

Weighted Goal Programming Approach for Solving Multi-Objective De Novo Programming Problems

^[1] Susanta Banik, ^[2] Debasish Bhattacharya
^{[1][2]} Department of Mathematics, NIT Agartala

Abstract - The De Novo Programming problems proposed by Zeleny is well known for its value in designing an optimal system by extending existed resources instead of finding the optimum in a given system with fixed resources. But still, now a General De Novo Programming Problem, having both maximizing and minimizing type of objectives, does not have a unique general algorithm for its solution. Especially when multi-objective problems are discussed in the light of De Novo hypothesis, different methods of solution directs decision maker to different solutions. This paper proposes a new method for solving General De Novo programming problem. In this approach, weighted goal programming technique has been used, where only one deviation variable has been taken. Problem-solving phases of the model are explained through illustrative example.

Keywords — De Novo Programming, Multi-objective Optimization, Weighted Goal programming.

I. INTRODUCTION

The De Novo Programming problem proposed by Zeleny [1] is popularly used to design an optimal system by extending resources instead of finding the optimum in a given system with fixed resources [1-5]. This method was first designed for single-criteria decision making [1, 2], later it has been extended to multi-criteria decision making, containing maximizing type of objectives only [3-5]. In the designed approach the computation could be handled easily and thus it has become a popular multi-criteria decision making technique for the optimal system design. But no method exists for the solution of General De Novo programming problem involving both maximizing and minimizing type of objectives. Only some special cases could be seen in the literature. To solve General De Novo programming problem, Li and Lee [6, 7], Chen and Hsieh [8], Nurullah [9], Chakraborty and Bhattacharya [10, 11] introduced several approaches. Li and Lee [6,7] first introduced a two phase fuzzy approach based on the ideal and negative ideal solution to solve the general De Novo programming problem. First phase of the approach is identical to max-min fuzzy model introduced by Zimmermann [12], and the possible solution obtained from the first phase is tested with an averaging operator to determine whether it is the only solution or not. Chen and Hsieh [8] introduced an innovative approach for solution of multistage general De Novo programming problem using fuzzy dynamic programming concept. Nurullah [9] employed min-max goal programming technique for the solution of multi-objective general De Novo Programming Problems and examined the closeness of the optimal objective values to the ideal values of the objectives. Chakraborty and

Bhattacharya in [10, 11] further studied De Novo programming problem and proposed a method for the solution of general De Novo programming problem in one step under fuzzy environment using usual Zimmermann's technique. Shi [13] introduced several optimum-path ratios for enforcing different budget levels of resources so as to find alternative optimal system designs for solving multi-objective De Novo Programming (MDNP) problems.

Shi [14] applied MDNP to formulate and solve problems of system design that involved multiple decision makers and a possible debt. Also Babic and Pavic [15], Huang, et al. [16], Zhang et al. [17], and Chen and Tzeng [18] have contributed De Novo Programming literature with their studies. Miao, D.Y et al [19] considered Interval-Fuzzy De Novo programming method for planning water resources systems under uncertainty in 2014. In 2015 Saeedi et al [20] utilized De Novo programming technique to determine the capacity in a closed-loop supply chain network when a queuing system is established at each recovery centre in the reverse flow. Thus De Novo programming and its application is a vibrant area of contemporary research. This motivated us to study the concept of De Novo programming further. In our quest we investigated the applicability of weighted goal programming technique in the solution of General De Novo programming problem. This resulted the present paper where we used weighted goal programming technique with one deviation variable for the solution of General De Novo programming.

The paper is organized as follows. In section II, we have discussed the basic concept and formulation of General De Novo programming problem. And then outline how to determine the ideal and pessimistic values.

In section III, the weighted goal programming method has been visited and how it could be used for solving General De Novo Programming problem has been delineated. A numerical example is also given to illustrate the approach in section IV. Finally, we address the conclusion.

II. CONSTRUCTION OF DE NOVO PROGRAMING PROBLEM

In this section we will discuss the construction of De Novo programming problem suggested by Zeleny (1986). In this construction process, we will consider a general type of problem involving both maximizing and minimizing types of objectives.

