Thermo Elastic Stress Analysis of Isotropic Thick Beam

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Abstract: **In this paper, thermal stress analysis of a thick isotropic beam is carried out using First order shear deformation Theory (FSDT). A First order shear deformation Theory (FSDT) taking into account transverse shear deformation effect, is presented for the bending analysis of thick isotropic beam subjected to non-uniform thermal load. A new shear deformation theory for the bending analysis of simply supported isotropic beams by using thermal load on it. The simply supported thick isotropic beams analyzed for the axial displacements, transverse displacements, axial bending stress and transverse shear stress. The most important feature of the theory is that, the transverse shear stresses can be obtained directly from the use of constitutive relations, satisfying the stress free boundary conditions at top and bottom surfaces of the beam. The present theory obviates the need of shear correction factor. Governing differential equations and boundary conditions of the theory are obtained using the principle of virtual work. The results obtained for bending analysis of isotropic beam subjected to non-uniform thermal load are compared with those obtained by other theories like Elementary Theory of Beam (ETB), to validate the accuracy of the presented theory.**

Keywords: **Isotropic thick beam, Principle of virtual work, Shear deformation, Thermal loading, Thermal stress and Transverse shear stress.**

I. INTRODUCTION

Thick beam and isotropic beam are basically forms two dimensional problem of elasticity theory. Thick beam or isotropic beams are being widely used in structure subjected to severe thermal environment which produce an intense thermal stress on it. Isotropic beam are structures operating in aggressive environment in various engineering projects. Such a cases are particularly observed in aerospace and aeronautical engineering. This structure are usually refer to as high temperature structure such as structure used in high speed aircraft and spacecraft & deterring realistic thermal load induced stress in concrete pavements and flexible pavements.

It is well-known that elementary theory of bending of beam based on Euler-Bernoulli hypothesis disregards the effects of shear deformation and stress concentration. The theory is suitable for slender beams and is based on the assumptions that the transverse normal to neutral axis remains so during bending and after bending, which means transverse shear strain is zero. Thus this theory underestimates the deflection in case of thick beams where shear strain is significant.

The first-order shear deformation theory (FSDT) is an improvement over the elementary theory of beam or the classical theory of plate as the case may be. It is based on the hypothesis that the normal to the mid-surface before deformation remain straight but not necessarily normal to the mid-surface after deformation. This is known as first order shear deformation theory because the thickness wise displacement field for the axial displacement is linear or of the first order. In FSDT transverse shear strain distribution is assumed to be constant through the thickness and thus shear correction factors are required to take into account appropriate strain energy due to shear deformation. Further the theory suffers from the boundary condition paradox in which the theory does not satisfy the kinematic boundary Condition of slope of deflection curve at the built-in end of the beam.

Thermal stresses of laminated plates subjected to linear thermal load across the thickness of the plate with classical plate theory are given by, Jones [19] and Reddy [25]. Thermal stress analysis of isotropic plates is given by Boley and Weiner [23]. A thin simply supported rectangular plate and the temperature distribution function subjected to heat supply is determined. Due to thermal bending moments, the thermal stress components are evaluated. The results are plotted in the form of series solutions K. C. Deshmukh et al. [11].

Thermal stresses in cross-ply laminated plates subjected to linear thermal load through the thickness of plate using refined shear deformation theory presented by Ghugal and Kulkarni [24].Trigonometric shear deformation theory (TSDT) for thermal analysis of composite plates is derived by Ghugal and Kulkarni [6].

Semi-analytical elasticity solutions for thermal deformations of functionally graded beams with various end conditions, using the state space method based on differential quadraturepresentedLu et.al [26].

