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Error Rate Analysis of PL Decoder in Dual Hop Dual Path Differential Cooperative System

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Abstract - **In this paper, we consider a Dual Hop Dual Path decode and forward based differential cooperative system and we derive the conditional probability density function (pdf), conditional cumulative distribution function (cdf) of the Piecewise Linear (PL) approximation of the decision variable of each Relay-Destination link used in the system. Utilizing this, we have also derived the average Bit Error Rate (BER) of the PL approximation for the case in which one of the Relay is in outage as compared to the other Relay and the data is received at the destination from the Relay closed to the source.**

Keywords: **Bit Error Rate, Decode and Forward, Differential Modulation, Log Likelihood Ratio, Performance Analysis, Piecewise Linear, Signal to Noise Ratio**

1.INTRODUCTION

In the cooperative communication system, it is not so easy to feedback/feed forward the values of the channel gains of the links involved in the system [1]. Differential modulation in cooperative system helps the destination to decode the data sent by the source without any knowledge of the channel gain of links used in the cooperative system. The data sent by the source is first differentially encoded and then it is transmitted, the relay receives and demodulates the differentially encoded data in a symbol wise manner [2] and transmits the data to the destination and this process is done in two phases to maintain orthogonality between the transmission. The destination possess the knowledge of average signal to noise ratio(SNR) of the source-relay link used in the cooperative communication system and the use of Piecewise Linear (PL) decoder improves the bit error rate(BER) and diversity performance in decode and forward (DF) based differential cooperative system [4]. The PL decoder also reduces the computational complexity in decoding of the differential data as compared to the maximum likelihood (ML) decoder [6]. In this paper, Differential modulation for a symbol wise decode and forward (DF) based cooperative system utilizing transmission with either of the relay present in the link is proposed, and the relay always forward the decoded data in the differential modulation based DF cooperative system. There is no direct source-destination link involved in the system. The conditional probability density function (pdf), conditional cumulative distribution function (cdf) of the data received at the destination from each relay-destination link is evaluated and plotted respectively. Also the average bit error rate (BER) of the PL approximation decision variable Y with BPSK constellation is evaluated and plotted for different average signal to noise ratio (SNR) of the relay-destination link for the case in which one of the relay is in outage and the other relay being close to the source receives the data correctly. The rest of the paper is organized as follows: In Section II, the system model is introduced. Section III explains the use of PL decoder of the BPSK data in the decode and forward based differential cooperative system at the destination. In Section IV, conditional CDF and PDF of each Relay-Destination link is evaluated and plotted. In Section V, performance analyses of the differential cooperative system is performed. Section VI concludes the paper.

II. SYSTEM MODEL

We design a cooperative system with one source (S), one destination (D) and two relays (R1 and R2) as shown in Fig1. Each of the node involved contains one antenna that can either receive or transmit the data at a time. The transmission of the symbols from the source (S) to the destination (D) is done in two orthogonal phases. The source broadcasts the data that is received by the relay close to the source during the first phase and the relay demodulates the data received from the source in a symbol-wise manner and transmits the data to the destination in the next phase with source being silent during the transmission from the relays. Let in the n-th time interval, the source transmits a M-PSK symbol $x[n] \in \{x1, x2, \ldots, xM\}$.The source before

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broadcasting the symbol performs the differential encoding as follows:

$$p[n]=p[n-1]x[n] \quad (1)$$

where p[0] = 1 (initialization symbols) and n = 1,2,…. As $|x[n]|^2 = 1$, therefore, $|p[n]|^2 = 1$. The symbol being broadcasted by S is received by either R_1 or R_2 in the n-th time interval can be written as $y_{s,r_1}[n] = h_{s,r_1}p[n] + e_{s,r_1}[n]$ (2)
$y_{s,r_2}[n] = h_{s,r_2}p[n] + e_{s,r_2}[n]$ (3) where h_{s,r_1} *and* h_{s,r_2} represents the circular complex Gaussian channel gain of the S-R_1 and S-R_2 link such that $h_{s,r_1} \sim \mathcal{CN}\left(0, \sigma^2_{s,r_1}\right)$ *and* $h_{s,r_2} \sim \mathcal{CN}\left(0, \sigma^2_{s,r_2}\right)$ where $\mathcal{CN}(\mu,\eta)$ denotes the complex Normal distribution with mean μ and variance η, $e_{s,r_1}[n]$and $e_{s,r_2}[n]$ is the AWGN noise with zero