General De Novo Programming Problem

Let there be r maximizing type of objectives and s minimizing type of objectives given by

$$\text{Max } z_k = c_{k1}x_1 + c_{k2}x_2 + \dots + c_{kn}x_n, \quad k = 1, 2, \dots, r.$$

$$\text{Min } w_l = c_{l1}x_1 + c_{l2}x_2 + \dots + c_{ln}x_n, \quad l = 1, 2, \dots, s.$$

Subject to, (1)

$$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n \leq b_i, \quad i = 1, 2, \dots, m.$$

Let p_i be the cost per unit of i^{th} resources, which are known. Then the total cost for managing the resource units b_1, b_2, \dots, b_m is given by,

$$p_1b_1 + p_2b_2 + \dots + p_mb_m = \sum_{i=1}^m p_i b_i$$

Suppose budget available to the decision maker is \mathcal{B} (in monetary unit). So, restriction on budgetary provision yields the constraint $\sum_{i=1}^m p_i b_i \leq \mathcal{B}$.

Unlike usual linear programming problem, in De Novo programming the resources b_1, b_2, \dots, b_m are not known rather they are variables and are subjected to the restriction, $\sum_{i=1}^m p_i b_i \leq \mathcal{B}$.

Let us define the following matrices:

$$C^1 = [c_{kj}]_{r \times n}; \quad C^2 = [c_{lj}]_{s \times n}; \quad A = [a_{ij}]_{m \times n};$$

$$b^T = [b_1 b_2 \dots b_m]; \quad x^T = [x_1 x_2 x_3 \dots x_n].$$

Here a general type of De Novo Programming problem could be represented as

$$\text{Max } Z = C^1 x$$

$$\text{Min } W = C^2 x$$

Subject to, (2)

$$x \leq b$$

$$pb \leq \mathcal{B}$$

$$x \geq 0$$

Where, C^1, C^2, A, b, x are as defined and

$$Z^T = [z_1 z_2 z_3 \dots z_r] \text{ and } W^T = [w_1 w_2 w_3 \dots w_s].$$

Now for the solution of general De Novo programming problem stated in (2), involving both maximizing & minimizing type of objectives, no general method exist.

As in Zeleny [5] pre-multiplying both side of the structural constraint $Ax \leq b$ by p we get, $pAx \leq pb$. Let us put $pA = v$, where v is a row vector, $v = [v_1 v_2 v_3 \dots v_n]$ say. Then using the budget constraint $pb \leq \mathcal{B}$, $pAx \leq pb$ could be re-written as, $vx \leq \mathcal{B}$ i.e.

$$v_1x_1 + v_2x_2 + \dots + v_nx_n \leq \mathcal{B}.$$

The variable v_j could be physically interpreted as follows.

Clearly, the constraint

$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n = b_i$ Suggest that for producing 1 unit of the product j , a_{ij} part of the resource b_i is expended. Thus $a_{1j}p_1 + a_{2j}p_2 + \dots + a_{mj}p_m = v_j$ denotes the cost of producing one unit of the j^{th} product.

The problem (2) thus further reduces to,

$$\text{Max } Z = C^1 x$$

$$\text{Min } W = C^2 x$$

Subject to, (3)

$$v_1x_1 + v_2x_2 + \dots + v_nx_n \leq \mathcal{B}$$

$$x_1, x_2, x_3, \dots, x_n \geq 0.$$

The problem has been reduced to an equivalent problem containing only one constraint involving n arguments. It can be proved that every solution of system (2) is also a solution of system (3) and conversely, Zeleny [2]. Thus with restructuring the generality of the system is not disturbed.

III. OPTIMISTIC AND PESSIMISTIC VALUES OF THE OBJECTIVES

To find the solution of the system (3) we first find the basic feasible solution of the system. For this we put $(n-1)$ variable to zero and find the values of the remaining variables. This can be done in $\binom{n}{1} = n$ ways.

Thus the basic feasible solutions are,

$$\left(\frac{\mathcal{B}}{v_1}, 0, 0, \dots, 0\right); \left(0, \frac{\mathcal{B}}{v_2}, 0, \dots, 0\right); \dots; \left(0, 0, 0, \dots, \frac{\mathcal{B}}{v_n}\right)$$

Substituting these basic feasible solutions one by one in all objectives we can find the optimum values of each of the objectives.