Thermoelastic stress analysis of perfectly clamped metallic rod using integral transform technique is given by Ghugal and G. R. Gandhe [15].Analytical solution for bending of cross-ply laminated plates under linear single sinusoidal thermal and mechanical load is presented by Zenkour [8] using of unified shear deformation plate theory. The two-dimensional therm-oelasticity solution for functionally graded thick beamspresented Lu et al [13]. Timoshenko [1] proposed a hypothesis for the development of first order shear deformation theory which states that the plane section which is perpendicular to the neutral axis before bending remains plane but not necessarily perpendicular to the neutral axis after bending. In this theory the transverse shear strain distribution over the crosssection of the beam is assumed to be constant through the thickness and thus require shear correction factor. Bresse [4], Rayleigh [3] and Timoshenko [2] were the pioneer investigators to include refined effects such as rotatory inertia and shear deformation in the beam theory. Timoshenko showed that the effect of transverse vibration of prismatic bars. This theory is now widely referred to as Timoshenko beam theory or first order shear deformation theory (FSDT) in the literature. In this theory transverse shear strain distribution is assumed to be constant through the beam thickness and thus requires shear correction factor to appropriately represent the strain energy of deformation. A study of literature by Ghugal and Shimpi [14] indicates that the research work dealing with flexural analysis of thick beams using refined trigonometric and hyperbolic shear deformation theories is very scarce and is still in infancy. Krishna Murty [12],Levinson [20], Bickford [27], Bhimaraddi and Chandrashekhara [28] presented parabolic shear deformation theories assuming a higher variation of axial displacement in terms of thickness coordinate. These theories satisfy shear stress free boundary conditions on top and bottom surfaces of beam and thus obviate the need of shear correction factor. Cowper [17] has given refined expression for the shear correction factor for different cross-sections of beam. The accuracy of Timoshenko beam theory for transverse vibrations of simply supported beam in respect of the fundamental frequency is verified by Cowper [21] with a plane stress exact elasticity solution. To remove the discrepancies in classical and first order shear deformation theories, higher order or refined shear deformation theories were developed and are available in the open literature for static and vibration analysis of beam.

II. THEOROTICAL FORMULATION

The variationally correct forms of differential equations and boundary conditions, based on the assumed displacement field are obtained using the principle of virtual work. The beamunder consideration occupies the following region:

$$0 \le x \le L; \quad 0 \le y \le b; \quad -h/2 \le z \le h/2$$

Consider a thick isotropic simply supported beam of length Lynx direction, Width b in y direction and depth h as shown in Figure 1. Where x, y and z are Cartesian coordinates. The beam is subjected to, thermal load of intensity T(x) on whole length of beam. Under this condition, the axial displacement, Transverse displacement, Axial bending stress and transverses hear stress are required to be determined.

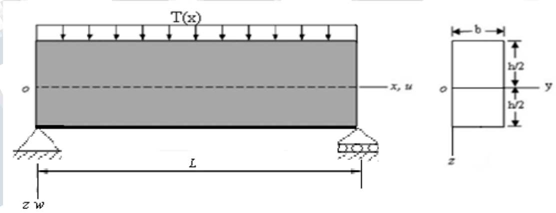

Figure 1: Simply supported beam bending under x-z plane

III. ASSUMPTIONS MADE IN THE THEORETICAL FORMULATION:

Theoretical formulation of present theory is based on the following assumptions.

1) The displacements are small in comparison with the beam thickness and therefore strains involved are infinitesimal.
2) Displacements given by elementary theory of bending.
3) The transverse displacement (w) in z direction is assumed to be function of x coordinate.
4) The beam is subjected to thermal load only.
5) The body forces are neglected.

IV. THE DISPLACEMENT FIELD

Based upon the before mentioned assumptions, the displacement field of the proposed beam theory is given as below

$$u(x,z) = -z\phi(x) \quad (1)$$

$$w(x,z) = w(x) \quad (2)$$

Where,

u = Axial displacement in x direction which is function of x and z.

w = Transverse displacement in z direction which is function of x.

ϕ = Rotation of cross section of beam at neutral axis which is function of x.

Normal strain:

Normal strains and shear strains are obtained within the framework of linear theory of elasticity using the displacement field given by Eq.(1)

$$\in_x = \frac{\partial u}{\partial x} = -z\frac{\partial \phi}{\partial x} \quad (3)$$

Shear strain:

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = -\phi + \frac{\partial w}{\partial x} \quad (4)$$

Stress:

The one dimensional Hooke's law is applied for isotropic material, stress is related to strain and shear stress is related to shear strain by the following constitutive relations.

