

Design of IIR Filter using Hybrid GA and BBO Algorithm

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Abstract:— A novel design strategy of Butterworth IIR filter is proposed in this paper. It considers two most effective hybrid optimization techniques GA and BBO. The results show that GA and BBO based filter designer is able to find transfer function required for given magnitude response. The proposed algorithm doesn't take unnecessary computation time and good in exploiting the solution as the solution doesn't die at the end of each generation. Hence, the performance of proposed hybrid algorithm outcomes the performances of previous proposed algorithms for designing of a digital filter. The simulated results show that the design filter is highly stable and the filter gain is exactly same as that of the ideal filter. The magnitude of the filter is less than one (<1), which is foremost objective of the design.

Index Terms— GA, BBO, Butterworth IIR Filter.

I. INTRODUCTION

Digital signal processing has a wide range of applications in the fields of Telecommunication, image processing, audio graphic equalizer etc. A digital filter is simply a discrete-time, discrete-amplitude convolver. Digital filter uses a digital processor to execute sampled values of the signal for numerical calculations. The processor can either be a general-purpose computer like PC, or a Digital Signal Processor (DSP) chip. Analog to digital converter (ADC) is used to sample and digitize the input analog signal. The resulting binary numbers, signify the sequential sampled values of analog input signal which are then reassigned to the processor, to perform numerical calculations on them. These calculations usually comprise of multiplying constant with the input values and then adding the products together. If required, the outcomes of these calculations, which now characterize sampled values of the filtered, signal i.e. output of a digital to analog converter (DAC) to transform the signal back to analog form. The block diagram in Fig. 1 shows the basic structure of such a system.

The process of converting of an analog signal into digital form is done using sampling with finite sampling frequency (f_s). It is necessary to do filtering of an input signal using a low-pass filter (LPF) that eradicates high-frequency components from input frequency spectrum of signal, hence it prevents aliasing. That's why this filter is known as anti-aliasing filter [1]. Subsequently after the filtering and sampling process is

accomplished, a digital signal is prepared for additional processing which, in this case, is filtering done by using the suitable digital filter. The output signal obtained is also a digital signal which may be reconverted back into analog form. After digital-to-analog conversion, frequency spectrum of signal may contain some frequency components higher than $f_s/2$ that must be removed. Hence again, it is essential to use a low-pass filter with the sampling frequency $f_s/2$.

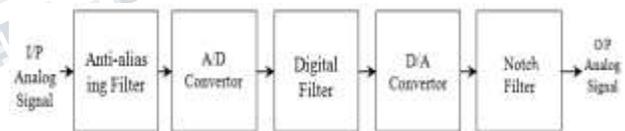


Fig. 1 Block Diagram of basic structure of Digital signal processing

There are two basic types of digital filters, Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) filters. In FIR filter impulse response is of finite duration. A Non Recursive filter (FIR) has no feedback and its input-output relation is defined by.

$$y(m) = \sum_{k=0}^M b_k x(m-k)$$

A discrete-time Recursive filter (IIR) has a z-domain transfer function which is the ratio of two z-transform polynomials. The ratio of two polynomials in the

variable z is the transfer function of the filter and may be written in a cascade form as [2]

$$H(z) = H_1(z)H_2(z)$$

The optimization of a low pass filters can be done by optimizing various parameters of the filter. There are certain algorithms available to optimize a filter like Genetic Algorithm (GA), Biogeography-Based Optimization (BBO) Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO). In this work hybrid of GA and BBO has been used. BBO is a population based evolutionary algorithm (EA) depending on mathematics of biogeography. Biogeography is the analysis of the geographical distribution of biological organisms. Biogeography-based optimization (BBO) is an evolutionary algorithm that optimizes a function by iteratively and stochastically for refining candidate solutions with respect to a given degree of quality, or fitness function. The BBO migration approach is alike to the global recombination method of the breeder GA and evolutionary approaches. In evolutionary approaches, global recombination used to generate new solutions, whereas BBO migration is used to alter existing solutions. Global recombination in evolutionary approach is a reproductive process, whereas migration in BBO is an adaptive process; it is used to revise existing islands. As with other population based optimization algorithms, we usually incorporate some sort of selectivity in order to hold the best solutions in the population. This averts the best solutions from being tainted by immigration. BBO is differ from Ant colony optimization because ACO generates a new set of solution with each iteration. But BBO maintains its set of solutions from one iteration of the next, relying on migration to probabilistically adopt those solution. BBO has the most in common with Particle Swarm Optimization and DE. In those approaches, solutions are maintained in one iteration to the next, but each solution is able to learn from its neighbours and adopt itself as the algorithm progress. PSO represents each solution as a point in a space, and represents the change over time of each solution as a velocity vector. PSO do not change its solution directly. DE changes its solution directly, but DE is not biological motivated. GA and ES reproduce children by crossover, namely their solution disappear at the end of each generation, while BBO solution are not rejected after each generation, but are rather improved by migration[3]. GAs are academic theoretical search techniques that can be used to search for an ideal