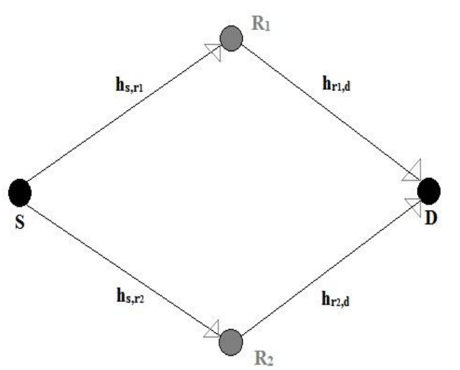

Fig1. Cooperative system with two relays

mean and N variance. It is assumed that the channel gains h_{s,r_1} *and* h_{s,r_2} remain constant for at least two consecutive time intervals 'n-1' and 'n'.From(1) and (2),we have

$$y_{s,r_1}[n] = y_{s,r_1}[n-1]x[n] + é_{s,r_1}[n] \quad (4)$$

where $é_{s,r_1}[n] = e_{s,r_1}[n] - e_{s,r_1}[n-1]x[n] \sim \mathcal{CN}(0,2N)$ is the AWGN noise.
From (1) and (3), we have

$$y_{s,r_2}[n] = y_{s,r_2}[n-1]x[n] + é_{s,r_2}[n] \quad (5)$$

where $é_{s,r_2}[n] = e_{s,r_2}[n] - e_{s,r_2}[n-1]x[n] \sim \mathcal{CN}(0,2N)$ is the AWGN noise.
Given $y_{s,r_1}[n-1]$, $y_{s,r_2}[n-1]$ and x[n] , it can be seen that $y_{s,r_1}[n] \sim \mathcal{CN}\left(y_{s,r_1}[n-1]x[n], 2N\right)$ and $y_{s,r_2}[n] \sim \mathcal{CN}\left(y_{s,r_2}[n-1]x[n], 2N\right)$. By maximizing the conditional pdf of $y_{s,r_1}[n]$ and $y_{s,r_2}[n]$ given that $y_{s,r_1}[n-1]$, $y_{s,r_2}[n-1]$ and x[n] are known in the relays, the ML decoder in the relay1 and relay2 can be obtained as

$$x_{r1}[n] = \max \mathrm{Re}\{y_{s,r_1}{}^*[n]y_{s,r_1}[n-1].x\} \quad (6)$$

$$x_{r2}[n] = \max \mathrm{Re}\{y_{s,r_2}{}^*[n]y_{s,r_2}[n-1].x\} \quad (7)$$

where x, $x_{r1}[n]$, $x_{r2}[n]$ ε BPSK constellation and $|x_{r1}[n]|^2 = |x_{r2}[n]|^2 = 1$. In the second phase, the relay differentially encodes $x_{r1}[n]$ into $v_{r1}[n]$ and $x_{r2}[n]$ into $v_{r2}[n]$ using (1) and transmits the encoded symbol into destination.

III. PL DECODER OF BPSK DATA IN THE DESTINATION

The destination receives the data from the relay in the n-th time interval will be

$$y_{r_1,d}[n] = h_{r_1,d}v_{r1}[n] + e_{r_1,d}[n] \quad (8)$$

and $y_{r_2,d}[n] = h_{r_2,d}v_{r2}[n] + e_{r_2,d}[n]$ (9)
$h_{r_1,d} \sim \mathcal{CN}\left(0, \sigma^2_{r_1,d}\right)$ *and* $h_{r_2,d} \sim \mathcal{CN}\left(0, \sigma^2_{r_2,d}\right)$ is the channel gain of the R_1-D link and R_2-D link and also $e_{r_1,d}[n]$ and $e_{r_2,d}[n]$ are the AWGN noise with 0 mean and N variance. It is assumed that the channel gains of the R-D links will remain constant for at least two consecutive time intervals 'n-1' and 'n'. From (1) and (8), we have

$$y_{r_1,d}[n] = y_{r_1,d}[n-1]x_{r1}[n] + é_{r_1,d}[n] \quad (10)$$

where $é_{r_1,d}[n] = e_{r_1,d}[n] - e_{r_1,d}[n-1]x[n] \sim \mathcal{CN}(0,2N)$ is the AWGN noise.From (1) and (9), we have