Let $Z_1^* = \text{Max } z_1$; $Z_2^* = \text{Max } z_2$;; $Z_r^* = \text{Max } z_r$

$W_1^* = \text{Min } w_1$; $W_2^* = \text{Min } w_2$;; $W_s^* = \text{Min } w_s$.

These optimum values of the objectives are called the ideal values and the vector of the ideal values is called the ideal point. Thus the ideal point is,

$$I^* = (Z_1^*, Z_2^*, \dots, Z_r^*, W_1^*, W_2^*, \dots, W_s^*)$$

Similarly, we can find the worst possible values of the objectives i.e. the negative ideal values. These are given by,

$$\bar{Z}_1 = \text{Min } z_1, \bar{Z}_2 = \text{Min } z_2, \dots, \bar{Z}_r = \text{Min } z_r;$$

$$\bar{W}_1 = \text{Max } w_1, \bar{W}_2 = \text{Max } w_2, \dots, \bar{W}_s = \text{Max } w_s;$$

Therefore, the vector of negative ideal values of the objectives are given by,

$$I^- = (\bar{Z}_1, \bar{Z}_2, \dots, \bar{Z}_r, \bar{W}_1, \bar{W}_2, \dots, \bar{W}_s)$$

In the formulation of Nurullah [9], ideal values have been used as the upper and lower aspiration levels for the maximizing and minimizing objectives respectively. In two-step fuzzy method of Li and Lee [6] these ideal and negative ideal values have been used to construct the membership function of the fuzzy objectives.

In our proposed approach the optimistic values of the objectives are taken as the ideal values, however, the pessimistic expectations for the objective functions are determined using Luhandjula's technique [27], instead of taking the worst possible choice of the objectives i.e. the negative ideal values. The determination of the optimistic and pessimistic expectations of the objectives functions in the proposed approach is explained by taking the following multi-objective LPP involving two maximizing objectives z_1, z_2 and one minimizing objective w_1 in three variables. Further let x_1^*, x_2^*, x_3^* are three basic feasible solutions of the problem such that,

Table 1. Calculation of Optimistic and Pessimistic objectives Values.

Solutions	Values of the objectives		
	z_1	z_2	w_1
x_1^*	$Z_1^* = z_1(x_1^*)$	$z_2(x_1^*)$	$w_1(x_1^*)$
x_2^*	$z_1(x_2^*)$	$Z_2^* = z_2(x_2^*)$	$w_1(x_2^*)$
x_3^*	$z_1(x_3^*)$	$z_2(x_3^*)$	$W_1^* = w_1(x_3^*)$

$Z_1^* = \text{max } z_1 = z_1(x_1^*)$ $Z_2^* = \text{max } z_2 = z_2(x_2^*)$, $W_1^* = \text{min } w_1 = w_1(x_3^*)$. The calculation has been placed in Table 1.

Clearly ideal point i.e. the vector of the optimistic expectations of the objective functions is given by $\alpha^* = (Z_1^*, Z_2^*, W_1^*)$.

Further, let $\hat{Z}_1 = \text{min } \{Z_1^*, z_1(x_2^*), z_1(x_3^*)\}$,

$$\text{if } \text{min } \{Z_1^*, z_1(x_2^*), z_1(x_3^*)\} \neq Z_1^*$$

$$\hat{Z}_2 = \text{min } \{z_2(x_1^*), Z_2^*, z_2(x_3^*)\}$$

$$\text{if } \text{min } \{z_2(x_1^*), Z_2^*, z_2(x_3^*)\} \neq Z_2^*$$

$$\hat{W}_1 = \text{max } \{w_1(x_1^*), w_1(x_2^*), W_1^*\},$$

$$\text{if } \text{min } \{w_1(x_1^*), w_1(x_2^*), W_1^*\} \neq W_1^*$$

Then the vector $\hat{\beta} = (\hat{Z}_1, \hat{Z}_2, \hat{W}_1)$ is taken as the vector of pessimistic expectation of the objective functions. The above calculation for finding pessimistic expectations could easily be extended to the more involved cases.

