

# Modeling and Analysis of Prosthetic Lower Limb for Artificial Limb Applications

<sup>[1]</sup>Sidharth Goutam Dash, <sup>[2]</sup> Dr. Sivanandan K.S.  
<sup>[1]</sup> Student, <sup>[2]</sup> Professor

<sup>[1][2]</sup>National Institute of Technology Calicut  
<sup>[1]</sup>sidharthgoutam03@gmail.com, <sup>[2]</sup>kss@nitc.ac.in,

---

**Abstract:** — this paper demonstrates an up rose mode of analyzing a contrived limb. It presents a brief description of human locomotion as well as dynamic behavior of the system. Assistive technology is one of the most emerging technology in this fast developing scientific era. This technology attempts to induce the ability of being self-dependent on the differently abled individuals. In order to fulfill the objectives of this technology it is needed to analyze the part of the human body and its function. The paper energizes the way of analysis of the contributions given by the lower limb of a human body. It facilitates to convert a physical system to a kind of analyzable system. This paper imparts the way of conversion of such systems. Here the focus is on the conversion of a physical lower limb system to mathematical system. The idea instructs the way of conversion of the following system.

---

## I. INTRODUCTION

One of the prior significant lineaments of human being includes locomotion. Upper limbs and lower limbs contribute locomotion to the human being. Huge numbers of actions are interrupted with the loss of these organs. Maintaining balance on the ground becomes a tough work to do. Movement is almost restricted with the absence of lower limb. In present days the development of science and engineering leads to the existence of solutions to the above problems.

Artificial walking aids the solution of locomotor impairment of human beings. This provides the assistance for compensating the restrictions of differently abled people deporting their daily work reducing dependencies. Artificial ambulation is inspired from the assistive technology.

The ease of analysis of human locomotion is not fully achieved despite of having closely replicated technology at hand. The human movement is a frequent physical activity occurring by electrical signals transferred by neurons from brain. Hence replacement of human lower limb by assistive technology is quite a difficult task. The most desirable maneuvering features of prosthetic system should be the system which demonstrates the features almost exchangeable to that of human ambulation.

Today a number of prosthetic limbs are available which do not entertain the precariousness of stimulant receptive data.

Some are quite expensive due to the complexities imparted. Now days the demands for the improvements in functional features are gradually touching the height. The weaknesses of the available prosthetic devices can be downplayed by the sound modeling of the system. The observation of lacuna in organized taxonomic approach towards control characteristics in the currently available system leads the path in proper modeling of system in this paper.

There exists a several number of modeling for bipedal ambulation. Some represent bipeds of a very simple kinematic structure and some are limited to two dimensions. On the fundament of kinematics data of dissimilar classes of neuropathic patients, some models are applied to optimize the orthosis operation in individuals with neuropathic feet. A well-formulated mathematical model translates the base reaction forces to the degree of disability. The assumption includes that the debase estimated at the stump socket interface will approximate those received by the proposed implant.

## II. MODELING PARAMETERS

There are number of parameters associated with the human lower limb. These parameters tend to vary in a Gait cycle according to walking environment. During the ambulation period the angle at the hip as well as the knee varies with respect to reference which is considered

to be perpendicular to the ground. The center of mass of femur as well as the tibio-fibula influences the stability of limb on the ground. The length of lower limb helps in determining the center of mass of the system. It is necessary to estimate the amount of force required for the linear movement and torque for the rotational movement of the hinges. The force exerted on the ground by the lower limb, reaction force from the ground and the frictional force present need to be considered for modeling. Hence in this paper angle, force, torques, position of center mass, length of the limb are considered as the variable for the modeling.