$$y_{r_2,d}[n] = y_{r_2,d}[n-1]x_{r2}[n] + é_{r_2,d}[n] \quad (11)$$

where $é_{r_2,d}[n] = e_{r_2,d}[n] - e_{r_2,d}[n-1]x[n] \sim \mathcal{CN}(0,2N)$ is the AWGN noise. Depending upon the erroneous demodulation of the data of the source in the relay, the conditional pdf of the data received in the destination can be written as

$$p_{y_{r_i,d}[n]}\left(y|y_{r_i,d}[n-1], x[n]\right) = (1 - \epsilon_i)p_{y_{r_i,d}[n]}\left(y|y_{r_i,d}[n-1], x_{ri}[n] = x[n]\right) + \epsilon_i\, p_{y_{r_i,d}[n]}\left(y|y_{r_i,d}[n-1], x_{ri}[n] \neq x[n]\right) \quad (12)$$

where ϵ_i is the average probability of error of the each S-R_i link, i=1 represents S-R_1 link and i=2 represents S-R_2 link.

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$p_{y_{r_i,d}[n]}\left(y|y_{r_i,d}[n-1], x_{ri}[n] \neq x[n]\right)$ gives the conditional pdf of Gaussian mixture R.V. and is written as

$$p_{y_{r_i,d}[n]}\left(y|y_{r_i,d}[n-1], x_{ri}[n] \neq x[n]\right) = \frac{1}{2\pi N_{r_id}} e^{\frac{-1}{2N_{r_id}}|y_{r_i,d}[n]-y_{r_i,d}[n-1]x_j|} \quad j\neq 1 \qquad (13)$$

where we assume that the source is transmitting the symbol $x[n] = x_1$ it can be seen that $p_{y_{r_i,d}[n]}\left(y|y_{r_i,d}[n-1], x_{ri}[n] = x[n]\right) \sim \mathcal{CN}(y_{r_i,d}[n-1]x[n], 2N_{r_id})$ and $y_{r_1,d}, y_{r_2,d}$ are independent . By maximizing the conditional pdf of $y_{r_i,d}[n]$ we have

$$z[n] = \arg\max \ \ln\left((1-\epsilon_i)e^{t_i} + \epsilon_i e^{t_j}\right) \qquad (14)$$

where j≠1 and i=1,2 represents S-R_1, S-R_2 and $t_i = e^{\frac{1}{N_{r_id}}Re\{y_{r_i,d}{}^*[n]\, y_{r_i,d}[n-1]x_1\}}$ and $t_j = e^{\frac{1}{N_{r_id}}Re\{y_{r_i,d}{}^*[n]\, y_{r_i,d}[n-1]x_j\}}$ follows the hermition quadratic form of Gaussian variates. Taking the log-likelihood ratio (LLR), symbol-wise decoder can be obtained to decide about x_1 =1 or x_2=-1 : $\Lambda_{1,2} \gtrless_{x_2}^{x_1} 0$ where $\Lambda_{1,2} = ln\frac{(a+s_1)}{(a+s_2)}$ (15) and

a= $\epsilon_i e^{\frac{1}{N_{r_id}}Re\{y_{r_i,d}{}^*[n]\, y_{r_i,d}[n-1]x_j\}}$ j≠1,2,

s_1=(1- ϵ_i) $e^{\frac{1}{N_{r_id}}Re\{y_{r_i,d}{}^*[n]\, y_{r_i,d}[n-1]x_1\}}$ + $\epsilon_i\, e^{\frac{1}{N_{r_id}}Re\{y_{r_i,d}{}^*[n]\, y_{r_i,d}[n-1]x_2\}}$ (symbol x_1 is transmitted) , s_2= (1- ϵ_i) $e^{\frac{1}{N_{r_id}}Re\{y_{r_i,d}{}^*[n]\, y_{r_i,d}[n-1]x_2\}}$ + $\epsilon_i e^{\frac{1}{N_{r_id}}Re\{y_{r_i,d}{}^*[n]\, y_{r_i,d}[n-1]x_1\}}$ (symbol x_2 is transmitted) . Since a, $s_1, s_2 > 0$, then after simple algebra the decision rule $ln\frac{(a+s_1)}{(a+s_2)} \gtrless_{x_2}^{x_1} 0$ is equivalent to $ln\frac{(s_1)}{(s_2)} \gtrless_{x_2}^{x_1} 0$. Therefore the LLR decoder at the destination is $\Lambda_{1,2} = f(t_i) = \ln\left(\frac{(1-\epsilon_i)e^{t_i}+\epsilon_i}{(1-\epsilon_i)+\epsilon_i e^{t_i}}\right)$ (16) is a PL function [3] that clips to –T for smaller values of t_i and clips to T for larger values of t_i which can be written as :

