

# Time-shift OQAM for GFDM

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**Abstract -** Generalized Frequency Division Multiplexing (GFDM) represents a new view on filtered multicarrier modulation with more degree of flexibility it can provide than OFDM and SC-FDE transmission and it has capability to fulfill the requirements of the next generation cellular system. In this paper GFDM performance analysis is done in AWGN channel and FSC using different types of receiver. A new approach called GFDM-OQAM is implemented to mitigate non orthogonality between subcarriers.

**Keywords:** GFDM, OQAM, OFDM, AWGN.

## 1. INTRODUCTION

Multi-carrier modulation has been widely adopted in recent decades for the implementation of the broadband communication system. Among the existing multicarrier modulation systems, OFDM is the most common. OFDM has attracted a lot of attention because of its robustness against multipath fading channel and its simple implementation using FFT and IFFT. Instead of many advantages of OFDM, it is not a desirable solution for future 5G system.

The problem with OFDM is that cyclic prefix is used for every symbol that reduces spectral efficiency and avoids latency reduction by shortening symbols. In OFDM time domain pulse is confined from 0 to T. The Fourier Transform of rectangular pulse is sinc function with lots of side lobes. If there is any synchronization problem then it will sample at side lobe frequency due to this there is lot of power leakage in frequency domain we call it Inter-Carrier Interference (ICI) and it is dominant in OFDM. Our proposed frame structure for GFDM, add a single prefix for the entire frame. The introduction of sub-symbols decouples the frequency resolution from the bandwidth of the subcarriers. In GFDM, the out-of-band radiation is limited by adjustable pulse shaping filter applied to the individual subcarriers.

The paper is organized as follows. Section II introduces the block structure of GFDM. In section III transceiver model using GFDM-OQAM is covered with mathematical expressions. Section IV analyzes the SER expression for OFDM and GFDM in AWGN and FSC channels. Section V deals with the

comparisons of GFDM and OFDM. Finally, section VI concludes the paper.

## II. GFDM BLOCK STRUCTURE

Transmitting and receiving data in quantities of blocks is a common concept in communication systems. In GFDM each block is designed to carry  $N = KM$  complex valued data symbols, which are scattered across  $K$  subcarriers and  $M$  sub-symbols. Figure 1(c) [1] shows an example time-frequency grid, which is annotated with the terminology that will be used. The smallest unit in this context is a data symbol. A subcarrier consists of  $M$  data symbols which are transmitted consecutively, using multiple time slots on a specific center frequency. A subsymbol denotes  $K$  data symbols that are transmitted in the same time slot using multiple center frequencies in parallel.

Based on this data block definition and assuming a digital signal that is sampled with the frequency  $B_s$ , the time and frequency dimensions of the signal can be determined. First, the sampling period is calculated as

$$T_s = \frac{1}{B_s} \quad (1)$$

Because there are  $K$  equidistant subcarriers in the system, the spacing between them is chosen as"

$$B_{sc} = \frac{B_s}{K} \quad (2)$$

This definition allows to support orthogonal and non-orthogonal configurations with GFDM. If it is not desired to have the option for orthogonality, the subcarrier distance could be smaller. Moreover, is the distance between the subcarrier center frequencies and

does not necessarily represent the actual bandwidth that is occupied by the subcarrier. The inverse of determines the duration of a sub-symbol as”

$$T_{sub} = \frac{1}{B_{sc}} = KT_s \quad (3)$$

As previously established, there are M sub-symbols in a

block, hence the total block duration sums up to

$$T_b = MT_{sub} = NT_s \quad (4)$$

$$B_{\Delta} = \frac{1}{T_b} = \frac{B_{sc}}{M} \quad (5)$$

denotes the frequency gap between two samples of the signal in the frequency domain.

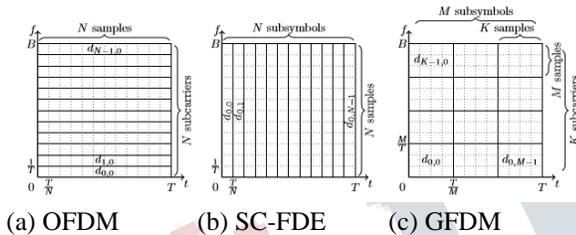


Figure 1: Time-frequency grid for different multicarrier system

### III. GFDM SYSTEM MODEL

In GFDM-QAM, transmission of signal with wellocalized pulse shaping filter assure Inter Symbol Interference (ISI) free transmission but non orthogonal subcarrier lead to undesirable Inter Carrier Interference (ICI) and degrade the system performance. To mitigate these effects, the concept of orthogonality of the offset quadrature amplitude modulation (OQAM) is explored. Figure 2 represents the block diagram for GFDM-OQAM with time offset.