solution to the development function of an optimization problem. In spite of focusing on a single solution, GAs work on group of sample solutions simultaneously where they deploy a population of individuals in each and every generation iterations here each individual, named as the chromosome, epitomizes one candidate solution to the problem [3] [4]. Fit individuals will endure within the population to imitate their genetic material. This then recombined to yield new individuals as off-springs.

II. PRESENT WORK

Digital low pass or high pass FIR and IIR filters are designed by using various Optimization algorithms like Genetic Algorithm (GA), Biogeography-Based Optimization (BBO), Particle Swarm Optimization (PSO) and Ant Colony Optimization (ACO). In 2010, Wenyin Gong [5] proposed a new hybrid Differential Evolution Biogeography-Based Optimization (DE/BBO) algorithm. The performance of proposed DE/BBO indicated that it was effective and efficient in comparison with other optimization algorithms as it performed better in terms of the quality of the final solutions and the convergence rate. But no one have used till date the hybrid algorithm composed of the two most effective optimization algorithm techniques, Genetic Algorithm (GA) and Biogeography-Based Optimization (BBO) i.e. (GA/BBO) for designing of digital filters. So problem formulates the motive and significance to design a Butterworth IIR filter by using hybrid algorithm composed of genetic and Biogeography-Based Optimization (GA/BBO) technique which would be more efficient algorithm for optimization, doesn't take unnecessary computation time, good in exploiting the solutions and solutions doesn't die at the end of each generation like other optimization algorithms and hence the performance of proposed hybrid algorithm outcomes the performances of the previous proposed algorithms for designing of a digital filter. The proposed algorithm comprises of GA and BBO combination in such a way that it requires for mapping of a digital filter to an element, assessing the fitness of a digital filter, generating an preliminary population of filters, and confirming that all filters are feasible. Initially, digital filter is designed to get magnitude response with acknowledged transfer function of a Butterworth band-pass filter. Lastly, filter is recycled to design a digital filter with a random magnitude response having a transfer function that is comparatively difficult to conclude with conventional techniques.

A. Filter design

In IIR filter, output is comprised of sum of two vector products; a weighted amalgamation of the input samples $[b_0, \dots, b_M][x(m), \dots, x(m-1)]^T$ and a weighted amalgamation of the output feedback $[a_1, \dots, a_N][y(m-1), \dots, y(m-k)]^T$ signify transpose. A recursive filter's output in general is a function of the previous output samples and the present and past input samples i.e. has feedback from output to input, and is defined by the following equation [6].

$$y(m) = \sum_{k=1}^N a_k y(m-k) + \sum_{k=0}^M b_k x(m-k)$$

The key difference between IIR filters and FIR filters is that an IIR filter is more compact than FIR, hence it can easily realize a set of frequency response with a lesser number of coefficients than an FIR filter. Hence a lesser number of filter coefficients entail less storage requirements and rapid calculation with higher throughput. IIR filters are more efficient in memory and computational necessities than FIR filters. Design starts by outlining the transfer function $H(z)$ for a digital filter, given as under,

$$H(z) = \frac{N(z)}{D(z)} = \frac{\sum_{i=0}^{\alpha} c_i z^{-i}}{1 + \sum_{i=1}^{\alpha} b_i z^{-i}} = K \times \frac{\prod_{i=1}^{\alpha} (z - z_i)}{\prod_{i=1}^{\alpha} (z - p_i)}$$

Where b_i and c_i are coefficients of a polynomial. p_i and z_i are poles and zeros of the factored form respectively. K , gain factor is essential for equality between the polynomial and factored form. Order of $H(z)$ is calculated by α .

Properties of $H(z)$ used in design and optimization of designed filter are:

- Linear Time Invariant (LTI) causal system with system function i.e. $H(z)$ is Bounded Input Bounded Output (BIBO) stable only if all the poles of $H(z)$ lie inside the unit circle i.e. $|p_i| < 1$.
- Stable and casual LTI system with system function i.e. $H(z)$ is real only if all complex zeros and poles of $H(z)$ have complex conjugate pairs or occur singularly on the real axis.