$$\sigma_x = E \in_x$$

$$= E\left(\in_x - \alpha T_{(x)}\right)$$

$$\text{put} \quad T_{(x)} = T_{0(x)} + \frac{z}{h}T_{1(x)}$$

$$\sigma_x = E\left[\in_x - \alpha\left(T_{0(x)} + \frac{z}{h}T_{1(x)}\right)\right]$$

$$= E\left(-z\frac{\partial\phi}{\partial x} - \alpha\, T_{0(x)} - \alpha\frac{z}{h}T_{1(x)}\right)$$

$$\sigma_x = \left(-zE\frac{\partial\phi}{\partial x} - \alpha\, ET_{0(x)} - \alpha E\frac{z}{h}T_{1(x)}\right) \quad (5)$$

$$\tau_{xz} = G\gamma_{xz} = G\left(-\phi + \frac{\partial w}{\partial x}\right) \quad (6)$$

Where E and G are young's modulus and shear modulus or the elastic constants of the beam material, and ∝ is the coefficients of non-linear in thermal expansion in x and z direction respectively and T_0 and T_1 is the thermal load.

The temperature field variation through the thickness is assumed to be

$$T(x,z) = T_0(x,z) + \frac{z}{h}T_1(x,z) \quad (7)$$

Where T_0 and T_1 are the thermal load.

V. GOVERNING DIFFERENTIAL EQUATIONS

Governing differential equations and boundary conditions are obtained from Principle of virtual work. Using equations for stresses, strains and principle of virtual work, variationally consistent differential equations for beam under consideration are obtained. The principle of virtual work when applied to beam leads to:

$$b\int_{x=0}^{x=l}\int_{z=-\frac{h}{2}}^{z=\frac{h}{2}}\left(\sigma_x\,\delta\in_x + \tau_{xz}\,\delta\gamma_{xz}\right)dz\,dx -$$

$$\int_{x=0}^{x=l} q_{(x)}\,\delta w\,dx = 0 \quad (8)$$

Where δ = variational operator

Employing Greens theorem in above Equation successively, we obtain the coupled Euler-Lagrange equations which are the governing differential equations and associated boundary conditions of the beam. The governing differential equations obtained are as follows:

$$GA\left(\frac{\partial\phi}{\partial x} - \frac{\partial^2 w}{\partial x^2}\right) = q_{(x)} \quad (9)$$

$$-EI\frac{\partial^2\phi}{\partial x^2} + GA\left(\phi - \frac{\partial w}{\partial x}\right) -$$

$$\left(\frac{\alpha}{h}\right)EI\frac{\partial T_{1(x)}}{\partial x} = 0$$

$$(10)$$

The associated consistent natural boundary conditions obtained are of following form along the edges x = 0 and x = L.

$$GA\left(\frac{\partial w}{\partial x}-\phi\right)=0 \qquad (11)$$

Where w is prescribed.

$$EI\frac{\partial \phi}{\partial x}+\left(\frac{\alpha}{h}\right)EI\frac{\partial T_{1(x)}}{\partial x}=0 \qquad (12)$$

Where ϕ is prescribed.

Where E is elastic constants of the material and I is moment of inertia of beam.

VI. THE SOLUTION SCHEME

Here we concern with the close form solutions of simply supported and rectangular steel beam. The boundary conditions for simply supported edges are

$$\text{At} \quad x=0, \text{and} \quad x=L$$
$$u_0=0 \ \text{and} \ w_0=0$$

The following is the solution form for $u_0(x,z), w_0(x,z)$ that satisfies above boundary conditions exactly. Such solution in theory of plates and shells is called as "closed-form" solution. This type of solution was suggested by Navier (1820) for the bending problem of simply supported rectangular beam.

$$w(x)=\sum_{m=1}^{\infty} w_{0m}\sin\frac{m\pi x}{L}$$
$$w=w_0\sin\frac{\pi x}{L} \qquad (13)$$
$$\phi(x)=\sum_{m=1}^{\infty} \phi_{0m}\sin\frac{m\pi x}{L}$$
$$\phi=\phi_0\cos\frac{\pi x}{L} \qquad (14)$$

To assess the performance of the present theory in the prediction of bending response of a beam under a thermal load, a simply supported isotropic beam of length L, width b, and thickness h is considered. The beam is subjected to thermal load which is given by

$$T_{1(x)}=\sum_{m=1}^{\infty} T_{0m}\sin\frac{m\pi x}{L}$$
$$T_{1(x)}=T_0\sin\frac{\pi x}{L} \qquad (15)$$

where T_{0m} is the coefficients of Fourier expansion

$$\mathrm{T}_{0m}=\frac{4\mathrm{T}_0}{m\pi} \quad for \quad m=1,2,3$$
$$\mathrm{T}_{0m}=0 \quad for \quad m=2,4,6$$

