

On the Output Voltage Harmonics of a Cuk Converter

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Abstract: -- This paper focuses on the analysis of the harmonic content in the output voltage of a non-isolated, non-synchronous, open loop Cuk converter. Further, it describes the design of a LC low pass filter which is to be used in cascade with the Cuk converter so as to achieve a reduction in the ripple content of the output voltage of the Cuk converter. In the end, this paper gives a comparison of the overall conversion efficiency of the Cuk converter and the ripple content in the output voltage at the load terminals with and without the LC low pass filter which would enable one to make a decision whether to use such a filter in cascade with a DC to DC converter or not.

Keywords:-- DC to DC converter, Harmonic distortion, Fourier series, Voltage ripple.

I. INTRODUCTION

DC to DC converters are desired to have constant output quantities (Output Voltage and Output Current). Non ideal sources and Non-linear loads add to generation of harmonics. The total harmonic distortion gives a measure of the variation or deviation of the output quantity from the desired value. Fourier series of the output signal describes the harmonic content of the signal (either voltage or current) in terms of its fundamental frequency. Harmonics are present in both transient and steady states of a system, but the transient state harmonic analysis isn't necessary in most of the cases as the transient state, typically, lasts only for a few milliseconds, whereas, the steady state typically lasts till the device is shut down by some means. Hence, steady state harmonic analysis becomes important, as these harmonics cause many ill effects to an electrical system, some of which are described below.

Harmonics can increase the neutral current and hence be harmful for the operation of a system. Harmonics in high voltage systems are more dangerous as they aid setting up of oscillations which can make the system unstable. In non-linear loads like DC motors, harmonics can lead to higher core losses than the supply frequency would. In induction motors, odd harmonics of higher order (typically, the 5th harmonic) set up torques in the direction opposite to the direction of rotation of the motor, which leads to the phenomenon of crawling.

The latest trends of power generation involve renewable sources and the power distribution involves design of more efficient power transmission systems. Harmonics present in the generated power can reduce the efficiency of

transmission to a great extent. This paper analyzes the harmonic content in the output voltage of an open loop, non – isolated and non-synchronous Cuk converter, referred to as the Cuk converter throughout the paper. A particular duty cycle is to be set for a given input voltage, to obtain the desired output voltage.

The output voltage waveform is obtained from simulations and its mathematical equation is derived, keeping in mind, the sampling rate of the simulation software. A frequency domain analysis of the signal gives the complete information about the harmonics present, that is, their magnitudes and frequencies (and also phases, if we go for a Fourier transform). Further, it aids us in deciding the cut-off frequency of the filters, which have to be designed in order to eliminate these harmonics, if need be.

Ideally, any DC to DC converter shouldn't have any harmonic content in the output, but practically, all of such converters produce harmonics. Approximation becomes essential for simplification of the analysis. The output voltage waveform obtained using simulation shows less harmonic content than the waveform obtained by practically rigging up the circuit, as a few parameters like distortions in the input DC voltage isn't considered during simulation, but, the mathematical analysis of harmonics to such a level of precision isn't necessary in this case, as most of the vital parameters which are responsible for the distortion of output voltage waveform are considered in the simulation model. Hence, such a trade-off, for the ease of mathematical analysis doesn't affect the efficacy of this paper.

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Cuk converter are small when used in conjunction with systems that run on Low Voltage DC (typically, systems that involve DC to DC conversion from 170V to 12V/5V/3.3V), but, usually DC to DC converters are followed by non-linear loads like computers and presence of harmonics could be detrimental to the functioning of such devices. Hence, harmonic analysis becomes important, even though the distortion in output voltage is small when compared to other DC to DC converters.

Usually, the ripples in the steady state are periodic. Any periodic function can be expressed as a sum of a certain number of sinusoidal waves (Fourier series). Therefore, by approximating the steady-state response of a Cuk converter to a periodic waveform, we can analyse the frequency components of the signal and make a decision on implementing additional circuitry for the elimination of harmonics.

This paper organization is as follows: In section II, the Simulink model of the Cuk converter is described and its output waveform is examined. Section III consists of Fourier Analysis. Section IV deals with filter design for the converter. Section V gives the comparison of various parameter based on which the requirement of a filter can be analyzed.