B. Fitness Function

The proposed method designs an optimized digital

filter with a random magnitude response. Therefore, the fitness function should be dependent on both the desired magnitude response with magnitude responses of the filter under assessment. A weighted squared error frequency method is projected to tackle it. Fitness of x_n is evaluated by mapping vectors of x_n with the zeros and poles of $H_n(z)$. Thereafter, the magnitude response $|H_n(e^{j\omega})|$ of $H_n(z)$ with by default gain of $K = 1$ is calculated for all frequency bands of ω .

To recompense for unity gain of $H_n(z)$, $|H_n(e^{j\omega})|$ is measured by K_n . Here K_n is selected for minimization of error of $K_n |H_n(e^{j\omega})|$ and $|H_d(e^{j\omega})|$. This is attained by driving the magnitude average value of $K_n |H_n(e^{j\omega})|$ equal to the magnitude average value of $|H_d(e^{j\omega})|$.

Equation for evaluating K_n is:-

$$K_n = \frac{\sum_{y=1}^Y |H_d(e^{j\omega_y})|}{\sum_{y=1}^Y |H_n(e^{j\omega_y})|}$$

Thereafter, squared error is evaluated with squaring the difference of $K_n |H_n(e^{j\omega})|$ and $|H_d(e^{j\omega})|$ for every value of ω . Squared error values are weighted and multiplied with Q , a weighting vector that allocates a weighting factor to every frequency band ω . This allows convinced frequency bands of magnitude response to subsidise more or less for the complete fitness of x_n . Lastly weighted values of the squared error are summed up and measured to yield the value of fitness of x_n . If $K_n |H_n(e^{j\omega})|$ is alike to $|H_d(e^{j\omega})|$, then only the fitness value will approx. to zero and minimization of error of $K_n |H_n(e^{j\omega})|$ and $|H_d(e^{j\omega})|$ will take place. The whole **fitness function** is represented by,

$$f(x_n) = \frac{1}{Y} \sum_{y=1}^Y [K_n |H_n(e^{j\omega_y})| - |H_d(e^{j\omega})|]^2 Q_y;$$

Here Y is the total no. of frequency bands, ω_y is a component of ω , and Q_y is a component of Q . As for $H_n(z)$, only real filters are deliberated, ω is normally stated in the range of 0 to π .

III. RESULTS AND DISCUSSION

The hybrid approach used in designing of the Butterworth filter has been proved efficient as the gain of the filter is almost matching to the gain of the ideal filter. Also, the magnitude response of the designed fourth order Butterworth filter is in the range of 0.01 to 0.9 with lower and upper 3-dB cut off points of $\omega_l = \frac{1}{4}$, and $\omega_u = \frac{3}{4}$. The frequency vector for specifying the desired magnitude response $|H_d(e^{j\omega})|$ consists of 10,000 frequency bins equally spaced between 0 and π .

A. Input parameters

The Butterworth filter identifies a well-known, mathematically optimized class of filters that are maximally flat in the pass band. The desired magnitude response $H_d(e^{j\omega})$ is a fourth-order Butterworth filter with lower and upper 3-dB cut-off points of $\omega_l = \frac{1}{4}$, and $\omega_u = \frac{3}{4}$, and unity passband gain. The filter designer program parameters are configured according to the Butterworth transfer function output.

The stopping criteria is set to GABBO_MaxGenerations (= 1500) and minimum fitness level is set to FF_MinFitLevel, so that the designer will search for the optimal solution for 1500 generations.

Table 1: System Parameters

Parameter	Value	Comment
GA_Alpha	4	Same order as $H_d(e^{j\omega})$.
GA_Crossover Probability	0.7	Probability of crossover.
BBO_Mutation Probability	0.01	Probability of mutation.
BBO_Keep	2	Elitism Parameter: Number of best habitats to keep from one generation to the next.
GABBO_Max Generations	1500	Maximum number of generations per problem.
GABBO_Population Size	200	Number of elements in population.
GABBO_Variable Per Population Member	GA_Alpha	Number of variables per population member.
FF_Wts	All 1's	Fitness Function Weights.
FF_MinFitLevel	0	Minimum fitness level

B. Filter design comparison

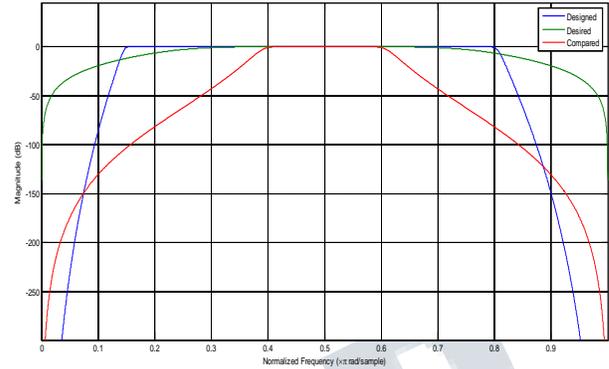


Fig 2. Filter design comparison

The graph shown in Fig. 2 compares magnitude response of the designed filter with respect to the desired filter and the compared filter. The graph clearly shows how the designed filter is better from the desired filter and the previous filter to which we are comparing our results.

