

Evaluation of Stress Intensity Factor for Pressure Loading In Cryogenic Tanks in Launch Vehicles

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Abstract:-- Fracture is a failure mode due to unstable propagation of a crack due to applied stress. Fracture mechanics provides a methodology for prediction, prevention, and control of fracture in materials, components and structures subjected to static, dynamic, and sustained loads. Fracture mechanics analysis is the basis of damage tolerance design methodology. The objectives of fracture mechanics analysis are determination of (1) Stress intensity factor (K), (2) Energy release rate (G), (3) Path independent integral (J), (4) Crack tip opening displacement (CTOD), and prediction of (1) Mixed mode fracture, (2) Residual strength and (3) Crack growth life. In this paper, Stress Intensity Factor(K) is analysed for a rectangular block containing a surface crack of elliptical profile and subjected to uniform tensile pressure. The stress intensity factor is evaluated using numerical approaches like Displacement Extrapolation method, Extended Finite Element method and Finite Element method. The results obtained from all the above methods are compared with the experimental method available. Using this an efficient approach is identified and has been used for analysing the stress intensity factor of a cryogenic cylinder which is considered as a case study.

Index Terms—Cryogenic tank, Stress Intensity Factor, XFEM, Displacement extrapolation method

I. INTRODUCTION

A complete estimation of structural failures revealed that the source of failures due to cracks to be Through thickness cracks, Surface cracks, Cracks emanating from fastener holes and Corner cracks. These cracks are in three dimensional crack configurations. Two dimensional approximations to these cracked bodies as plane strain/plane stress are generally inaccurate and unacceptable. This work focuses on the determination of mode I stress intensity factors and variations along a surface crack front in structures and components. By solving 3D boundary value problems accurate solutions for stress intensity factor can be obtained. S Raju and Newman [1] studied coefficients influencing stress intensity factor for semielliptical cracks on the inside and outside region of a cylinder. The crack surfaces were subjected to uniform, linear, quadratic and cubic stress distributions. By using 3D finite element method stress intensity factor were calculated and compared with other analysis of surface cracks in cylinder. In designing for fracture components and structures, surface cracks are encountered for the three dimensional configurations. Current advances in the FEA, FEM programs and availability of fast, large computers have led to more refined fracture mechanics analysis and finite element modeling of complex cracked bodies.

II. FINITE ELEMENT MODEL DEVELOPMENT

Finite element modeling is described here as the analyst's choice of finite elements (type/shape/order), material models, meshes, constraint equations, pre and post processing options, governing matrix equations and their solution methods available in a commercial FEA program for the analysis. The anticipated finite element model includes a very fine mesh of singular isoperimetric pentahedral solid elements (SPENTA15) with length Δa and specified number NS from number of segments (NSEG) and one crack face to another along the surface crack front. A compatible mesh of regular elements (NREG) namely isoparametric hexahedral solid element (HEXA20) and isoparametric pentahedral solid element (PENTA15) are used to discretize. Fig. 1 shows hexahedral solid element of Serendipity family. It has twenty nodes. There are eight nodes located at the vertices and the others are at the mid-side points of the parent element.