So, let the pessimistic values of the objectives be represented by, $\hat{\beta} = (\hat{Z}_1, \hat{Z}_2, \dots, \hat{Z}_r, \hat{W}_1, \hat{W}_2, \dots, \hat{W}_s)$

IV. WEIGHTED GOAL PROGRAMMING

Goal Programming (GP) is one of the most important methods in the Multi Objective Decision Making (MODM) system. This method is regarded as an extension of classical linear programs which includes achievement of target values for each objective, in place of maximizing or minimizing the objective functions. The term Goal Programming was first introduced by Charnes and Cooper [21]. With the studies of Lee [22], Ijiri [23], and Ignizio [24]; Goal Programming has become a strong and well-accepted method in literature.

The general purpose of Goal Programming is to minimize the deviation between the achievement of the goals and their aspiration levels (target values). The minimization procedure can be accomplished with different methods (Romero [25]). One such method is weighted goal programming method (WGP) which was introduced by Tamiz et. al. [26]. In WGP the unwanted deviation are allotted weights according to their virtual importance and is minimized as an Archimedean sum.

The algebraic formulation of a WGP is as follows,

$$\text{min } a = \sum_{k=1}^r (\alpha_k d_k^+ + \beta_k d_k^-) + \sum_{l=1}^s (\alpha_l d_l^+ + \beta_l d_l^-)$$

Subject to, (4)

$$f_k(\mathbf{x}) + d_k^- - d_k^+ = G_k, \quad k=1, 2, \dots, r$$

$$f_l(\mathbf{x}) + d_l^- - d_l^+ = g_l \quad l=1, 2, \dots, s$$

$$\mathbf{x} \in C_s$$

where $f_k(\mathbf{x})$ and $f_l(\mathbf{x})$ are linear functions (objectives) of \mathbf{x} , and G_k (resp. g_l) are the target values for k^{th} maximizing (resp. l^{th} minimizing) objective $f_k(\mathbf{x})$ (resp. $f_l(\mathbf{x})$). d_k^- and d_k^+ respectively are the negative and positive deviations from the target value for the k^{th} maximizing objective $f_k(\mathbf{x})$. Similarly, d_l^- and d_l^+ are defined for the l^{th} minimizing objective $f_l(\mathbf{x})$. Also α_k (resp. α_l) ≥ 0 and β_k (resp. β_l) ≥ 0 are the weights respectively attached to the positive and negative deviations in the achievement function a . These weights take the value zero if the minimization of the corresponding deviational variable is unimportant to the decision maker. C_s is the feasible set of solutions for the constraints $a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n \leq b_i, i=1, 2, \dots, m; \mathbf{x}^T = x_1 x_2 x_3 \dots x_n$

A. Normalisation technique

A normalisation technique is required if goals are measured in different units, in order to overcome the problem of incommensurability. In a WGP, occurs when deviational variables are measured in different units and summed up directly. This simple summation will cause an unintentional bias towards the objectives with a larger magnitude. This bias may lead to erroneous or misleading results. One suggestion to overcome this difficulty is to divide each objective through by a constant pertaining to that objective. This ensures that all objectives have roughly the same magnitude. Such a constant is known as a normalisation constant. This leads to the revised algebraic format for a WGP:

$$\min a = \sum_{k=1}^r \left(\frac{\alpha_k d_k^+ + \beta_k d_k^-}{\gamma_k} \right) + \sum_{l=1}^s \left(\frac{\alpha_l d_l^+ + \beta_l d_l^-}{\gamma_l} \right)$$

Subject to, (5)

$$f_k(x) + d_k^- - d_k^+ = G_k, \quad k=1, 2, \dots, r$$

$$f_l(x) + d_l^- - d_l^+ = g_l, \quad l=1, 2, \dots, s$$

$$\mathbf{x} \in C_s$$

where γ_k (resp. γ_l) is the normalisation constant for the k^{th} maximizing (resp. l^{th} minimizing) objective.