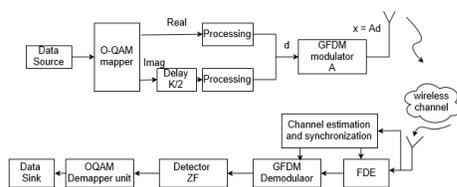


Figure 2: Block diagram of GFDM-OQAM transceiver

In GFDM-QAM, in-phase and quadrature phase data ( $d_{k,m} = d_{k,m}^{(i)} + jd_{k,m}^{(q)}$ ) is transmitted with the time offset of K/2 sample i.e half of the subcarrier spacing for interference free transmission and additional phase rotation of  $\Pi/2$  is provided among adjacent subcarrier and sub-symbols which is given as

$$g_{k,m}^{(i)}[n] = j^k g_{k,m}[n] \quad (6)$$

$$g_{k,m}^{(q)}[n] = j^{k+1} g_{k,m+\frac{1}{2}}[n] \quad (7)$$

The in-phase pulse shape is same as prototype filter used in GFDM-QAM with additional phase rotation term  $j^k$  and quadrature pulse shape is shifted version of prototype filter with half of the sub-symbol duration with additional phase rotation term  $j^{k+1}$ .

Therefore, GFDM-OQAM transmitted signal given by

$$x(n) = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} d_{k,m}^{(i)} g_{k,m}^{(i)}[n] + \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} d_{k,m}^{(q)} g_{k,m}^{(q)}[n] \quad (8)$$

Let  $h = [h_0 \dots h_{N_{ch}}]^T$  be the channel impulse response of length  $N_{ch}$ . The received signal after propagation through the wireless channel can be modelled as

$$r = Hx + w \quad (9)$$

where  $H = \text{circ} \{ \tilde{h} \}$  is a circulant channel convolution matrix and  $\tilde{h}$  is the zero padded version of  $h$  of the same length as  $x$ . The vector  $w \sim (0, \sigma_w^2 I_{KM})$  denotes the complex additive white Gaussian noise (AWGN) samples with noise variance  $\sigma_w^2$  and  $I_{KM}$  is the identity matrix of order  $KM$ .

By appending cyclic prefix (CP) frequency domain channel equalization is carried out at the receiver side. FDE is done at the receiver side to avoid the multipath channel impairments. It is given as”

$$y = \text{IFFT} \left\{ \frac{\text{FFT}(r)}{\text{FFT}(h)} \right\} \quad (10)$$

from the received and equalized signal the estimated data is given as

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$$\tilde{d} = By \quad (11)$$

The matrix B for different receiver is given as follows

$$\text{MF: } B_{MF} = A^H \quad (12)$$

where  $(.)^H$  denotes Hermitian conjugate. Matched filter receiver increases SNR at the output of receiver but problem with matched filter receiver is that it generate intercarrier interference when non orthogonal subcarriers are used, which will degrade SER performance.

$$\text{ZF: } B_{ZF} = (A^H A)^{-1} A^H \quad (13)$$

The ZF receiver eliminates self generated interference at the cost of enhancing the noise, as the receiver filter collects noise outside the desired bandwidth. In the flat channel, the noise enhancement factor (NEF) determines the reduction in the signal-to-noise ratio when using the ZF receiver. NEF is equal for every k and is given as

$$\xi = \sum_{n=0}^{MK-1} |[B_{ZF}]_{k,n}|^2 \quad (14)$$

$$\text{MMSE: } B_{MMSE} = (\sigma_w^2 I + A^H A)^{-1} A^H \quad (15)$$

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where I is a unit matrix. MMSE receiver is having performance between ZF receiver and MF receiver. MMSR receiver acts as MF receiver at low SNR and avoid self interference and for high SNR, it act as ZF receiver that will minimize the effect of noise. At the receiver side matched filtering is used to recover the data and is given as

$$\tilde{d}_{k,m}^{(i)} = R\{r[n] \circ g_{k,m}^{(i)*}[-n]\}_{n=0} \quad (16)$$

where  $\circ$  represents circular convolution with period N. The real orthogonality is specified by the projection between any  $m'$ th subsymbol and the  $k'$ th subcarrier at all k, m i.e.