### III. ASSUMPTIONS

The assumptions made in this modeling is as follows

- i) The center of rotation remains at a definite point.
- ii) The centre of mass of every segment is invariable.
- iii) Every segment is considered to be rigid bodies.
- iv) The mass of the trunk is unattended

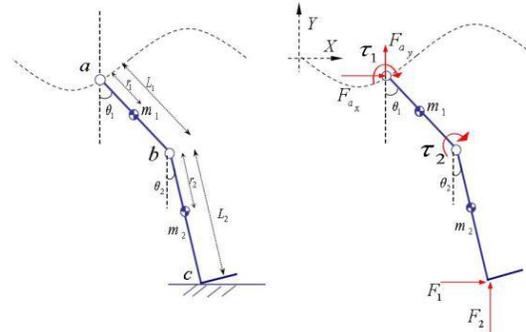
### IV. MODELING METHODOLOGY

The target is to gain the schemes of ordinary differential equation regulating the motion of human limb/prosthetic limb. A prosthetic limb can be modeled as a serial manipulator with rigid links. In order to get the equations of motion for a system with kinematic chain two methods can be applied i.e. Newton Euler and Lagrangian formulation.

Although both methods give similar equations of motion Lagrangian dynamics is simpler than Newton Euler. The Newton-Euler is derived from Newton's Second Law of Motion, and on analysis of forces and moments of constraints working between contiguous links. The resultant equations let in the coupling forces and moments, hence surplus mathematical operations are required to obviate these excess terms. Conversely, the straightforward Lagrangian conceptualization is an energy-based access to dynamics and reflexively all workless forces such as internal forces are disregarded in this approach. Therefore, this paper applies Lagrangian formulation in the following concept to drive the equation of motion and figure out the inverse dynamics of the system necessary to have command over torque of actuators.

In inverse dynamic model of a robotic system, the inputs are the required trajectories that concerns to a time history of the yielded position, velocity, and

acceleration of every articulate. Implementing the knowledge of these histories, the torque/force expected to be enforced at different actuated joints are found out as the result of this inverse dynamic model. Figure 1 exemplifies the simplified model of human leg. Sinusoidal centre is conceived as hip.



**Figure 1: Model of Limb**

In Figure 1  $r_1$  and  $r_2$  are the space among the centre of mass of link and its upper articulate,  $L_1$  and  $L_2$  are the length of femur and tibia, respectively and  $\theta_1$  and  $\theta_2$  are angular posture of femur and tibia with reference to the global y-axis. So, when the body is conceived to be upright, the hip angle becomes  $\theta_1$ , knee angle is the relative angular posture of femur and waist. Therefore  $\theta_1$  is hip angle (1)  $\theta_2$  is knee angle (2). The free body diagram of the model is pictured in Figure 1. Hip and knee torques, enforced by forces working through the tendons and ligaments, are symbolised by  $\tau_1$  and  $\tau_2$ .  $F_{1x}$  and  $F_{1y}$  symbolises the horizontal and vertical elements of base reaction force employed at the Center Of Pressure (COP). The COP is the plantar position of vertical GRF.  $F_{1x}$  and  $F_{1y}$  symbolise the forces working on the femoral head utilized by the socket.

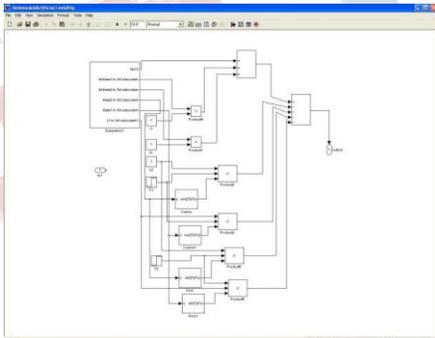
The points a, b, c tag the hip joint, knee joint, and COP respectively, in Figure 1 and  $(a_x, a_y)$ ,  $(b_x, b_y)$  and  $(c_x, c_y)$  specify their positions with respect to the global frame XY. To employ the Lagrangian formula, the Cartesian ordinates of center of mass for every link are  $(x_1, y_1)$  and  $(x_2, y_2)$ . From the above parameters the hip and knee torque formula can be obtained as