II. THE CUK CONVERTER

A Cuk converter is similar to the buck-boost converter in operation, with the output voltage having opposite polarity. Here, a capacitor acts as the energy storage system, in contrast to a buck boost converter, where an inductor acts as the energy storing element. At the end of each commutation process, the energy of the inductor must remain the same. And hence the average voltage across the inductor over a commutation cycle must be zero. This is the basic principle of operation of a Cuk converter.

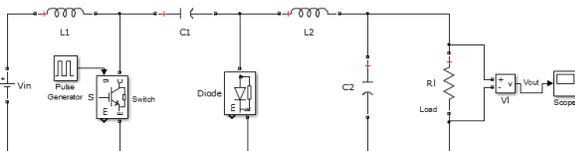


Figure 1: Circuit diagram and simulation model of a Cuk converter.

The circuit diagram of a Cuk converter is as shown in Figure 1. It uses an IGBT as a switch and a pulse generator to control the switch. The output is read across the load resistance R_L . The Cuk converter has been designed to have a maximum voltage ripple of 5% and a maximum current ripple of 10%. To achieve the same, the required parameters of the Cuk converter are as shown in table 1.

Table 1: Parameters of the Cuk converter

L1	6.5mH
L2	5.1mH
C1	6.8mF
C2	10μF
RL	1.48Ω

The expected output voltage of the converter for an input DC voltage of 12V, is -12V DC. This is achieved by setting the duty cycle of the control signal to 55.58% instead of 50%, owing to the non-ideal components in the circuit.

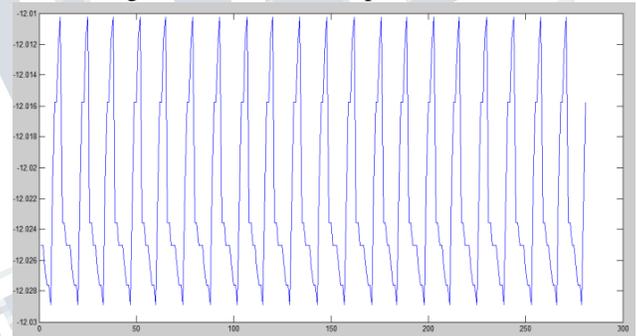


Figure 2: Actual output voltage of a Cuk converter, for an input voltage of 12V (DC)

The actual output of the Cuk converter, in steady state is as shown in Figure 2.

As it is evident from figure 2, the output voltage has a ripple of around 30mV. The output voltage waveform can be approximated to a triangular wave, in other words, as a combination of two ramp functions, without a significant loss of accuracy.

The mathematical equation describing the above wave for one particular time period is as described by equations (1) and (2). It is a piecewise function of two ramps which approximate the ripple in the output voltage of the Cuk converter.

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III. ANALYSIS USING FOURIER SERIES

As stated earlier, if a function is periodic, it can be expressed as a sum of a certain number of sinusoidal waves, which is famous as the Fourier series of the function.

The general form of a Fourier Series is given by:

$$V(t) = \begin{cases} 857.3613 - 869.5652 \cdot t & (0.999777s < t < 0.999800s) \text{ (1)} \\ 1333.3333 \cdot t - 1345.0966 & (0.999800s < t < 0.999815s) \text{ (2)} \end{cases}$$

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The general form of a Fourier Series is given by:

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right) \quad (3)$$

where a_n, b_n are called Fourier Coefficients and n is the order of the harmonic.

In the above equation, the value of a_0 is the average value of the function. For evaluating the Fourier coefficients, we have Euler's Formulae. For a periodic function $v(t)$ with a period of T ,

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$$a_0 = \frac{1}{T} \int_{kT}^{(k+1)T} v(t) dt \quad (4)$$

Evaluating the above integral for the given function defined in a time-period, we get the value of a_0 as -12.02 V. Thus, the average value of DC output of the Cuk converter is -12.02V.

