta t)^2} m$ .
- $a = \frac{1}{\beta \Delta t} m + \frac{\gamma}{\beta} c$ ; and  $b = \frac{1}{2\beta} m + \Delta t (\frac{\gamma}{2\beta} - 1) c$

2. Calculations for each time step,  $i$

$$2.1 \Delta p_{capi} = \Delta p_i + a\dot{u}_i + b\ddot{u}_i .$$

$$2.2 \Delta u_i = \frac{\Delta p_{capi}}{k_{cap}} .$$

$$2.3 \Delta \dot{u}_i = \frac{\gamma}{\beta \Delta t} \Delta u_i - \frac{\gamma}{\beta} \dot{u}_i + \Delta t \left(1 - \frac{\gamma}{2\beta}\right) \ddot{u}_i .$$

$$2.4 \Delta \ddot{u}_i = \frac{1}{\beta (\Delta t)^2} \Delta u_i - \frac{1}{\beta \Delta t} \dot{u}_i - \frac{1}{2\beta} \ddot{u}_i .$$

$$2.5 u_{i+1} = u_i + \Delta u_i , \dot{u}_{i+1} = \dot{u}_i + \Delta \dot{u}_i , \ddot{u}_{i+1} = \ddot{u}_i + \Delta \ddot{u}_i .$$

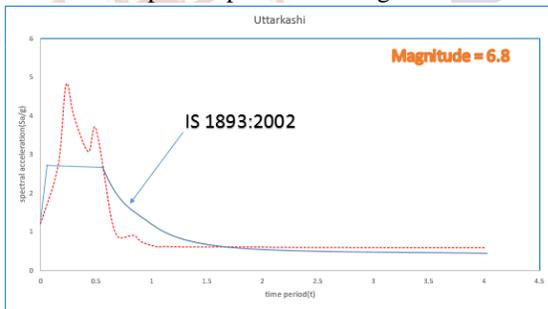
3.0 Repetition for the next time step. Replace *i* by *i*+1 and implement steps 2.1 to 2.5 for the next time step.

By using the above steps a program was developed in MATLAB software. This program is very useful to find the spectral acceleration, velocity, and displacement. A list of 40 structures was modeled in such way that their time periods varies from 0.1 to 4 sec.

**IV. RESULTS AND DISCUSSION**

The peak responses of 40 structures idealized as SDOF is calculated using Newmarks method for past 8 Indian Earthquakes. The 8 Earthquakes are of medium to low magnitudes highest of 6.8 magnitudes. Earthquakes which are taken in this study are Uttarkashi, Chamoli, Barkot, Dharmshala, Northeast India, India-Bangladesh, Xizang-India, Chamba.

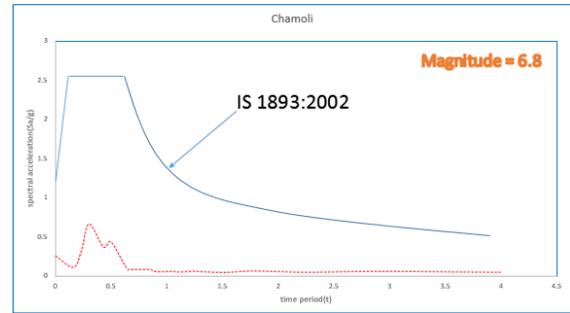
Figure 4.1 shows peak response of 40 structures due to Uttarkashi Earthquake and compared with IS 1893:2002 response spectrum. This graph shows the clear dissimilarity between response spectrum obtained by Newmarks method and IS 1893:2002 response spectrum. The peak value in Newmarks method is 4.8g where as in IS 1893:2002 response spectrum is 2.5g.



**Fig (4.1) Uttarkashi response spectrum.**

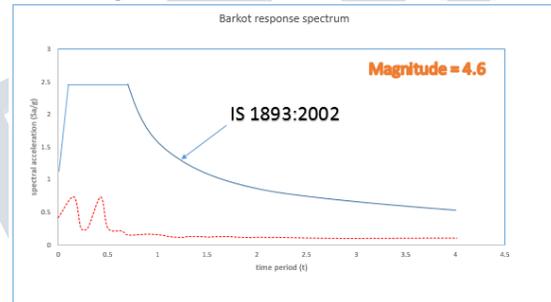
Figure 4.2 shows peak response of 40 structures due to Chamoli Earthquake and compared with IS 1893:2002 response spectrum. This graph shows the clear dissimilarity between response spectrum obtained by Newmarks method and IS 1893:2002 response spectrum. The peak value in Newmarks method is 0.65g where as in IS 1893:2002 response spectrum is 2.5g.

Figure 4.3 shows peak response of 40 structures due to Barkot Earthquake and compared with IS 1893



**Fig (4.2) Chamoli response spectrum.**

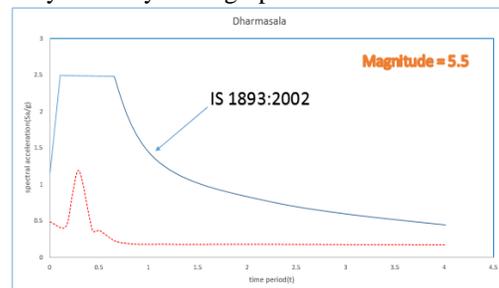
response spectrum. This graph shows the clear dissimilarity between response spectrum obtained by Newmarks method and IS 1893:2002 response spectrum. The peak value in Newmarks method is 0.7g where as in IS 1893:2002 response spectrum is 2.5g.