$$\begin{aligned} \tau_1 = & m_1 r_1 [ \ddot{\theta}_1 + x_a'' \cos \theta_1 + y_a'' \sin \theta_1 + g \sin \theta_1 ] + \\ & m_2 L_1 [ L_1 \ddot{\theta}_1 + x_a'' \cos \theta_1 + y_a'' \sin \theta_1 + g \sin \theta_1 ] + \\ & m_2 r_2 \\ & [ -r_2 \ddot{\theta}_2 + g \sin \theta_2 + x_a'' \cos \theta_2 + y_a'' \sin \theta_2 + \\ & L_1 (\dot{\theta}_1 + \dot{\theta}_2) \cos(\theta_1 - \theta_2) + L_1 ((\dot{\theta}_1)^2 + \dot{\theta}_2^2) \sin(\theta_1 - \theta_2) ] \\ & + \\ & I_2 \ddot{\theta}_2 + I_1 \ddot{\theta}_1 - L_2 F_1 \cos \theta_2 - L_1 F_1 \cos \theta_1 - L_2 F_2 \sin \theta_2 \\ & - L_1 F_2 \sin \theta_1 \end{aligned} \quad (3)$$

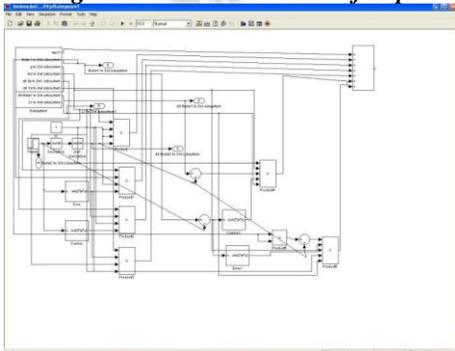
$$\begin{aligned} \tau_2 = & m_2 r_2 [ \\ & r_2 \ddot{\theta}_2 + x_a'' \cos \theta_2 + y_a'' \sin \theta_2 + L_1 \ddot{\theta}_1 \cos(\theta_1 - \theta_2) \\ & + L_1 (\dot{\theta}_1)^2 \sin(\theta_1 + \theta_2) + g \sin \theta_2 ] \\ & + I_2 \ddot{\theta}_2 - L_2 F_1 \cos \theta_2 \\ & - L_2 F_2 \sin \theta_2 \end{aligned} \quad (4)$$

**V. MODELLING SIMULATIONS**

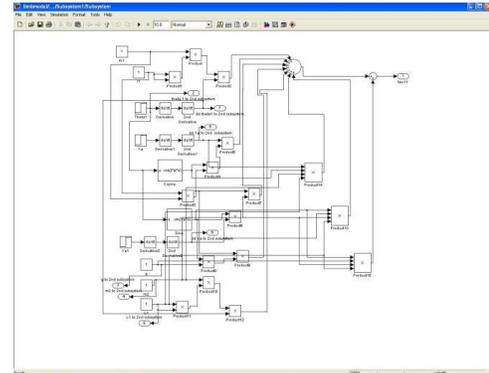
The model of limb has been simulated in the Matlab Version 2014a.



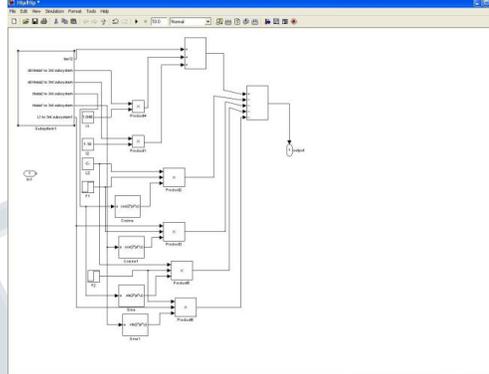
**Figure 2: Simulink Model of Hip**



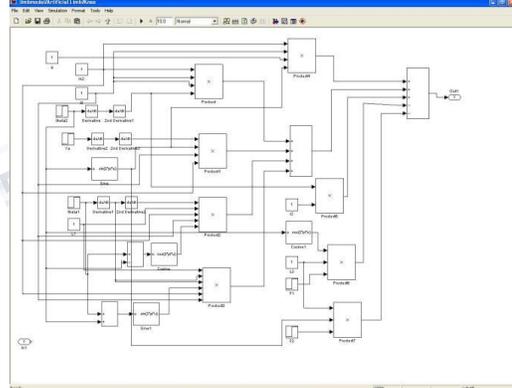
**Figure 3: Simulink Model of Subsystem1 of Hip**