C. Stability comparison

The corresponding pole-zero plot of the designed filter is shown in Fig. 3. As all the poles and zeros are situated inside the unit circle, the designed filter is stable.

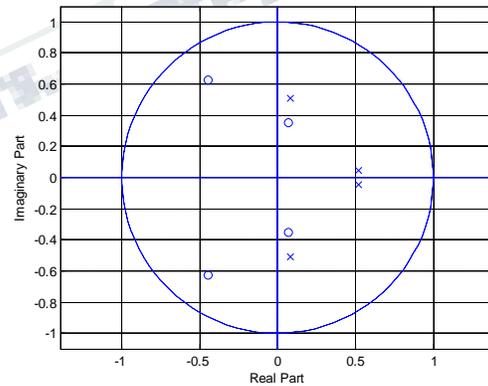


Fig. 3. Stability Comparison (Pole-zero Plot)

Table 2: Gain-Pole-Zeros

Gain	Poles	Zeros
0.2929	$0.5194 + 0.0459i$	$-0.4487 + 0.6243i$
	$0.0786 + 0.5060i$	$0.0680 + 0.3546i$
	$0.5194 - 0.0459i$	$-0.4487 - 0.6243i$
	$0.0786 - 0.5060i$	$0.0680 - 0.3546i$

D. Fitness Curve

The fitness curve in Fig.4 shows that the ending fitness level of the designed filter is approximately 10^{-32} and that major fitness improvements ceased after about 1,200 generations.

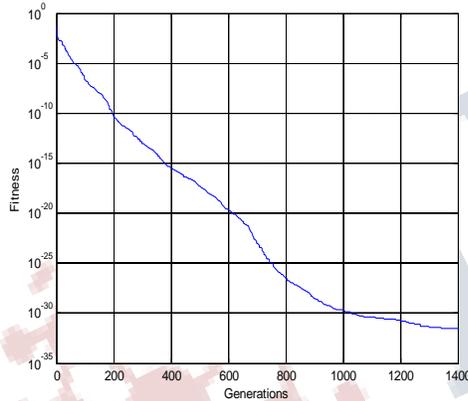


Fig. 4. Fitness Curve

E. Comparison of proposed work

Table 3 shows how the designed value is better than the compared value. The iterations of the proposed work are less, thus time taken to execute the program code will be much less as compared to the traditional filter design using GA.

Table 3: Designed filter Vs Compared filter

Parameter	Compared Value	Designed Value
Iterations	1498	1200
Order	4	4
ω_L and ω_H	$\frac{1}{4}$ and $\frac{3}{4}$	$\frac{1}{4}$ and $\frac{3}{4}$
Magnitude Response $ H(e^{j\omega}) $	Less than 1 $ H(e^{j\omega}) \leq 0.1654$ $0.8920 \leq H(e^{j\omega}) \leq 1$	Less than 1 $ H(e^{j\omega}) \leq 0.01$ $0.9 \leq H(e^{j\omega}) \leq 1$

E. Comparison with ideal filter

The following table compares the designed filter with the ideal Butterworth filter. It clearly shows that gain of the designed filter is exactly same as that of the ideal filter. Also the poles and zeroes of the designed filters closely resemble or approach towards ideal pole and zero values.

Table 4. Designed filter Vs Ideal filter

Parameter	Ideal Value	Design Value
Gain	0.29289	0.2929
Zeros	1	$-0.4487 + 0.6243i$
	-1	$0.0680 + 0.3546i$
	1	$-0.4487 - 0.6243i$
	-1	$0.0680 - 0.3546i$
Poles	$0.45509 + j0.45509$	$0.5194 + 0.0459i$
	$-0.45509 + j0.45509$	$0.0786 + 0.5060i$
	$0.45509 - j0.45509$	$0.5194 - 0.0459i$
	$-0.45509 - j0.45509$	$0.0786 - 0.5060i$

IV. CONCLUSION

By studying Table 1-4 and Fig.1-4, we have arrived at the following conclusions: The magnitude response performance of the designed filter using hybrid approach (GA/BBO) is better than the filter designed only with GA. As the number of iterations using hybrid approach is less, thus the time required to execute the program code will be much less. The designed Butterworth IIR filter is stable as the poles and zeroes of the filters are located inside the unit circle. The gain of the designed Butterworth IIR filter is exactly same as that of the ideal filter.

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