$$f(t_i) = f_{PL}(t_i) = \begin{cases} -T\,, if & t_i < -T \\ t_i\,, if & -T \leq t_i \leq T \\ T, if & t_i > T \end{cases} \qquad (17)$$

where $t_i = e^{\frac{1}{N_{r_id}}Re\{y_{r_i,d}{}^*[n]\, y_{r_i,d}[n-1]x_1\}}$ follows the hermition quadratic form of gaussian variates $and\ T = ln\frac{(1-\varepsilon_i)}{\epsilon_i}$. From (14) and (17), the following PL decoder in the destination with two relays and no direct link between source and destination can be written as: $\Lambda_{1,2}^d = \sum_{i=1}^{2} f_{PL}(t_i)$ Here i=1 represents R_1-D link and i=2 represents R_2-D link. The PL decoder is applied in a pair-wise

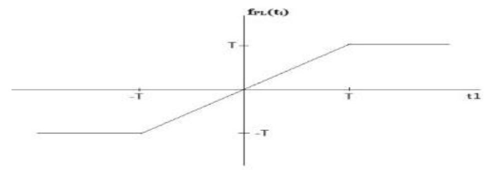

Fig2. Plot of PL function $f_{PL}(t_i)$ vs t_i

manner to the constellation points for taking a decision about the symbol transmitted by the source S. The conditional probability density function (pdf), conditional cumulative distribution function (cdf) of the data received from each of the relay-destination link is evaluated and plotted and average BER is also evaluated at the destination for the case in which one of relay is in outage and the data is received at the destination from the relay close to the source.

IV. CONDITIONAL CDF AND PDF OF THE PL APPROXIMATION DECISION VARIABLE OF EACH R-D LINK

Let Y be a R.V where Y= $\Lambda_{1,2}^d = \sum_{i=1}^{2} f_{PL}(t_i)$. Here, we consider that one of the Relay is in outage as compared to the other Relay and the Relay closed to the source will receive the data sent from the source, differentially encodes it and transmits it to the destination. Therefore, the decision variable Y≈ $f_{PL}(t_i)$ where 'i' can be either 1 or 2. $F_Y(y)|\gamma_i$ represents the conditional CDF of the decision variable 'Y' which is a PL function of one of the Relay-Destination link obtained from equation (17) for y < -T, $f_{PL}(t_i) \leq$ y for no t_i , $F_Y(y)$=0 for y > T, $f_{PL}(t_i) \leq$ y for ∀ t_i , $F_Y(y)$= 1 for –T< y < T, $f_{PL}(t_i)\leq$ y for $t_i \leq$ y ,$F_Y(y)$= $F_{t_i}(y)$ and $F_Y(y)$ is discontinuous for y= -T and y= T.

1) **for y = -T,** $F_Y(-T)$ can be obtained from [1,equation (27)] which is written as $F_Y(-T)$=

$$\frac{1}{2}\sum_{k_i=0}^{\infty}\sum_{n_i=0}^{k_i} e^{-\left(2|x^2|-\frac{b_i}{8}\right)\gamma_i} \cdot \frac{\gamma_i{}^{k_i} c_i{}^{k_i}}{k_i!4^{k_i}} \frac{\Gamma(k_i-n_i+1,2T)}{2^{n_i}\,(k_i-n_i)!} L_{n_i}\left(\frac{-b_i\gamma_i}{8}\right) \qquad (18)$$

where x= x_1-x_2, b_i=$2(2|x|^2 + x_i^*x + x_ix^*)$, c_i= $2(2|x|^2 - x_i^*x - x_ix^*)$ from [1] , γ_i represents the instantaneous SNR of the R_i-D link , $\Gamma(.,.)$ denotes the incomplete gamma function, $L_{n_i}(.)$ denotes the Laguerre polynomial and i=1 represents the R_1-D link, i=2 represents R_2-D link.