The value of a_n can be evaluated by the following formula:

$$a_n = \frac{2}{T} \int_{kT}^{(k+1)T} v(t) \cos\left(\frac{n\pi t}{T}\right) dt \quad (5)$$

The value of b_n can be evaluated by the following formula:

$$b_n = \frac{2}{T} \int_{kT}^{(k+1)T} v(t) \sin\left(\frac{n\pi t}{T}\right) dt \quad (6)$$

The expressions of Fourier coefficients depends on the value of n . As n varies from 1 to the series converges to the defined function. To study the frequency components

present in the output voltage, one must observe the nature of Fourier coefficients. The Fourier series coefficients for various values of n are as shown in table 2.

Table 2: Values of Fourier series coefficients for different values of n , obtained using Wolfram Alpha.

n	a_n	b_n
1	14.7998	3.94053
2	-0.0158774	-0.0120833
3	-3.62513	-3.58544
4	0.00745662	0.016747
5	0.816591	2.94881
6	0.000302744	-0.0124716
7	0.540164	-2.11802
8	-0.00607658	0.0111309
9	-1.18326	1.22016
10	0.0104329	0.00629569

Table 2 gives the values of the Fourier series coefficients for different values of n .

Therefore, the output voltage of the Cuk converter can be expressed as

$$V(t) = -12.02 + (14.9778\sin(82673.5 \cdot t) - 0.01587\sin(2 \cdot 82673.5 \cdot t) + (3.94053\cos(82673.5 \cdot t) - 0.012083\cos(2 \cdot 82673.5 \cdot t)) \dots \quad (7)$$

As it is evident from the equations for

and, as value of n increases, the value of the coefficients decreases. Consequently, the amplitude of the harmonics decreases, as the order increases. Therefore, harmonics of very high order contribute very less to the total harmonic distortion of the output voltage of the Cuk converter. Hence, we can set a limit for the order of harmonics, beyond which, we can assume that there is no contribution from those harmonics to the total harmonic distortion of the output voltage of the Cuk converter.

Introducing passive low pass filters in cascade with the Cuk converter not only results in the attenuation of

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significant harmonics but also increases the order of the system and decreases the overall conversion efficiency (Due to the introduction of new passive elements). Increase in the order of the system directly results in increased complexity for the control of the system. Therefore, a trade-off has to be made between the total harmonic distortion in the output voltage and the conversion efficiency and complexity of control of the system. Simulations come in handy, when one has to make a decision on introducing a filter in cascade with the Cuk converter or to live with the harmonics.

IV. FILTER DESIGN

In the previous section on analysis using Fourier series, it was found that the fundamental frequency of the output voltage of the Cuk converter is 13.157kHz. As we need only the DC component of the output voltage to be present at the load terminals, the filter being introduced should function such that even the fundamental frequency is attenuated to a considerable extent. As the fundamental frequency is 13.157kHz, the cut-off frequency of the passive low pass filter can be taken to be around 15% to 20% less than the fundamental frequency, so that a considerable attenuation is achieved for the fundamental frequency. In this case, the cut-off frequency of the filter is set to 10kHz. Also, it is evident that if the fundamental frequency gets attenuated to a considerable extent, all the other harmonic components will get attenuated to a higher extent, as they have a higher frequency and lower amplitude when compared to the fundamental frequency. The cut-off frequency of an LC low pass filter is given by:

$$f = \frac{1}{2\pi\sqrt{LC}} \text{ Hz} \tag{8}$$

For a cut-off frequency of 10kHz, the value of C3 is taken to be 1μF. Substituting that in equation (8), the value of L3 is calculated to be 253.30μH. The circuit diagram of the Cuk converter in cascade with the LC filter is as shown in figure 3.

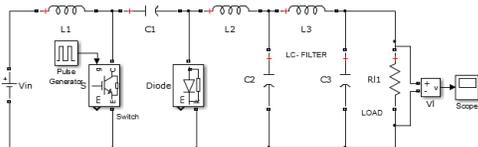


Figure 3: Circuit diagram and simulation model of a Cuk converter cascaded with a LC low pass filter.

The output voltage, as seen across the load terminals is as shown in figure 4.