Here T_o is the intensity of thermal load.

$$q_{(x)}=q_0\sin\frac{\pi x}{L} \qquad (16)$$

$$\text{At} \quad x=0, \text{and} \quad x=L$$
$$u_0=0 \ \text{and} \ w_0=0$$

Put the value eq. (13), (14), (15) and (16) in eq. (9) and (10), Where w_m *and* ϕ_m are the unknown coefficients of the respective Fourier expansion and m isthe positive integer. Substituting this form of solution and the load $q(x)$ into governingequations and discarding all the terms containing time derivatives yields the two algebraic simultaneous equations which can be written in following matrix form.

$$GAw_0\left(\sin\frac{\pi\,\mathrm{x}}{l}\right)\frac{\pi^2}{l^2}-GA\phi_0\left(\sin\frac{\pi\,\mathrm{x}}{l}\right)\frac{\pi}{l}$$
$$=q_0\sin\left(\frac{\pi\,\mathrm{x}}{l}\right)$$
$$w_0\left[GA\frac{\pi^2}{l^2}\right]-\phi_0\left[GA\frac{\pi}{l}\right]=q_0 \qquad (17)$$
$$(K_{11}) \qquad (K_{12})$$
$$-GAw_0\left(\cos\frac{\pi\,\mathrm{x}}{l}\right)\frac{\pi}{l}-EI\phi_0\left(\cos\frac{\pi\,\mathrm{x}}{l}\right)\frac{\pi^2}{l^2}+$$
$$GA\phi\left(\cos\frac{\pi\,\mathrm{x}}{l}\right)-\left(\frac{\alpha}{h}\right)EIT_1\left(\cos\frac{\pi\,\mathrm{x}}{l}\right)\frac{\pi}{l}=0$$
$$-GAw_0\frac{\pi}{l}-EI\phi_0\frac{\pi^2}{l^2}+GA\phi-\frac{EI\alpha T_1\pi}{hl}=0$$

$$-w_0\left[Gbh\frac{\pi}{l}\right]+\phi_0\left[EI\frac{\pi^2}{l^2}+Gbh\right]=\frac{EI\alpha T_1\pi}{hl}$$
$$(K_{21}) \qquad (K_{22}) \qquad (18)$$

$$K_{11}=GA\frac{\pi^2}{l^2}$$

$$K_{12}=K_{21}=GA\frac{\pi}{l}$$

$$K_{22}=EI\frac{\pi^2}{l^2}+GA$$

$$\begin{pmatrix}K_{11} & K_{12}\\ K_{21} & K_{22}\end{pmatrix}\begin{Bmatrix}w_0\\ \phi_0\end{Bmatrix}=\begin{bmatrix}q_0\\ T_1\end{bmatrix} \qquad (19)$$

But $q_{(x)}=0$ (mechanical load is absents) and consider transverse load is zero (pure thermal load)

VII. ILLUSTRATIVE EXAMPLE

In order to prove the efficiency of the present theory, the following numerical examples are considered. The following material properties for steel beam is used having E = 210 GPa, T_1=50^0, μ=0.3, α=12x10^{-6} and G=E/2(1+μ)where E is the Young's modulus and μ is the Poisson's Units. Consider Cross section of beam is as follows Length=3m, Width=0.23m, Depth=0.30m and subjected to thermal load on it.

$$w_0=1.824E^{-3}$$

$$\phi_0=1.910E^{-3}$$

Example 1: Simply supported beam with thermal load T (x)

A simply supported beam with the origin of beam on left end supported at x=0. The beam is subjected to, thermal load of intensity T (x) over the span L on surface z = h/2 acting in the z direction.