2) **for -T < y < 0,** $F_Y(y)$ is written as $F_Y(y)=$

$$\frac{1}{2}\sum_{k_i=0}^{\infty}\sum_{n_i=0}^{k_i} e^{-\left(2|x^2|-\frac{b_i}{8}\right)\gamma_i} \cdot \frac{{\gamma_i}^{k_i}{c_i}^{k_i}}{k_i!4^{k_i}}\frac{\Gamma(k_i-n_i+1,-2y)}{2^{n_i}\,(k_i-n_i)!}L_{n_i}\left(\frac{-b_i\,\gamma_i}{8}\right) \quad (19)$$

3) **for 0 ≤ y < T,** $F_Y(y)$ can be obtained from [1,equation (28)] which is written as

$$F_Y(y)= 1-\frac{1}{2}\sum_{k_i=0}^{\infty}\sum_{n_i=0}^{k_i} e^{-\left(2|x^2|-\frac{c_1}{8}\right)\gamma_i}\frac{{\gamma_i}^{k_i}{b_1}^{k_i}}{k_i!4^{k_i}}\frac{\Gamma(k_i-n_i+1,2y)}{2^{n_1}\,(k_i-n_i)!}L_{n_i}\left(\frac{-c_1\,\gamma_i}{8}\right) \quad (20)$$

4) **for y = T** , $F_Y(T)$ is written as $F_Y(T)=$

$$1-\frac{1}{2}\sum_{k_i=0}^{\infty}\sum_{n_i=0}^{k_i} e^{-\left(2|x^2|-\frac{c_1}{8}\right)\gamma_i}\frac{{\gamma_i}^{k_i}{b_1}^{k_i}}{k_i!4^{k_i}}\frac{\Gamma(k_i-n_i+1,2T)}{2^{n_1}\,(k_i-n_i)!}L_{n_i}\left(\frac{-c_1\,\gamma_i}{8}\right) \quad (21)$$

using equation (18),(19),(20),(21) we evaluate the conditional CDF $F_Y(y)|\gamma_i$ which is written as , $F_Y(T)|\gamma_i= \int_{-\infty}^{T}(f_{\Lambda_{1,2}^d}(y)|_{\gamma_i})dy = \int_{-\infty}^{-T}(f_{\Lambda_{1,2}^d}(y)|_{\gamma_i})dy + \int_{-T}^{0}(f_{\Lambda_{1,2}^d}(y)|_{\gamma_i})dy + \int_{0}^{T}(f_{\Lambda_{1,2}^d}(y)|_{\gamma_i})dy$ where $f_{\Lambda_{1,2}^d}(y)|_{\gamma_i}$ is the conditional pdf of the PL function $f_{PL}(t_i)$.

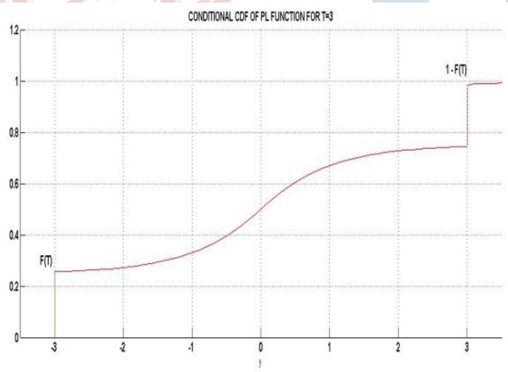

Fig3.1.Conditional CDF $F_Y(y)|\gamma_i$ vs y of Decision variable 'Y' for T=3($\epsilon_i=10^{-2}$)

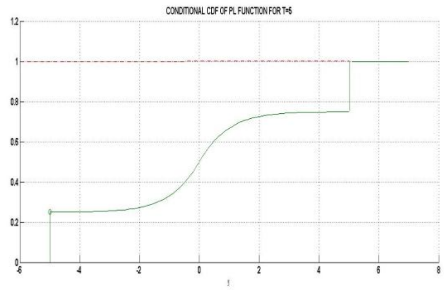

Fig3.2.Conditional CDF $F_Y(y)|\gamma_i$ vs y of Decision variable 'Y' for T=5($\epsilon_i=10^{-3}$)

