

Complexity Reduction of MIMO Decoder

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Abstract: -- The data rates and the supported range in communication systems, can be increased using MIMO (Multiple input and multiple output) technique. MIMO technique uses multiple antennas at both transmitter and receiver. MIMO systems uses Orthogonal frequency division multiplexing (OFDM) technique for multicarrier modulation. QR decomposition (QRD) is the first step in the decoding of the MIMO receiver. Gram Schmidt, Householder and Givens described QR decomposition method which are computationally intensive as these involve division operation for normalization. The computation complexity of these methods for MIMO-OFDM systems is difficult to handle because QR decomposition is performed for each subcarrier. Sphere decoder is an efficient decoder for MIMO systems. In this paper we use Modified Householder's method for reducing the computation complexity without affecting the system packet error rate (PER) performance. The simulation process is carried out in all different models of 802.11 TGAC channels.

Keywords: -- MIMO decoder, QR decomposition, Sphere decoder.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) is one of the main techniques for achieving high throughput in wireless communication. Currently many upcoming cutting edge wireless communication technologies like, IEEE 802.11ac, IEEE 802.16e and LTE adapted MIMO technology. With MIMO technology, the complexity of the signal processing to detect the transmitted signal with low probability of error increases. This process is usually termed as MIMO detector or decoder. The Singular Value Decomposition (SVD) [1] technique is used in closed loop MIMO systems. In the open loop MIMO systems, QR decomposition (QRD) of the channel matrix is generally used in the MIMO detection module due to its simplicity. The QR decomposition can be used in numerous MIMO decoders like Zero Forcing (ZF), minimum mean squared error (MMSE), and maximum likelihood performance achieving sphere decoder [2]. To support all the MIMO decoders mentioned above, a high efficient reduced complex QR decomposition module is needed. The QRD factorizes the channel matrix H in to the product of a unitary matrix Q and an upper triangular matrix R . The authors in [3] have proposed a QR decomposition method that requires fewer multiplications compared to conventional methods. But, there is no mention about the number of divisions required for QR decomposition, which consume significant resources when implemented. The Sorted QR decomposition as discussed in [4] reduces the complexity in terms of multipliers, but it

requires several square root operations and divisions. There have been numerous papers in the literature, focusing on reducing the complexity of MIMO decoders in terms of multipliers, adders and not on divisions and square root operations required. Modified Householder method reduces complexity for MIMO decoder using QRD without any degradation in system performance. We provide packet error rate (PER) simulation results with proposed method and the conventional method for wireless LAN systems based on IEEE 802.11ac.

II. MIMO SYSTEM MODEL AND DETECTION

A. System Model

MIMO system model with n_T transmit and n_R receive antennas as show in Fig. 1. The matrix H_c describes the $n_R \times n_T$ complex MIMO channel, x_c is the $n_T \times 1$ modulated complex transmitted vector and is given by $x_c = [x_1 \dots \dots \dots x_{n_T}]$ The complex received vector $y_c = [y_1 \dots \dots \dots y_{n_T}]$ of dimension $n_T \times 1$ can be expressed as

$$y_c = H_c x_c + n_c \quad (1)$$

where the channel H_c can be described as

$$H_c = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1n_T} \\ h_{21} & h_{22} & \dots & h_{2n_T} \\ \cdot & \cdot & \dots & \cdot \\ h_{n_R 1} & \cdot & \dots & h_{n_R n_T} \end{bmatrix} \quad (2)$$

and n_c is a $n_R \times 1$ complex additive white Gaussian noise vector and subscript 'c' indicates that the matrices and the vectors have complex elements.

B. MIMO Detection based on QR decomposition

In MIMO systems with spatial mapping scheme, multiple signals are transmitted independently over the channel to increase the data transmission rate. At the receiver, separating these signals combined in the channel poses a big challenge. Here, x_c has to be estimated from y_c assuming channel information is known. Using QR decomposition, the channel matrix H_c can be decomposed in to an upper triangular matrix R_c and a unitary matrix Q_c . The complex channel can be represented as $H_c = Q_c R_c$. Pre-multiplying Eq. (1) with Q_c^* on both sides,

$$Q_c^* y_c = Q_c^* Q_c R_c x_c + Q_c^* n_c$$

$$\hat{y} = R_c x_c + \hat{n}_c \quad (3)$$

where $\hat{y} = Q_c^* y_c$ and * in superscript indicates Hermitian transpose operation. Using Eq. (3), x_c can be obtained with linear techniques.

III. QR-DECOMPOSITION METHODS

Gram-Schmidt [5], Householder (HH) transformation [6], [7] and Givens rotation (GR) [7] are three well known algorithms used to perform QR decomposition. We briefly describe each of these techniques.