There are several different normalisation methods, each with its own normalisation constant. (3.1)

For obtaining a satisfactory solution of the De Novo programming problem based on Weighted Goal Programming, the problem (3) is reformulated. According to the new formulation we take $G_k = Z_k^*$, the maximum value of the k^{th} maximizing objective and $g_l = W_l^*$, the minimum value of the l^{th} minimizing objective. Since the ideal values have been used as aspiration level for the maximizing (resp. minimizing) type of objectives we must have $d_k^+ = 0$ and $d_l^- = 0$. Now the ideal values (Optimistic values Z_k^*, W_l^*) and pessimistic values (\hat{Z}_k, \hat{W}_l) obtained by Luh and Jula's comparison technique [23] will be used for the normalization of objectives. This will restrict goal deviations to unit less numbers. Now for solving the general De Novo programming problem (3) by using weighted goal program (5) we have the following system

$$\min a = \sum_{k=1}^r \left(\frac{\beta_k d_k^-}{\gamma_k} \right) + \sum_{l=1}^s \left(\frac{\alpha_l d_l^+}{\gamma_l} \right)$$

Subject to, (6)

$$z_k + d_k^- = Z_k^*$$

$$w_l - d_l^+ = W_l^*$$

$$\mathbf{v} \leq \mathcal{B}$$

$$k=1, 2, \dots, r; l=1, 2, \dots, s; z_k = f_k(x) \text{ and } w_l = f_l(x); \gamma_k = Z_k^* - \hat{Z}_k \text{ and } \gamma_l = \hat{W}_l - W_l^*.$$

For solution of General De Novo programming problem we propose the system (6).

The proposed weighted programming approach for solving De Novo Programming problem is illustrated by following example.

EXAMPLE

Let us first consider the numerical problem from Zeleny [2].

$$\text{Max } z_1 = 50x_1 + 100x_2 + 17.5x_3 \quad (\text{profits})$$

$$\text{Max } z_2 = 92x_1 + 75x_2 + 50x_3 \quad (\text{Quality})$$

$$\text{Max } z_3 = 25x_1 + 100x_2 + 75x_3 \quad (\text{Workers Satisfaction})$$

Subject to the available time restriction on different machine,

$$(7)$$

$$\begin{aligned} 12x_1 + 17x_2 &\leq 1400 && \text{(Milling Machine)} \\ 3x_1 + 9x_2 + 8x_3 &\leq 1000 && \text{(Lathe)} \\ 10x_1 + 13x_2 + 15x_3 &\leq 1750 && \text{(Grinder)} \\ 6x_1 + 16x_3 &\leq 1325 && \text{(Jig Saw)} \\ 12x_2 + 7x_3 &\leq 900 && \text{(Drill Press)} \\ 9.5x_1 + 9.5x_2 + 4x_3 &\leq 1075 && \text{(Band saw)} \end{aligned}$$

$$x_1, x_2, x_3 \geq 0.$$

With the unit prices of the resources (\$100 per hour)

$p_1 = 0.75$, $p_2 = 0.6$, $p_3 = 0.35$, $p_4 = 0.50$, $p_5 = 1.15$, and $p_6 = 0.65$.

The problem is to find the optimal portfolio of machine capacities under the present budget level $B = \$4658.75$.

The problem has been solved by using our proposed approach and the result obtained has been compared with those obtained by Zeleny [2], Li and Lee [6], and Nurullah [9].

To solve the problem by our proposed method, we first put the problem in the form (2) of a multi-objective De Novo programming model as follows:

Here, $\mathbf{pA} = \begin{bmatrix} 23.475 & 42.675 & 28.7 \end{bmatrix}$, \mathbf{A} is the coefficient matrix of the constraints.

Thus (7) takes the form,

$$\text{Max } z_1 = 50x_1 + 100x_2 + 17.5x_3$$

$$\text{Max } z_2 = 92x_1 + 75x_2 + 50x_3$$

$$\text{Max } z_3 = 25x_1 + 100x_2 + 75x_3$$

Subject to, (8)

$$23.475x_1 + 42.675x_2 + 28.7x_3 = 4658.75$$

Here the basic feasible solutions are: (198.455, 0, 0);

(0, 109.168, 0); (0, 0, 162.325).

The calculation of the optimistic and pessimistic objective values are shown in the Table 1.