$$R\{g_{k,m}^{(i)}[n] \circ g_{k,m}^{(i)*}[-n]\}_{n=0} = \delta[k, m] \delta[k' m'] \quad (17)$$

$$R\{g_{k,m}^{(q)}[n] \circ g_{k,m}^{(q)*}[-n]\}_{n=0} = \delta[k, m] \delta[k' m'] \quad (18)$$

$$R\{g_{k,m}^{(i)}[n] \circ g_{k,m}^{(q)*}[-n]\}_{n=0} = 0 \quad (19)$$

$$R\{g_{k,m}^{(q)}[n] \circ g_{k,m}^{(i)*}[-n]\}_{n=0} = 0 \quad (20)$$

#### IV. SER CALCULATION FOR OFDM AND GFDM

Symbol Error Rate (SER) – In digital communication, symbol rate is the number of symbols changes across the transmitting medium per unit time using digitally modulated signal. Using SER we can study the performance of any scheme in different channel. AWGN Channel: In GFDM, NEF regulates the equivalent signal to noise ratio (SNR) at the receiver side. Thus, SER performance for GFDM and OFDM under AWGN only differs in the equivalent SNR. So, SER for GFDM under AWGN is given by”

$$P(e) = 2\left(\frac{\sqrt{\beta}-1}{\sqrt{\beta}}\right) \text{erfc}(\sqrt{\gamma}) - \left(\frac{\sqrt{\beta}-1}{\sqrt{\beta}}\right)^2 \text{erfc}^2(\sqrt{\gamma}) \quad (21)$$

$$\text{where } \gamma = \frac{3R_T}{2(2^H-1)} \frac{E_s}{\xi N_0} \text{ and } R_T = \frac{KM}{KM+N_{ep}+N_{cs}} \quad (22)$$

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where  $p=2^\mu$  and  $\mu$  is the number of bits per QAM symbols.  $N_{CP}$  and  $N_{CS}$  are the length of cyclic prefix and cyclic suffix, respectively,  $E_s$  is the average symbol energy and  $N_0$  is noise power spectral density.

Frequency-selective channel: A small SER performance on FSCs is a major requirement for multicarrier modulation. The noise variance for the  $l^{th}$  subcarrier is given by

$$\sigma_l^2 = \frac{1}{MK} \sum_{k=0}^{MK-1} \left| \frac{G_{R_{l,m}}[-k]}{H[k]} \right|^2 \sigma_n^2 = \xi_l \sigma_n^2 = \xi_l \frac{N_0}{2} \quad (23)$$

where  $G_{R_{l,m}}[k]$  is the filter frequency response for the  $l^{th}$  subcarrier and  $m^{th}$  sub-symbol and  $\xi_l$  is the resultant NEF. for every  $m$ , equivalent noise variance is same. Therefore, the position of the filter in the frequency domain changes the NEF because the frequency response of multipath channels is not uniform.

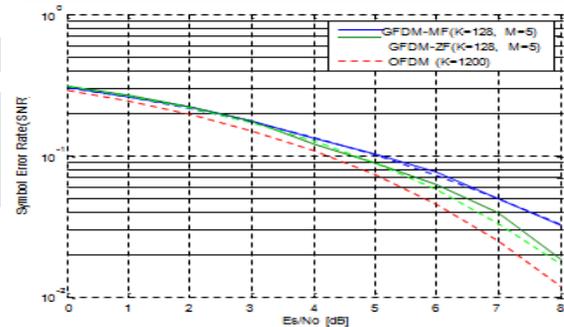
**V. RESULTS AND DISCUSSIONS**

The simulation results are presented in this chapter. In Figure 3, OFDM is having less SER than GFDM for the same signal to noise ratio in AWGN channel i.e. SER is 0.07 for OFDM and 0.1 for GFDM-MF for 5dB SNR. Orthogonality between subcarriers is lost due to cyclic pulse shaping filter. MF receiver cannot completely remove ICI and suffers from performance loss due to residual ICI while in ZF solution ICI is completely eliminated, since  $(A^H A)^{-1} A^H$  can have large value and its multiplication with  $y$  can result in noise in detected symbols, which increases SER. In Figure 4, the matched filter receiver shows the poorer performance since it is highly affected by ICI. In selective channel, equalizer also presents a noise enhancement and for lower value of SNR, this factor is more significant than the noise enhancement caused by the ZFR. The Figure 6 and 7 shows the SER performance of GFDM-OQAM is same to the OFDM system in AWGN channel. SER performance of ZF receiver is better than MF receiver because it can completely remove self-interference with the inverse transmission matrix, but it also introduces

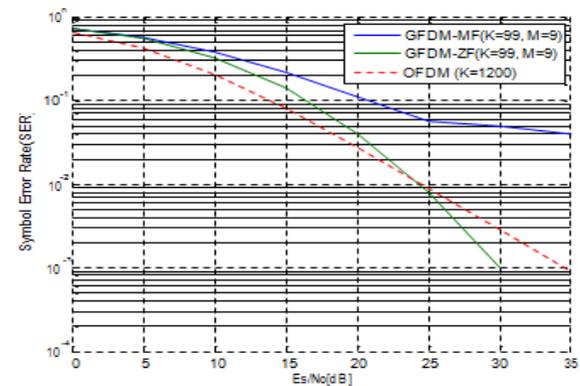
noise that is given by noise enhancement factor. The MMSE receiver has the best performance over MF and ZF receiver and it act as MF in low SNR and ZF receiver in high SNR, because the MMSE receiver has been designed by taking into account self-interference and noise enhancement.