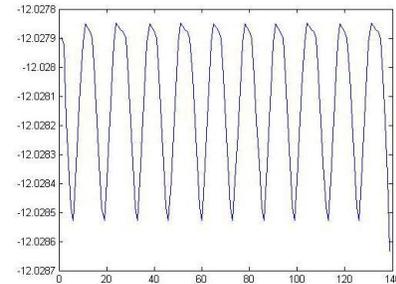


Figure 4: Actual output voltage of the Cuk converter cascaded with a LC low pass filter, for an input voltage of 12V (DC).

The output voltage waveform shown in figure 4 can also be approximated to a periodic wave and analysis similar to what was done in the section III is repeated.

The mathematical expression for the filtered ripple wave for a defined time period is:

$$V(t) \equiv \begin{cases} 16.0866 - 28.125 * t & (0.999625s < t < 0.999657s) \tag{9} \\ 100 * t - 111.9944 & (0.999657s < t < 0.999666s) \tag{10} \end{cases}$$

Table 3 gives the Fourier series coefficients of the above waveform.

Table 3: Fourier series coefficients of the output voltage of the Cuk converter cascaded with a LC low pass filter, obtained using Wolfram Alpha.

n	a_n	b_n
1	9.4328	11.8491
2	-0.51506	2.2399
3	4.1340	-1.9782
4	1.9722	0.9580
5	-0.4843	-2.1636
6	1.2691	-1.5750
7	-1.0573	-0.0062
8	-1.1114	-1.4132
9	-0.7752	0.3286
10	-1.3844	0.6523

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It is evident from table 3 that the Fourier series coefficients of the output voltage waveform after filtering is lesser than the non-filtered output. Also, it follows the same trend, that is, with increasing values of n , the value of the coefficients decrease. Hence, the higher order harmonics of the filtered output can be completely neglected as they do not reflect in the output waveform significantly.

Also, the ripple in the output voltage after filtering is around 0.9mV, which is evident from figure 4. This is around 3% of the original ripple, which is a significant decrease when compared to the original value of 30mV.

V. CALCULATION AND COMPARISON OF EFFICIENCY

One of the main concerns before introduction of the filter was the anticipated reduction in efficiency. Hence, a comparison of the overall conversion efficiency with and without the filter is required to make a decision on implementing the filter.

The overall efficiency of the converter is defined as the ratio of output power and input power. Table 4 gives a comparison of the overall efficiency of the filter in both the cases.

Table 4: Input voltage and current of the Cuk converter with and without the LC low pass filter.

Parameter/Case	Input Voltage	Input Current
Without filter	12V	10.90A
With filter	12V	10.94A

Table 5: Output voltage and current of the Cuk converter with and without the LC low pass filter.

Parameter/Case	Output Voltage	Output Current
Without filter	12.02V	8.175A
With filter	12.02V	8.108A

Table 6: Input and output power of the Cuk converter with and without the LC low pass filter.

Parameter/Case	Input Power	Output Power
Without filter	130.8W	98.2635W
With filter	131.28W	97.4581W

Table 7: Efficiency and ripple in output voltage of the Cuk converter with and without the LC low pass filter.

Parameter/Case	Efficiency	Ripple
Without filter	75.125%	30mV
With filter	74.237%	0.9mV

As it is evident from table 7, for a dip in efficiency by less than 1%, 97% of the ripple in the output voltage of the Cuk converter could be eliminated.

VI. CONCLUSIONS

When DC to DC converters are cascaded with LC low pass filters, there is a significant decrease in the ripple content of the output voltage waveform, which could be achieved as a result of a small dip in conversion efficiency. A decision has to be made about introducing the filter, keeping in mind the increased complexity of control, increase in bulkiness of the circuit as a result of the extra inductor added, and increase in cost.

Also, many systems which run on DC have a specific amount of tolerance for the ripple. If the ripple content in the output of the converter is within the limits, there is no need to add additional circuitry for the suppression of the ripple content in the output. This would even have the added advantage of simpler control and marginally higher efficiency.

Therefore, usage of a LC low pass filter to suppress the harmonic content in the output of a DC to DC converter has to be decided on a case by case basis.

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