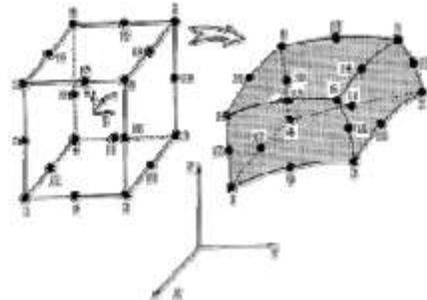


Fig.1 Quadratic order hexahedral solid element

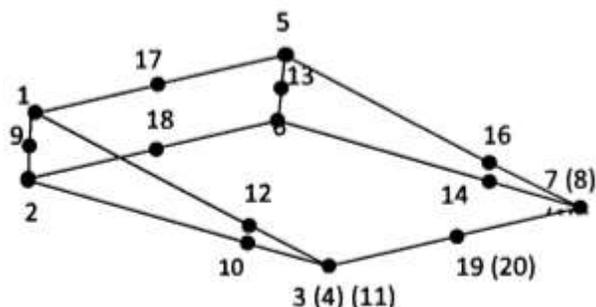


Fig.2 Singular pentahedral element

Any variable Φ is estimated over the domain of parent element domain by considering the incomplete quadratic order polynomial. Explicit shape functions $N_i(\xi, \eta, \zeta)$ ($i = 1 \dots 20$) have been derived for polynomial basis which are readily available. Using iso-parametric formulation, the parent element can be distorted to have curved or straight edges and curved flat or faces as illustrated in Fig.1. Use of a three-point Gauss quadrature formula in each of the ξ, η, ζ coordinate directions is suggested to compute the element matrices and vectors. The required weighting factors and gauss point locations are readily available. The HEXA20 element is widely used in practice and is executed in each commercial FEM system. Pentahedral solid element of the serendipity family of quadratic order (15Nodes) is shown in Fig.2. This is intended by further distorting the HEXA20 element as illustrated in Fig.2. Specifically, it includes giving way a face and constraining the nodes that are assembled to have identical degrees of freedom. This element is also called as PENTA15. One more distortion of the PENTA15 element creates SINGULAR element which is called as SPENTA15 [2]. In the curved crack front the edge with nodes 3-19-7 are located.

The mid-side nodes 10, 12, 14, 16 are advanced to quarter point locations near the crack front. The number of the SPENTA15 elements (NS) from the number of segments (NSEG) along a crack front and one crack face to the other can be increased progressively and the singular elements size Δa can be decreased. The mid-side nodes 10, 12, 14, 16 are moved to quarter point locations closer to the crack front. The number of the SPENTA15 elements (NS) from one crack face to the other and the number of segments (NSEG) along a crack front can be progressively increased and the size of the singular elements Δa can be decreased to attain convergence in computed stress intensity factors. In the present work ANSYS program is used to create finite element model. However, the required singular element are not listed in the

element library in ANSYS. Therefore the user experience and pre-processing commands is required for the creation of SPENTA15 element mesh.

III. DISPLACEMENT EXTRAPOLATION

The displacement extrapolation method is one which is based on the nodal displacements near the region of crack tip. Quarter-point Iso-parametric elements are used to obtain a good representation of the crack-tip field [4]. By shifting a quarter to the crack tip the mid-side nodes of all surrounding elements linear-elastic singularity for the stresses and strains is obtained.

The asymptotic expression for the displacement which is normal to the crack plane, for a bi-dimensional crack under mode I loading, v , is given by [9]:

$$v = k \frac{1+\nu}{4E} \sqrt{\frac{2r}{\pi}} \left[(2k+1) \sin \frac{\theta}{2} - \sin \frac{3\theta}{2} \right] + A \frac{(1+\nu)r}{E} (k-3) \sin \theta \dots \dots \dots (1)$$

Where ν the Poisson's ratio, E is the modulus of elasticity, k an elastic parameter equal to $3+4\nu$ for plane strain and $(3-\nu)$ for plane stress, A_i are parameters depending on the $(1+\nu)$ specimen geometry and load acting on it, and r and θ are the polar coordinates, defined in Fig. 3.

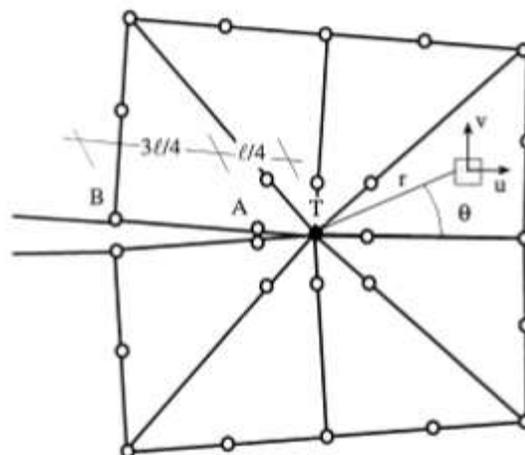


Fig.3 Quarter-point singular elements and coordinates for near crack-tip field description

When the displacement v is assessed along the crack faces ($\theta = \pm \pi$), equation (1) only contains terms in $r^{1/2}, r^{3/2}, r^{5/2}$ etc. making the extrapolation more accurate [Chan et al. 1992]. Particularizing equation (1) for nodes A and B on the singular element at the upper face of the crack we have:

$$V_A = K_I \sqrt{\frac{2}{\pi}} \frac{(1+\nu)(\kappa+1)}{4E} \sqrt{l} - \frac{A_2(1+\nu)(\kappa+1)}{12E} l^{3/2} + O(l)^{5/2} \text{ ---- (a)}$$

$$V_B = K_I \sqrt{\frac{2}{\pi}} \frac{(1+\nu)(\kappa+1)}{4E} \sqrt{l} - \frac{2A_2(1+\nu)(\kappa+1)}{3E} l^{3/2} + O(l)^{5/2} \text{ ---- (b)}$$