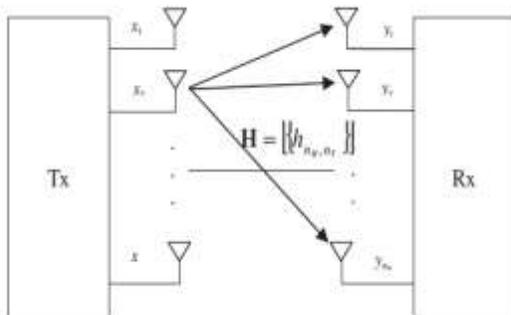


Fig. 1 MIMO system modal

A. Gram-Schmidt Orthogonalization

In numerical analysis, we come across the Gram Schmidt process [5] for orthonormalization of a set of vectors by subtracting the projection of one on the others. The Gram Schmidt procedure is numerically unstable for the system

with finite precision and usually not preferred in system implementation.

B. Householder Transformation

Householder transformation is a QRD technique to decompose any matrix in to an upper triangular matrix R_c and a unitary matrix Q_c . The main idea of this Householder reflections technique is to find reflection matrix P [6] and also known as Householder matrix. The Householder matrix annihilates all elements in a vector except the first element, which will be replaced with the norm of the corresponding vector.

Algorithm	I	Householder	Transformation
step 1:	Channel	Matrix	Input: H_c
step 2:	Decomposition	outputs:	Q_c, R_c
step 3:	Default	Initials:	$R_c \leftarrow H_c, Q_c \leftarrow I_{nr}$
step 4:	for	$k=1:$	$\min(n_R-1, n_T)$
step 5:	$a = [R_{k:n_R, k}]$		
Step 6:	$e_1 = \begin{bmatrix} 1 \\ 0_{k-1} \end{bmatrix}$		
step 7:	$\alpha = \ a\ $	column	vector norm
step 8:	$v = a \mp \alpha e_1$	(the sign of the α depends	up on sign of the first element in the vector a)
step 9:	$p = I_{nr-k+1} - \frac{2vv^H}{v^H v}$	(house	holder matrix)
step 10:	$qck = \begin{bmatrix} I_{k-1} & 0_{k-1, nr-k+1} \\ 0_{nr-k+1, k-1} & P \end{bmatrix}$		
step 11:	$R_c = qck * R_c;$		
step 12:	$Q_c = Q_c * qck;$		
step 13:	end for		

C. Givens Rotation

Givens Rotation is also used to decompose the channel matrix H_c using a plane rotation [8]. Using a sequence of such Givens rotations in the form of a matrix, we can selectively manipulate the entries of a matrix H_c to reduce into upper triangular form (R_c).

IV. MODIFIED HOUSEHOLDER TRANSFORMATION

The system model in Eq. (3) has complex terms, the direct method of implementing QR decomposition requires multiplications, square root and division operations. The Eq. (1) can be reframed in to real system Eq. (4) with transmitted real vector X, real channel matrix H and received real vector y of order $(2n_R \times 1)$, $(2n_R \times 2n_T)$ and $(2n_T \times 1)$ respectively. Reframed MIMO equation:

$$Y = HX + n. \quad (4)$$

where y, x, H and n are computed from the complex quantities y_c, x_c and H_c as follows. Where

$$y = [Re\{y_1\} \quad Im\{y_1\} \quad \dots \quad Re\{y_{n_R}\} \quad Im\{y_{n_R}\}]^T$$

$$H = \begin{bmatrix} H_{11} & H_{12} & \dots & H_{1n_T} \\ H_{21} & H_{22} & \dots & H_{2n_T} \\ \vdots & \vdots & \ddots & \vdots \\ H_{n_R 1} & \dots & \dots & H_{n_R n_T} \end{bmatrix},$$

$$X = \begin{bmatrix} Re\{x_1\} \\ Im\{x_1\} \\ \vdots \\ Re\{x_{n_T}\} \\ Im\{x_{n_T}\} \end{bmatrix}, \quad n = \begin{bmatrix} Re\{n_1\} \\ Im\{n_1\} \\ \vdots \\ Re\{n_{n_R}\} \\ Im\{n_{n_R}\} \end{bmatrix}$$

and each element of H can be written as

$$H_{ij} = \begin{bmatrix} Re\{h_{ij}\} & -Im\{h_{ij}\} \\ Im\{h_{ij}\} & Re\{h_{ij}\} \end{bmatrix}$$

where $Re\{\cdot\}$ and $Im\{\cdot\}$ denotes the real and imaginary parts of the channel element respectively and T in superscript indicates transpose operation.