Therefore, optimistic values (ideal values) and the pessimistic values (obtained by Luhandjula's Comparison Technique) of the objectives are indicated below:

$$I^* = \{ 10916.813, 18257.86, 12174.375 \} \text{ and}$$

$$\hat{\beta} = \{ 2840.7012, 8116.289, 4961.375 \}$$

Therefore the weighted goal programming form (6) of the given De novo programming problem becomes,

Table 2. Calculation of optimistic and pessimistic value of the objectives

	BFS			Objective Functions		
	x_1	x_2	x_3	z_1	z_2	z_3
1						
2	198.455	0	0	9922.790	18257.86 (Z_2^*)	4961.375 (Z_3^*)
3	0	109.168	0	10916.813 (Z_1^*)	8187.6098	10916.8

Table 4. Comparison of Optimized Objectives Values Obtained by Different Methods						
Objective Functions	Zeleny (1990)	Zimmerman (1978)	Li and Lee (1990)	Nandlal approach	Method 1	Method 2
Z_1	7656.87	6837.348	6837.348	8788.43	9922.811	10916.811
Z_2	12855.89	13135.11	13135.11	12912.43	18257.46	8187.609
Z_3	8572.40	8530.93	8530.92	8372.341	4961.43	10916.81
Closeness to the ideal	-	$\lambda = 0.49487$	$\lambda = 0.49487$	$d = 0.13177$	$a = 1.12307$	$a = 0.13177$
Weights	-	-	-	$\alpha_1 = 0.50$, $\alpha_2 = 0.25$, $\alpha_3 = 0.25$	-	$u_1 = 0.50$, $u_2 = 0.25$, $u_3 = 0.25$
Variables	x_1	92.48	98.125	98.125	93.401	198.4543
	x_2	20.90	6.691	6.691	35.343	0.0008109
	x_3	55.61	72.116	72.116	33.376	0

$$\text{Min } a = \frac{\beta_1 d_1^-}{z_1^* - \bar{z}_1} + \frac{\beta_2 d_2^-}{z_2^* - \bar{z}_2} + \frac{\beta_3 d_3^-}{z_3^* - \bar{z}_3}$$

Subject to,
(9)

$$50x_1 + 100x_2 + 17.5x_3 + n_1 = 10916.813$$

$$92x_1 + 75x_2 + 50x_3 + n_2 = 18257.86$$

$$25x_1 + 100x_2 + 75x_3 + n_3 = 12174.375$$

$$23.475x_1 + 42.675x_2 + 28.7x_3 = 4658.75$$

$$x_1, x_2, x_3 \geq 0$$

At first the weights attached to the normalized negative deviation variables are all taken as 1 and we call it as method 1. The solution obtained by using Lingo 13.0 of the weighted goal programming (9) of the given De Novo programming is equipped in Table 3. The solution yields objective values lying between ideal (optimistic) and pessimistic values. The value of a comes out to be 1.123072.

Table 3. Solution of De Novo program (9)

Decision variables	Value	Value of Objectives		
x_1	198.4543	Z_1	Z_2	Z_3
x_2	0.0008109	9922.811	18257.46	4961.43
x_3	0			

Next we find the solution of the above problem with weights attached to the normalized negative deviation variable such that the sum of the weights is unity. This method will be termed as method 2. Now varying the weights, we can find various solutions of the problem.

One of the typical solution is obtained for the values

$u_1 = 0.50, u_2 = 0.25, u_3 = 0.25$, are given in Table 4.

The solution of the problem (9) obtained by the techniques of Zeleny [2], Zimmerman [12], Li and Lee [6] and Nurullah [9] have been shown in Table 4 and are compared with those obtained by proposed methods presented in this paper.

CONCLUSION

An one phase approach for solving Multi-objective De novo programming problem (MODNPP) is presented here. In this reformulation less number of deviation variables has been used. For the large problems, this will certainly reduce the processing time. Further instead of using worst possible objective values (negative ideal values), another set of objective values based on Luhandjula's comparison technique have used as pessimistic values for solving MODNPP with weighted goal programming approach. These formulation gives the better objective values, which are seen in the Table 4. As a future prospect of research, the solution of MODNPP using other suitable method may be investigated.

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