Where l is the length of the element side TB. Ignoring higher order terms, equations (a) and (b) can be solved for K_I and A_2 . The value of the stress intensity factor is then:

$$K_I = \frac{E'}{3(1+\nu)(\kappa+1)} \sqrt{\frac{2\pi}{l}} (8V_A - V_B) \text{ ---- (2)}$$

Or

$$K_I = \frac{E'}{12} \sqrt{\frac{2\pi}{l}} (8V_A - V_B) \text{ ---- (3)}$$

Where E' is the effective elastic modulus defined as equal to E for plane stress and $E/(1-\nu^2)$ for plane strain.

A simpler estimation of K_I can be made by means of the quarter node displacement, V_A , if terms in $l^{3/2}$ and higher are neglected in Eq. (3):

$$K_I = \frac{2E'}{(1+\nu)(\kappa+1)} \sqrt{\frac{2\pi}{l}} V_A \text{ ---- (4)}$$

The displacement field which is along the crack edge $\theta = \pi$ for a singular six-node or eight-node Iso-parametric element is a function of the nodal displacements V_A and V_B , and is given by:

$$V(r) = (4V_A - V_B) \sqrt{\frac{r}{l}} - (4V_A - 2V_B) \frac{r}{l} \text{ ---- (5)}$$

By setting $\theta = \pi$ in Eq. (1) and identifying terms with \sqrt{r} in equations (2) and (7) we obtain:

$$K_I = \sqrt{\frac{2}{\pi}} \frac{(1+\nu)(\kappa+1)}{2E'} \sqrt{r} - (4V_A - V_B) \sqrt{\frac{r}{l}} \text{ ---- (6)}$$

And the stress intensity factor now is,

$$K_I = \frac{E'}{(1+\nu)(\kappa+1)} \sqrt{\frac{2\pi}{l}} (4V_A - V_B) \text{ ---- (7)}$$

Equations (3), (4) and (7) are three different estimates of K_I based on the nodal displacements of the quarter point element located on the upper face of the crack. Due to symmetry, a similar result would be obtained for the lower face element. In the following section we evaluate the performance of these three K_I estimates.

IV. EXTENDED FINITE ELEMENT METHOD (XFEM)

The XFEM is a numerical technique which is also known as generalized finite element method (GFEM), that spreads the basic finite element method (FEM) approach for solving many differential equations with discontinuous functions. To overcome the problems which are complexity in nature, the XFEM was established. Modeling of fractures were used in earlier applications. In this method there is no need to keep the track on the crack path which is one of the advantage of this method. Many researchers has observed

that this method is used for problems such as regular meshing of micro structural features such as singularities voids, material interface etc. For issues such as for interfacing of the issue's feature into the estimation region can apparently improve accuracy and convergence rates.

V. VALIDATION FOR CHECKING RELIABILITY OF SOFTWARE

Fig indicates Vonmises stress for rectangular block subjected to crack

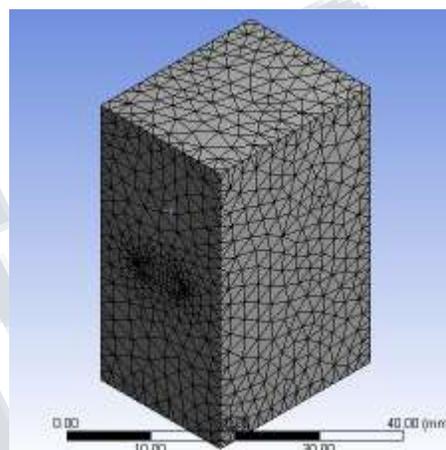


Fig.4 Meshed part of the rectangular block in ANSYS

Fig.4 shows meshed region of rectangular block in ANSYS. Fine meshing has been used.