Modified Householder (MHH) transform is computationally improved compared to the HH algorithm by changing the Householder matrix. MHH eliminates the division operation in computing QR decomposition [9].

Algorithm 2 Modified Householder Transformation

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step 1: Channel Matrix Input: H
step 2: Decomposition outputs: Q, R
step 3: Default Initials: R ← H, Q ← InR
step 4: for k=1: min (nR-1, nT)
step 5: a = [Rk:nR,k]
step 6: e1 = [ 1 ]
           [ 0k-1 ]
step 7: α = ||a|| column vector norm
step 8: v = a / α e1 : the sign of the α depends
           up on sign of the first element in the vector a
step 9: p = InR-k+1 * vH * v - 2v * vH (modified house
holder matrix)
step 10: qk = [ (vh * v) * Ik-1   0k-1, nR-k+1 ]
           [ 0nR-k+1, k-1         P ]
step 11: R = qk * R
step 12: Q = Q * qk
step 13: end for
    
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V. MIMO DECODERS USING QR DECOMPOSITION

There are many MIMO decoders, some of them are listed below. The output vector y in Eq. (4) pre-multiplying with Q^T on both sides, we get

$$\hat{y} = Rx + \hat{n} \quad (5)$$

Where $\hat{y} = Q^T y$, $\hat{n} = Q^T n$ and R is an upper triangular matrix formed from QR decomposition of channel matrix.

A. Zero-Forcing Detector

ZF solution is obtained from Eq. (5) as

$$W = (R^T R)^{-1} R^T \quad (6)$$

From Eq. (5) and Eq. (6)

$$W\hat{y} = x + W\hat{n} \quad (7)$$

The computation saving here increases with increased MIMO size.

B. MMSE Detector

MMSE solution is obtained from Eq. (5) as
 $W = (R^T R + \sigma_n^2 I_{n_T})^{-1} d^2$ (8)
 As in ZF we can reduce the complexity in finding W.

C. Sphere Decoder

Sphere decoder solution is obtained from Eq. (5) as
 $||\hat{y} - Rx||^2 \leq d^2$ (9)
 The hypothesis value Rx which lies inside the sphere of radius d and the real received vector \hat{y} can be represented as [2], the cost function of sphere decoder is defined as
 $\arg_{x \in D_L^n} \min \left(\sum_{i=1}^n (\hat{y}_i - \sum_{j=1}^n r_{i,j} x_j)^2 \right) \leq d^2$ (10)
 we can save few multiplications in the sphere decoder where the entries of $r_{i,j}$ are zero in Eq. (10).

VI SIMULATION RESULTS

Simulation is carried, in the multipath fading channels for IEEE 802.11ac (VHT) system. We considered VHT mixed format packet consisting of preamble, signal fields and data as in [5] and [6]. We have modeled Analog Front End (AFE) section with analog filter, DAC/ADC, TGac channel [10] and RF impairments. We selected simulation parameters as given in Table I.

Table I. configurations of simulated systems

MCS Index	Parameters		
	Modulation	Code rate	TX x RX antennas
3	16 QAM	1/2	2 x 2
5	64 QAM	2/3	4 x 4

We considered different SNRs in dB with block fading using TGac channel models A, B, C, D, E, F. For each SNR, we run simulations for 1000 realizations of channel and noise. In each realization, we generate a different transmit frame and pass through Analog Front End (AFE), the channel. We add AWGN noise and introduce other impairments like frequency offset, clock offset and phase noise. At the receiver, we perform timing and frequency corrections and estimate the channel using long training field

(LTF). The estimated channel can be used in the detection of QR based MIMO. We performed PER simulations with both modified HH (MHH) and conventional HH using a channel bandwidth of 20MHz.