**TABLE I Simulation Parameters**

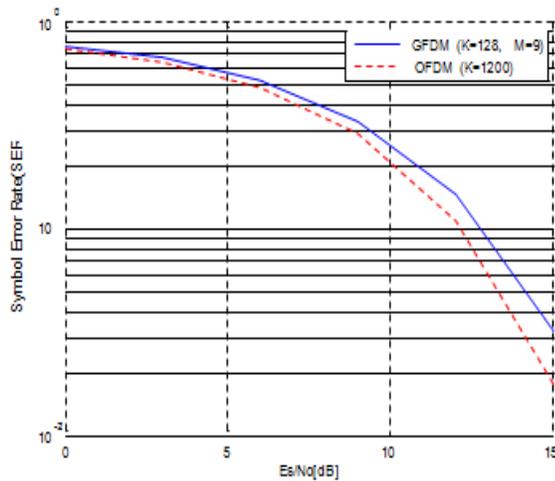
Specification	GFDM	OFDM
No. of subcarriers K	128	2048
No. of sub-symbols M	5 and 9	1
Subcarrier Spacing	240KHz	15KHz
Pulse shaping filter	RRC	Rectangular
Modulation index	4	4
Roll-off factor	0.3, 0.5	-
Modulation type	QAM	QAM
Delay(μs)	0, 0.52, 1.56, 3.13	1.56, 3.13



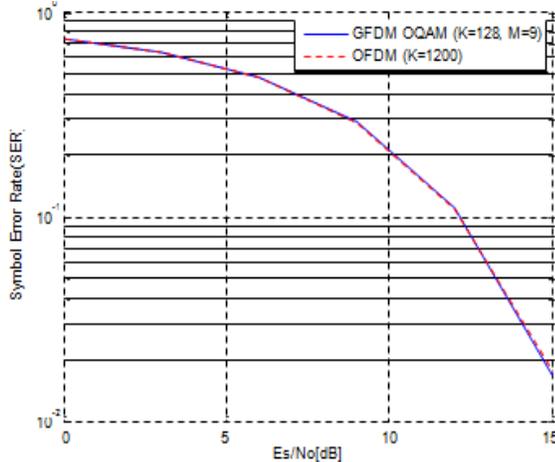
**Figure 3: SER performance of OFDM compared with GFDM using Matched Filter and Zero Forcing Receiver in AWGN channel**



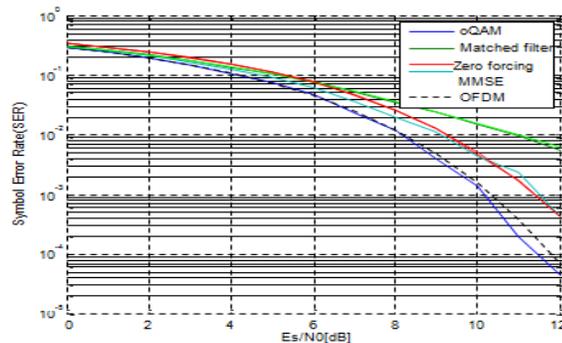
**Figure 4: SER performance of OFDM compared with GFDM using Matched Filter and Zero Forcing Receiver in FSC channel**



**Figure 5: SER performance of OFDM compared with GFDM in AWGN channel**



**Figure 6: SER performance of OFDM compared with GFDM-OQAM in AWGN channel**



**Figure 7: SER performances of GFDM-OQAM, MF, ZF, MMSE and OFDM in AWGN channel**

**VI. CONCLUSION**

GFDM has been introduced as a candidate waveform modulation scheme for the air interface of 5G networks. GFDM is a block type multicarrier modulation and allows to implement long filters or to reduce the total number of subcarriers which makes it more flexible. Due to non orthogonality of the subcarriers, OFDM outperforms conventional GFDM in AWGN channel and in FSC for above high SNR i.e. above 20 dB GFDM-ZF performs better than OFDM. Problem of non orthogonality is resolved by using GFDM offset quadrature amplitude modulation. SER performance of the GFDM-OQAM is same to that OFDM in AWGN channel. It has been shown that the modulation system can meet the needs of the different scenarios with its flexibility. However, GFDM is a new modulation technology with the potential to meet the needs of the next generation wireless mobile networks.

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