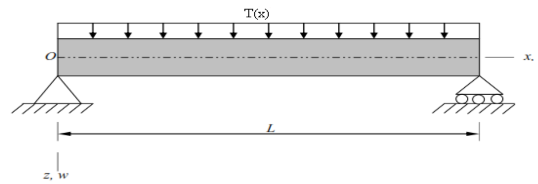

Figure 2: A simply supported uniform beam subjected to thermal load

VIII. NUMERICAL RESULTS

In this paper the numerical results for axial displacement, transverse displacement, bending stress, transverse shear stress and thermal stress are determined for simply supported isotropic beam subjected to thermal load across the thickness of beam.

$$w=w_0\sin\left(\frac{\pi x}{l}\right)$$

$$w=1.824E^{-3}\sin\left(\frac{\pi x}{l}\right) \qquad (20)$$

$$\phi=\phi_0\cos\left(\frac{\pi x}{l}\right)$$

$$\phi=1.910E^{-3}\cos\left(\frac{\pi x}{l}\right)$$

$$u=-z\phi$$

$$u=-z\,1.910E^{-3}\cos\left(\frac{\pi x}{l}\right) \qquad (21)$$

$$\sigma_x = \frac{\partial u}{\partial x}$$

$$\sigma_x = -z\,1.910E^{-3} - \cos\left(\frac{\pi x}{l}\right)\left(\frac{\pi}{l}\right)$$

$$\sigma_x = \frac{1.910\,E^{-3}\,\pi}{l} z \sin\left(\frac{\pi x}{l}\right) \qquad (22)$$

$$\tau_{xz} = \frac{1.910\,E^{-3}\,\pi^2}{l^2} \cos\left(\frac{\pi x}{l}\right) \left[-\left(\frac{z^2}{2}\right)+\left(\frac{h^2}{8}\right)\right] \qquad (23)$$

Table1.Axial displacements u at $\left(x=0, z=\pm h/2\right)$, axial stress σ_x at $\left(x=L/2, z=\pm h/2\right)$, Maximum Transverse shear stress τ_{xz} at $\left(x=0, z=\pm h/2\right)$, simply supported isotropic beam subjected to thermal load for ETB Theory

z/h	u	σ_x	τ_{xz}
0.15	-0.0000458	0.000048	0
0.12	-0.0000366	0.0000384	0.00000135
0.08	-0.0000244	0.0000256	0.00000269
0.04	-0.0000122	0.0000128	0.0000035
0.02	-0.00000611	0.0000064	0.0000037
0	0	0	0.00000376
-0.02	0.00000611	-0.0000064	0.0000037
-0.04	0.0000122	-0.0000128	0.0000035
-0.08	0.0000244	-0.0000256	0.00000269
-0.12	0.0000366	-0.0000384	0.00000135
-0.15	0.0000458	-0.000048	0

Table2. Axial displacements u at $\left(x=0, z=\pm h/2\right)$,axial stress σ_x at $\left(x=L/2, z=\pm h/2\right)$, Maximum Transverse shear stress τ_{xz} at $\left(x=0, z=\pm h/2\right)$, simply supported isotropic beam subjected to thermal load for FSDT Theory.

z/h	u	σ_x	τ_{xz}
0.15	-0.0002865	0.0003	0
0.12	-0.0002292	0.00024	0.00000848
0.08	-0.0001528	0.00016	0.00001686
0.04	-0.0000764	0.00008	0.00002188
0.02	-0.0000382	0.00004	0.00002314
0	0	0	0.00002356
-0.02	0.0000382	-0.00004	0.00002314
-0.04	0.0000764	-0.00008	0.00002188
-0.08	0.0001528	-0.00016	0.00001686
-0.12	0.0002292	-0.00024	0.00000848
-0.15	0.0002864	-0.0003	0

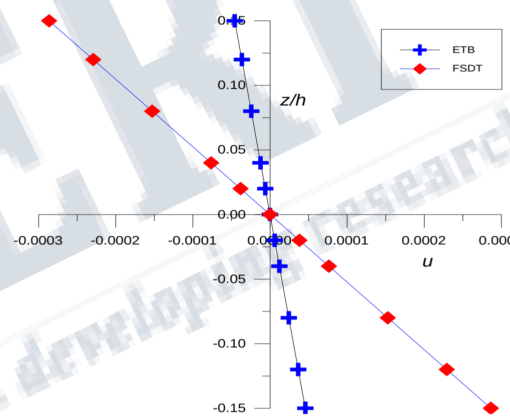

Figure 3: Variation of Maximum Axial Displacement (u)