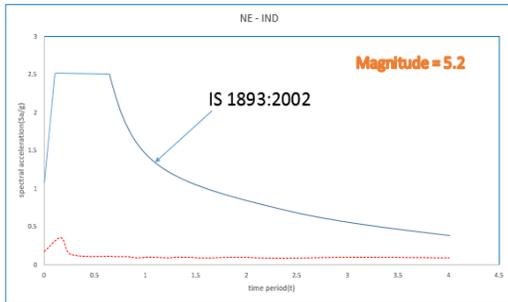
**Fig (4.3) Barkot response spectrum.**

Figure 4.4 shows peak response of 40 structures due to Dharmshala Earthquake and compared with IS1893:2002 response spectrum. This graph shows the clear dissimilarity between response spectrum obtained by Newmarks method and IS 1893:2002 response spectrum. The peak value in Newmarks method is 1.2g where as in IS 1893:2002 response spectrum is 2.5g.

Figure 4.5 shows peak response of 40 structures due to Northeast India Earthquake and compared with IS1893:2002 response spectrum. The peak value in Newmarks method is 0.3g where as in IS 1893:2002 response spectrum is 2.5g. Also from 0.2sec to 4 sec the Sa/g value is almost 0.1sec which is very less value when compared to IS 1893:2002 response spectrum, this graph shows the most dissimilarity than any other graphs.



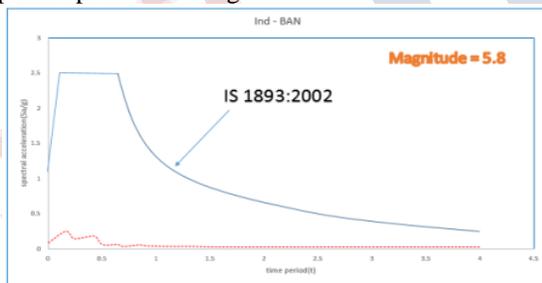
**Fig (4.4) Dharmshala response spectrum.**



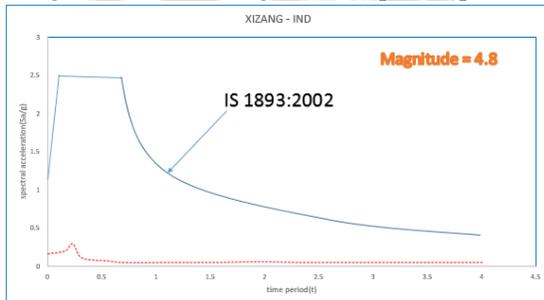
**Fig (4.5) NE India response spectrum**

Figure 4.6 shows peak response of 40 structures due to India-Bangladesh Earthquake and compared with IS1893:2002 response spectrum. This graph shows the clear dissimilarity between response spectrum obtained by Newmarks method and IS 1893:2002 response spectrum. The peak value in Newmarks method is 0.25g where as in IS 1893:2002 response spectrum is 2.5g.

Figure 4.7 shows peak response of 40 structures due to Xizang-India Earthquake Earthquake and compared with IS1893:2002 response spectrum. This graph shows the clear dissimilarity between response spectrum obtained by Newmarks method and IS 1893:2002 response spectrum. The peak value in Newmarks method is 0.2g where as in IS 1893:2002 response spectrum is 2.5g.



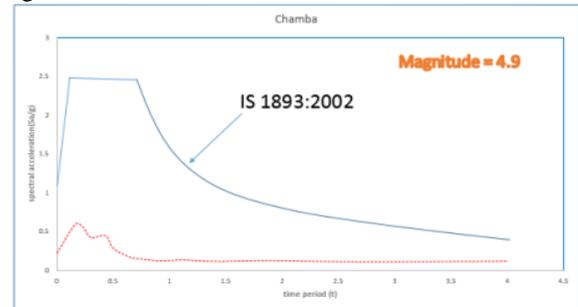
**Fig (4.6) India - Bangladesh response spectrum.**



**Fig (4.7) Xizang-India response spectrum**

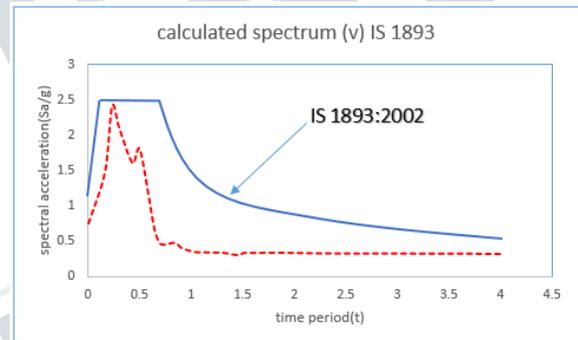
Figure 4.8 shows peak response of 40 structures due to Chamba Earthquake Earthquake and compared with IS1893:2002 response spectrum. This graph shows the clear dissimilarity between response spectrum obtained by Newmarks method and IS

1893:2002 response spectrum. The peak value in Newmarks method is 0.2g where as in IS 1893:2002 response spectrum is 2.5g.



**Fig (4.8) Chamba response spectrum**

Average peak response of 8 earthquakes for each structure is calculated and one standard deviation is added for each average peak response of structure. A response spectrum is developed for average peak responses of 40 structures. The obtained response spectrum is compared with IS1893:2002 response spectrum (Fig.4.9). The Sa/g values from time period 0.5 to 1 sec are not identical.



**Fig (4.9) calculated response spectrum and IS 1893 response spectrum.**

**V. CONCLUSION**

- 1) A comparison between the response spectrum developed by 8 Indian Earthquakes using Newmark's method and the response spectrum given in IS1893 (part1): 2002 have done.
- 2) Sa/g values of Uttarkashi has higher peak value than IS 1893.
- 3) Sa/g values from time period 0.5 to 1 sec are also not identical, this displays the response spectrum provided in IS 1893 are not so accurate for these Earthquakes.
- 4) Sa/g values dissimilarity is clearly presented in this paper.

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