Fig3.shows the plot of conditional CDF of the PL function $f_{PL}(t_i)$ where i=1 represents Relay1-Destination link and i=2 represents Relay2-Destination link. The conditional PDF of the PL function $f_{PL}(t_i)$ can be obtained by differentiating its conditional CDF $F_Y(y)|\gamma_i$ which is written as

$$f_Y(y)|\gamma_i = \begin{cases} F_Y(-T),\ y=-T \\ f_{t_i}(y), -T<y<T \\ 1-F_Y(T),\ y=T \\ 0, otherwise \end{cases} \quad (22)$$

where $F_Y(-T)$ and $1-F_Y(T)$ are evaluated using equation (18) and (21). For range $-T<y<T$, $f_Y(y) = f_{t_i}(y)$ is the conditional pdf of t_i which can be obtained from [1,equation(15)] and [7] is written as, 1) for y> 0 , $f_Y(y)|\gamma_i=$

$$\sum_{k_1=0}^{\infty}\sum_{n_1=0}^{\infty} e^{-\left(2y+\frac{|x^2|\gamma_{r_i,d}}{2}+\frac{c_1\gamma_{r_i,d}}{8}\right)} y^{(k_1-n_1)}\frac{{\gamma_{r_i,d}}^{k_1} b_1^{k_1}}{k_1!(k_1-n_1)!2^{(k_1+n_1)}}L_{n_1}\left(\frac{-c_1\gamma_{r_i,d}}{8}\right) \quad (23)$$

2)for y<0, $f_Y(y)|\gamma_i=$

$$\sum_{k_1=0}^{\infty}\sum_{n_1=0}^{\infty} e^{-\left(-2y+\frac{|x^2|\gamma_{r_i,d}}{2}+\frac{b_1\gamma_{r_i,d}}{8}\right)} (-y)^{(k_1-n_1)}\frac{{\gamma_{r_i,d}}^{k_1} c_1^{k_1}}{k_1!(k_1-n_1)!2^{(k_1+n_1)}}L_{n_1}\left(\frac{-b_1\gamma_{r_i,d}}{8}\right) \quad (24)$$

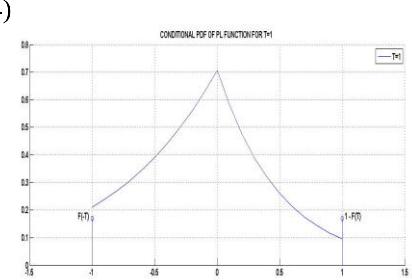

Fig4.1 Conditional pdf $f_Y(y)|\gamma_i$ vs y[-T,T] of Decision variable'Y' for T=1($\epsilon_i=10^{-1}$)

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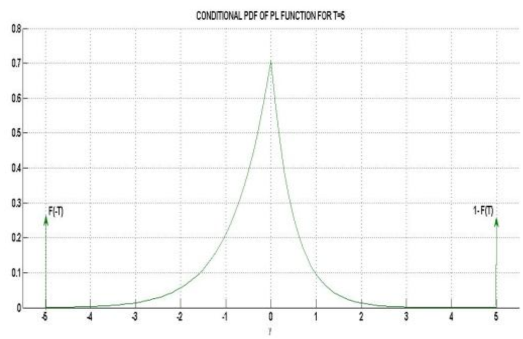

Fig4.2 Conditional pdf $f_Y(y)|\gamma_i$ vs y[-T,T] of Decision variable'Y' for T=5(ϵ_i=10^{-3})

using equation (18), (21),(23) and (24), the conditional probability density function of the PL approximation decision variable is evaluated and is shown in Fig.4 for the complete range of y.