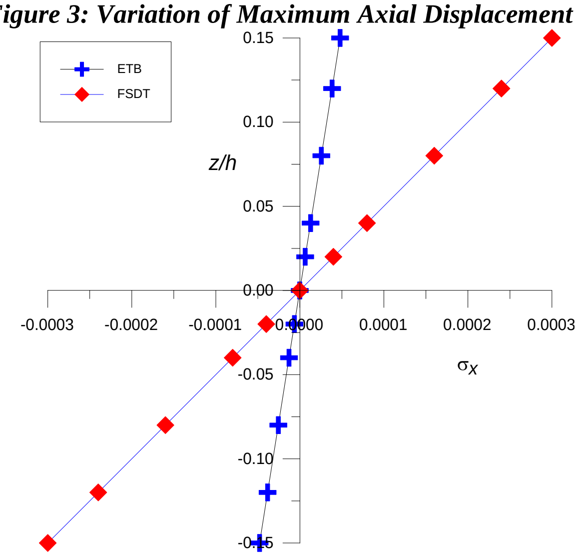

Figure 4: Variation of Maximum Axial Stress (σ_x)

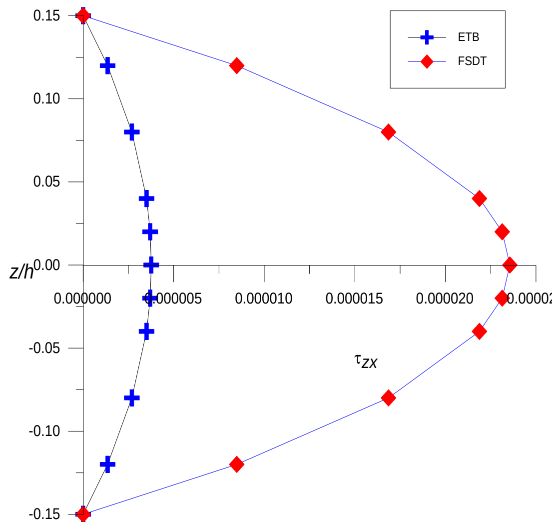

Figure 5: Variation of Maximum Transverse Shear Stress (τ_{xz})

IX. DISCUSSION OF RESULTS

Thermal stress analysis of a thick beam using First order shear deformation theory is carried out in the present research work and result obtained are discussed as follows. The theory has several features as given below:

a) The number of unknown variables is same as that in ETB.
b) The shear deformation in the beam is properly accounted.
c) The theory obviates the need of shear correction factor.
d) The governing differential equations and the associated boundary conditions are variationally consistent.

The results obtained for axial displacements, stresses and Transverse shear stress for simply supported isotropic beam subjected to non-linear thermal load are presented in Table 1 and Table 2. The Table 1.and Table 2. shows the comparison of maximum displacements, stressesand Transverse shear stress for the isotropic beam (steel) subjected to thermal load through thickness variation of displacements and stress. The comparison of theories show that the value of maximum displacements, stressesand Transverse shear stress obtained by FSDT are higher compared to the values given by ETB. The result of maximum axial displacement u obtained by present theory. The variation of u is presented as shown in Figure 3.The variation of maximum dimensional axial stresses σ_x for a beam as shown in Figure 4. The maximum transverse shear stress τ_{xz} obtained by present theory using constitutive relation. The through thickness variation of this stress obtained via constitutive relation obtained are presented in Figures 5.

X. CONCLUSION

Thermal response of isotropic beam underthermal load across the thickness of beam has been studied by using First order shear deformation Theory (FSDT)).Present theory gives good prediction of the thermal response of beam in respect of displacements and stresses. The present theory obviates the need of shear correction factor and theory is variationally consistent. The present theory satisfies the shear stress free surface conditions on the top and bottom surfaces of the beam.

APPENDIX

LIST OF NOMENCLATURE

A = Cross sectional area of beam= bh
b = Width of beam in y direction;
E, G and μ= Elastic constants of the material;
E = Young's modulus;
G = Shear modulus;
h= Thickness of beam;
I =Moment of inertia of cross section of beam;
L =Span of the beam;
q =Intensity of transverse Load;
u =Axial displacement in x direction;
w =Transverse displacement in z direction;
x, y, z =Rectangular Cartesian coordinates;
δ = Variational operator;
μ=Poisson's ratio of the beam material;
σ =Axial stress in x direction;
τ=Transverse shear stress in zxplane;
σ = Transverse normal stress in z direction;
w=Non-dimensional transverse displacement;
T= Thermal load

LIST OF ABBREVIATIONS

ETB	Elementary theory of beam
FSDT	First order shear deformation Theory

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