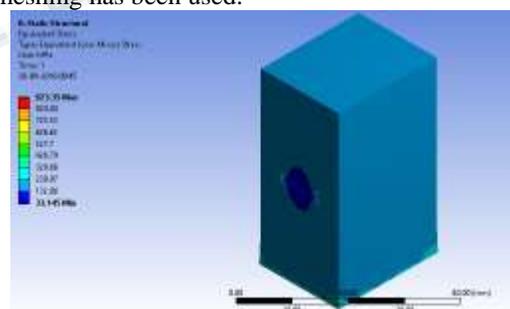


Fig.5 Vonmises stress in the rectangular block

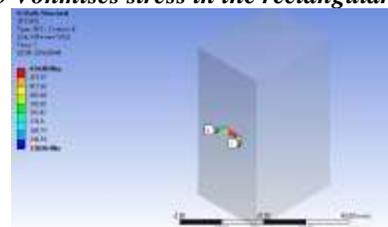


Fig.6 Stress Intensity Factor along the crack front

Fig 4 and 5 indicates Vonmises stress for the rectangular block and Stress Intensity Factor along the crack front. It is observed that maximum value of Stress Intensity Factor is 450 MPa mm^{1/2} and minimum value is 350 MPa mm^{1/2}.

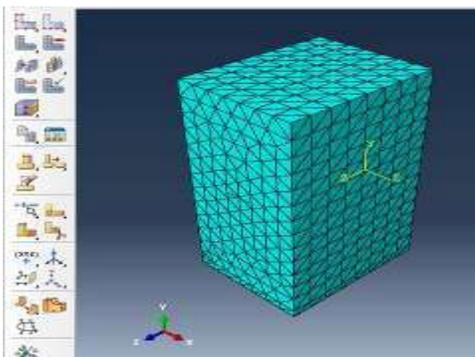


Fig.7 Meshed part of the rectangular block in ABAQUS

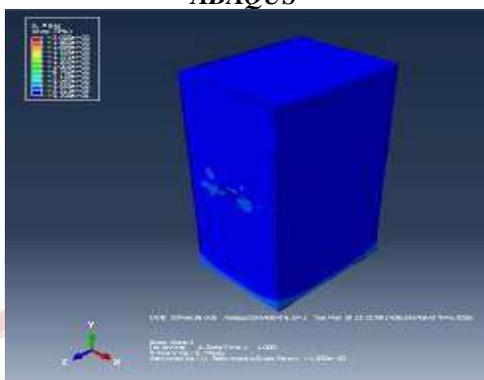


Fig.8 Vonmises stress for rectangular block in ABAQUS

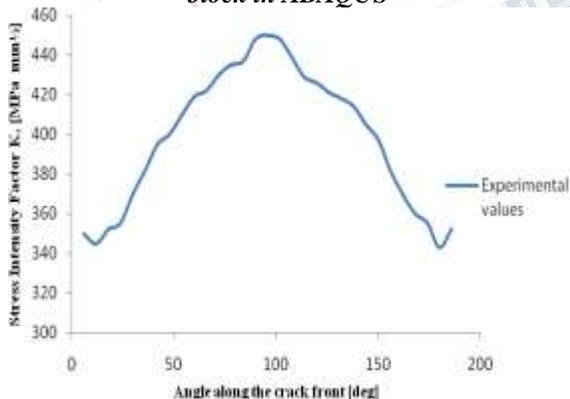


Fig.9 Experimental values of Stress Intensity Factor Vs Angle of the crack

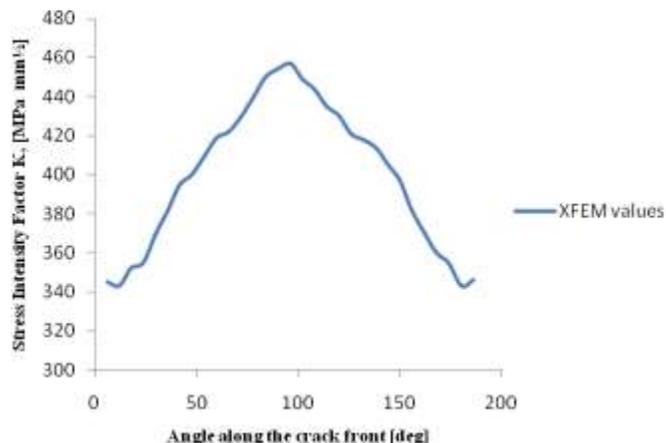


Fig.10 Stress Intensity Factor Vs Angle of the crack front obtained from XFEM

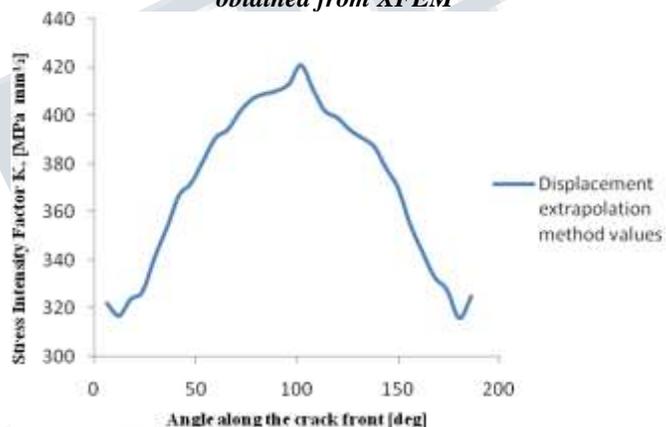


Fig.11 Stress Intensity Factor Vs Angle of the crack front obtained from Displacement extrapolation method

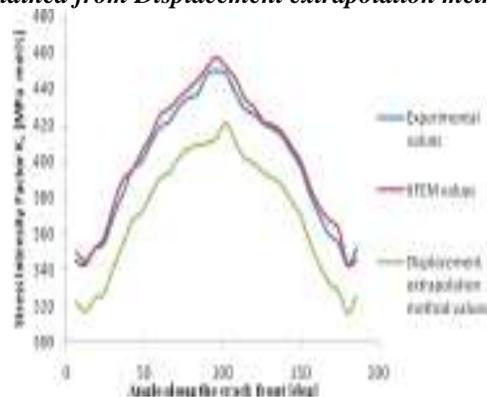


Fig.12 Comparison of Stress Intensity Factor Vs Angle of the crack front obtained from different methods.

Validation of the Stress intensity factor values of the rectangular block is made by comparing Stress intensity