V.PERFORMANCE ANALYSIS OF THE DIFFERENTIAL COOPERATIVE SYSTEM WITH BPSK CONSTELLATION

In this section, we will find the bit error rate (BER) for the decision variable Y of the DF based differential cooperative system with BPSK constellation. Let x_1=1 be the transmitted BPSK symbol and the destination wrongly decides that the transmitted symbol is ẋ=-1 then the conditional Bit Error Rate at the destination receiver is given as

$P_e^{x1,x2}|\gamma_i = P_r\{\Lambda_{1,2}^d < 0\}=F_{\Lambda_{1,2}^d}(0)$ (25)

Considering the case in which Relay 1 is in outage and Relay 2 is close to the source receiving the data correctly, then the decision variable Y = $\Lambda_{1,2}^d \approx f_{PL}(t_2)$ and the conditional BER is written as $P_e^{x1,x2}|\gamma_i$= $F_Y(0)$ which is obtained from equation (18) and (19). The average BER is calculated as $P_e^{x1,x2}$=$\int_0^\infty P_e^{x1,x2}|\gamma_i\, p_{\gamma_{r2,d}}(\gamma_2)d\gamma_2$ (26) where $p_{\gamma_{r2,d}}(\gamma_2) = \frac{1}{\bar{\Upsilon}_{r_{2,d}}} e^{\frac{-\gamma_2}{\bar{\Upsilon}_{r_{2,d}}}}$ (27) is the pdf of instantaneous SNR of the second relay-destination link and $\bar{\Upsilon}_{r_{2,d}}$ is the average SNR . (If the channel gains are complex-valued Gaussian random variables then the pdf of instantaneous SNR of the relay-destination link will be Xi-square distributed).Therefore, $P_e^{x1,x2}$=$\int_0^\infty F_Y(0)\, p_{\gamma_{r2,d}}(\gamma_2)d\gamma_2$ where

$F_Y(0)$=

$$\frac{1}{2}\sum_{k_i=0}^{\infty}\sum_{n_i=0}^{k_i} e^{-\left(2|x^2|-\frac{b_i}{8}\right)\gamma_i} . \frac{{\gamma_i}^{k_i}{c_i}^{k_i}}{k_i!4^{k_i}} \frac{\Gamma(k_i-n_i+1,2T)}{2^{n_i}\,(k_i-n_i)!} L_{n_i}(\frac{-b_i\,\gamma_i}{8})$$

$+$

$$\frac{1}{2}\sum_{k_i=0}^{\infty}\sum_{n_i=0}^{k_i} e^{-\left(2|x^2|-\frac{b_i}{8}\right)\gamma_i} . \frac{{\gamma_i}^{k_i}{c_i}^{k_i}}{k_i!4^{k_i}} L_{n_i}(\frac{-b_i\,\gamma_i}{8}) \int_{-T+0.1}^{0} \frac{\Gamma(k_i-n_i+1,-2y)}{2^{n_i}\,(k_i-n_i)!} dy$$

,

(28) On substituting the values of $F_Y(0)$ and $p_{\gamma_{r2,d}}(\gamma_2)$ in equation (24), the equation comes out to be of the form

$\int_0^\infty x^{\alpha-1}e^{-px}L_v(cx)dx = \frac{\Gamma(\alpha)}{(p-c)^\alpha}\,{}_2F_1(\alpha,v+1;1;\frac{c}{(c-p)})$ (29)

where ${}_2F_1(\alpha,\beta;1;z)$ is the Gauss hypergeometric function, $\Gamma(.)$ represents the complete gamma function, $L_v(.)$ is the Laguerre polynomial . The average BER of the differential cooperative system for the case in which relay 2 being close to the source is receiving the data and the other relay is far from the source is calculated and the plot is shown in Fig5 for different values of average SNR of relay-destination link.

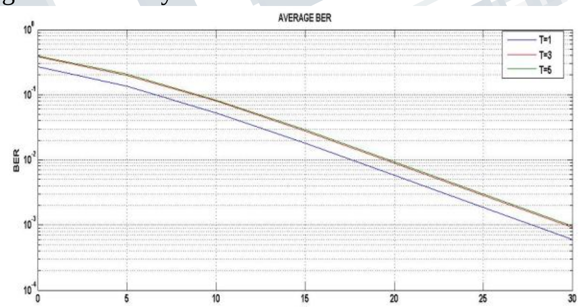

Fig5.Performance Analysis of the PL Decoder where 0 dB ≤ $\bar{\Upsilon}_{r_2,d}$ ≤ 30 dB,for T=1,3 and 5

VI.CONCLUSIONS

We have derived the conditional cumulative distribution function (cdf) , conditional probability distribution function (pdf) of the Piecewise Linear (PL) approximation of the decision variable of each Relay-Destination link. Moreover, we have also derived the average Bit Error Rate of the PL decoder for the case in which one Relay is in outage and other Relay is close to the Source.

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