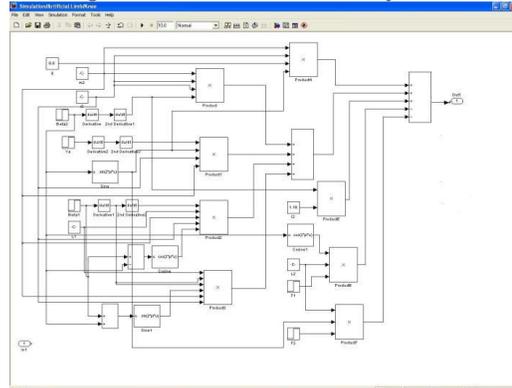
**Figure 4: Simulink Model of Subsystem2 of Hip**



**Figure 5: Simulink Model of Subsystem 3 of Hip**



**Figure 6: Simulink Model 1 of Knee**



**Figure 7: Simulink Model 2 of Knee**

---

## VI. CONCLUSION

The mathematical model for an artificial limb is formulated structured on inverse dynamics model. The torques required at the different joint of a prosthetic lower limb can be estimated from this modeling without considering the biological signals. The limitations of the nominated model include the assumptions conceived. The system is believed to be linear and hence modeling is the linear approximation of the system. This estimation may bring in its own advantages as well as disadvantages in the analytical solutions/conclusions.

## REFERENCES

- [1] T. S. Sirish, K. S. Sivanandan, Jobin Varghese, P. K. Rajendrakumar, "Modeling, analysis and fabrication of supporting system for lower limb", International Journal of Scientific Engineering and Technology, Volume No.3 Issue No.4, pp: 428-432
- [2] Suresh L. G. R. S. Nagakumar, M. V. Sudarsan, "AT89C51 Microcontroller based control model for hybrid assistive limb", Selected Works, Winter January 10, 2013
- [3] Takahiko Nakamura, Kazuhiro Kosuge, "Model based walking support system with wearable walking helper", 2003 IEEE International Workshop on Robot and Human Interactive Communication Millbrae, California, USA, Oct 31 -Nov 2 2003
- [4] Jerry E Pratt, Benjamin T. Krupp, Christopher J Morse, Steven H. Collind, "The Roboknee: An exoskeleton for enhancing strength and endurance during walking", IEEE, International Conference on Robotics & Automation, 2004, IEEE
- [5] Jamshidi N, Rostami M. Najarian S, Menhaj M B Saadatnia M, Firooz S, "Modelling of human walking to optimize the function of ankle-foot orthosis in Guillan-Barre patients with foot drop", Singapore Medical Journal, 2009; 50(4): 413
- [6] Paul Stephenson, Bahaa B. Seedhom, "Estimation of forces at the interface between an artificial limb and an implant directly fixed into the femur in above knee amputees", Journal of orthopedic science, J Orthop Sci (2002) 7:292-297
- [7] Nicola Sancisi, Vincenzo Parenti-Castelli, "A 1-DoF parallel spherical wrist for the modelling of the knee passive motion",
- [8] [https://en.wikipedia.org/wiki/Inverse\\_dynamics](https://en.wikipedia.org/wiki/Inverse_dynamics)

[9] T. S. Sirish, Sujalakshmy V., Dr. K. S. Sivanandan, "A new approach for modeling the system dynamics", Global Journals Inc., Volume 12 issue1 version 1.0 january 2012.