values obtained from experimental method, XFEM and Displacement extrapolation method. From the validation a conclusion was made that which method is suitable for finding Stress intensity factor value. It can be seen from fig 12 that XFEM value is giving almost near value of stress intensity factor compared to that of the values obtained from that of the displacement extrapolation method.

VI CASE STUDY

A case study has been done on extracting the Stress Intensity Factor for surface cracks in a cryogenic cylinder. The mesh size has been varied to achieve refinement. The meshed cylinder is shown in fig.13. The analysis was carried out for extracting SIF for different crack lengths using XFEM and Displacement Extrapolation Method.

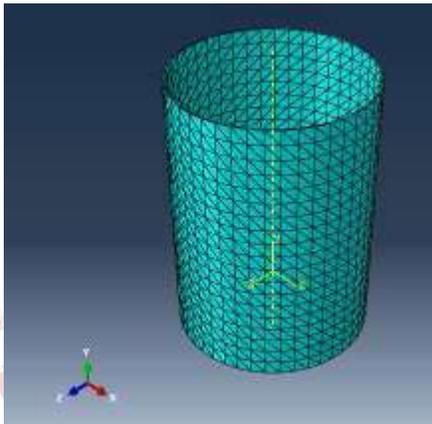


Fig.13 Meshing region of cryogenic tank in ABAQUS

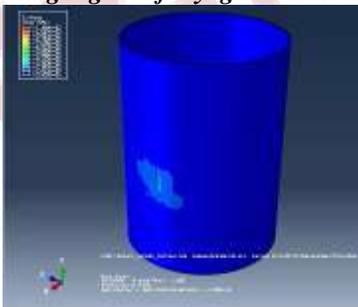


Fig.14 Vonmises stress for cryogenic tank in ABAQUS

Fig shows the vonmises stress distribution on cryogenic tank subjected to internal pressure of 2.75MPa. Stress intensity factor value obtained from XFEM method is 1205 MPa mm^{1/2} and displacement extrapolation method in 1160 MPa mm^{1/2}.

VII CONCLUSION

From this research work on crack analysis, stress intensity factors are determined using ANSYS and ABAQUS which are validated using benchmarks. Convergence is considered in this work. Based on refinement procedure we conclude that XFEM is accurate for finding Stress intensity factor value and it is observed that as plastic zone increases resistance also increases. Because of that Stress Intensity Factor also increases.

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