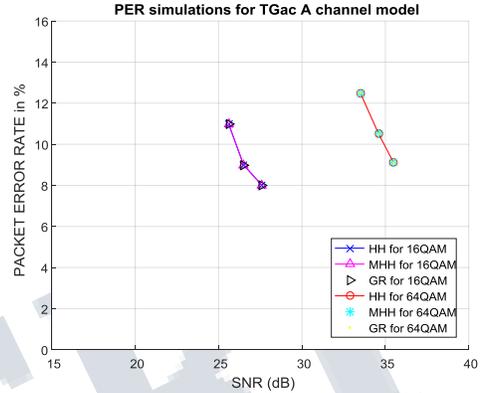


Fig.2 Packet error rate for TGAC A model

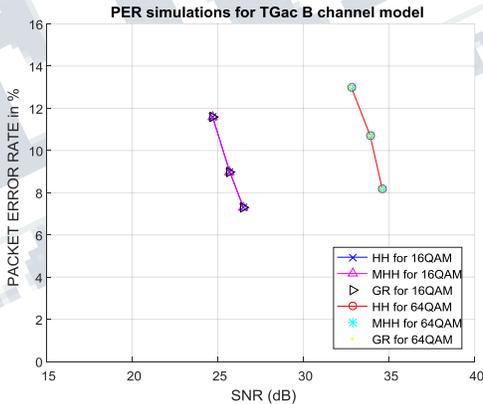


Fig.3 Packet error rate for TGAC B model

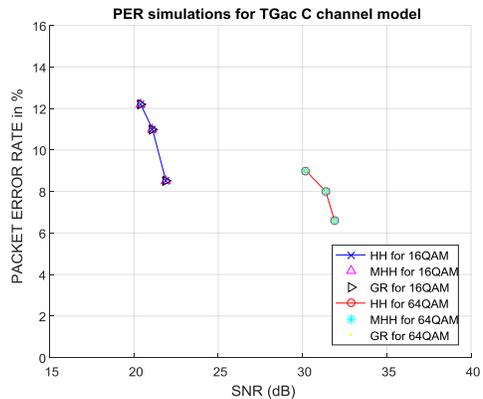


Fig.4 Packet error rate for TGAC C model

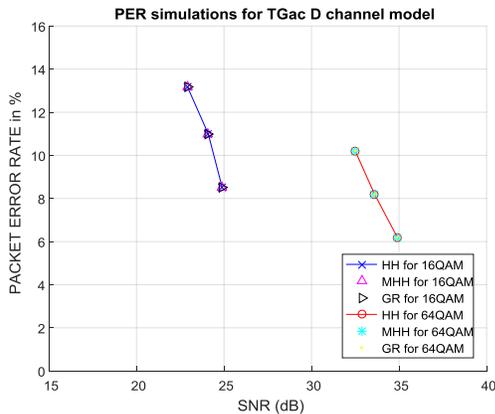


Fig.5 Packet error rate for TGAC D model

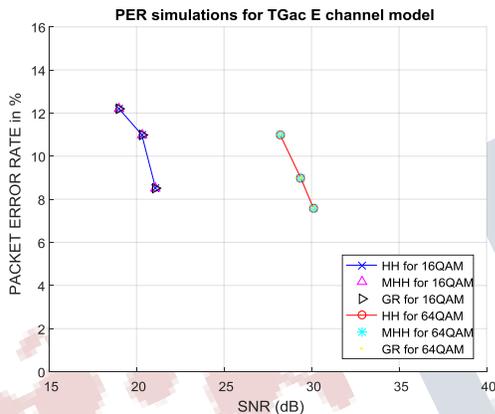


Fig.6 Packet error rate for TGAC E model

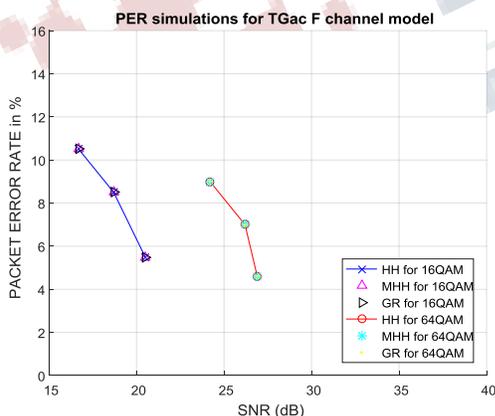


Fig.7 Packet error rate for TGAC F model

VII. COMPUTATIONAL COMPLEXITIES

A. Computational Complexities for proposed Method

Computational complexities of the QRD techniques are obtained from the Algorithm 1 and Algorithm 2 and, are tabulated in Table II for $n_R = n_T = n$. In Table II, it is clearly shown that, The HH requires several division operations compared to that of modified HH. For a MIMO channel matrix of size 2×2 , modified HH saves around 30 division operations. Similarly, we can show that, for other values of n_R and n_T , the modified HR is computationally less intensive.

Table II. Saving in complexities for proposed methods

QRD	Basic method	Modified method	Saving in computations
HH	DIV	0	$\frac{8}{3}n^3 + 2n^2 + \frac{n}{3} - 1$
	MUL	$\frac{40}{3}n^3 - 18n^2 + 11n$	$n^2 + n$

VIII. CONCLUSION

Complexity of MIMO decoder is reduced based on modified QR decomposition. The method was simulated using VHT wireless LAN system under different channel conditions. MIMO decoder gives same PER performance as that of the conventional QR based MIMO decoder.

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