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Optimal Load Flow Solution for Active Power Loss Minimization in Transmission System

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***Abstract* - Optimal power flow analysis has been rapidly developing from last five decades and has gained an extreme importance in modern power system economics and operation. Traditionally, it is widely used for economic load dispatch and various methods are available in literature to solve these problems. These methods generally need conventional load flow to be carried out at the beginning for solving the optimal load flow equations. The proposed method directly calculates unknowns in power system without doing conventional iterative load flow analysis. In this paper, minimization of power loss in transmission lines is taken as objective function while maintaining active and reactive power balance and voltage limits using Lagrange nonlinear optimization approach. The solution determines bus voltage magnitudes and angles such that objective is attained while satisfying equality and inequality constraints. Complete analysis is carried out using proposed method on a simple three bus system and compared the results with Newton-Raphson load flow solution. It is observed that power loss can be reduced using proposed method while maintaining voltage at all the buses within the limit. The optimization method is also tested on IEEE 5 bus system and results are presented in this paper.**

***Keywords:*--- Equality constraints, Lagrange multipliers, Inequality constraints, Objective function, Optimal power flow.**

I. INTRODUCTION

Load flow studies are necessary to ensure reliable, economic and quality power transfer from generating station to consumer end. Load flow analysis provides solution for the systems with specified generation, demand and network topology which has wide applications in system parameter control, economic load scheduling, planning and designing future power systems. The digital computer solution for load flow problem is first presented in [1], using admittance matrix. The load flow analysis has been done by many researchers using numerical methods like Gauss-Seidal method and the most favored Newton-Raphson method [2]. Further, impedance matrix method [3] is developed to improve convergence however, it requires high memory. To increase computation speed, decoupled load flow [4] is carried out with simplified algorithm but with some sacrifice in accuracy. All these methods could not avoid iteration process to get power flow solution and can be compared in terms of computational speed, convergence characteristics, accuracy and memory occupation [5]. Though conventional load flow gives solution for various unknown parameters, it need not results in an optimal solution in terms of cost of generation, power loss etc. Therefore, optimal power flow (OPF) solutions were developed to tackle this issue. There are various OPF methods available [6]-[8] to minimize cost of generation, transmission loss, transmission line reactive power flow etc.

Over half a century from pioneer work by Carpentier [9], OPF studies continue to evolve till date in rapidly developing modern deregulated and competitive electrical markets with integration of non-conventional generating units and nonlinear loads. Traditionally, deterministic optimization techniques have been widely used to solve nonlinear optimization problems with certain assumptions. Gradient method [10] solves OPF problem with 10 to 20 computations of Jacobean matrix depending upon number of control variables and used penalty functions to handle inequality constraints. Newton method [11], [12] handles inequality

constraints very well with fast convergence and not adversely
affected by number of control variables or inequalities. B. Stott and E. Hobson have proposed a fast and reliable linear programming (LP) iterative technique by taking advantage of sparse matrix [13] with sacrifice in accuracy due to linearization. Then after, a quadratic objective function is formed and solved by quadratic programming method with much higher accuracy than LP [14]. Further, a more efficient and fast algorithm of interior point method is developed in 1994[15].

Recently, non-deterministic or intelligent optimization methods like genetic algorithm(GA) and particle swarm optimization methods(PSO) are becoming popular to solve multi-objective problems with global optima with objective functions such as generation cost minimization,

loss minimization etc.[16], [17]. The intelligent methods need complex algorithms with large dimensions. All OPF methods developed have been successfully proved their capabilities to ensure optimal system operations, however a simple load flow solution approach is presented in this paper.
The classical objective function for all above OPF is generally to minimize the generation cost. Due to deregulation of power sector, the generated power exchange is made possible between the utilities. This requires huge transmission of power over a long distance which substantially increases transmission losses in the transmission lines. As per survey by Ministry of Power, the transmission and distribution losses in India in last year is above 20%, which is very high compared to global standards. Thus, though power sector has shown significant growth in power availability, losses are still a huge concern. Therefore, minimization of transmission loss is taken as the main objective for the proposed algorithm. The OPF problem is solved using Lagrange multiplier technique so that loss is minimized satisfying system constraints. The algorithm developed is simple and can be easily implemented. Compared to other methods, it does not require normal load flow to be carried out. But, it directly solves the Lagrange function to get the solution.
This paper is organized as follows:
The mathematical formulation of proposed method of OPF is described in section II. In section III, the method is implemented on a three bus system and simulation results are presented. Finally, section IV summarizes the conclusions of the paper.

II. PROPOSED OPF FORMULATION

A Lagrange multiplier technique [18] is used for problem formulation which consists of objective function, equality constraints and inequality constraints. Mathematically, the Lagrange function can be written as:

$$\mathcal{L} = f(\bar{x}) - \lambda_1\big(g_1(\bar{x})\big) - \lambda_2\big(g_2(\bar{x})\big) - \dots - \mu_1\big(h_1(\bar{x})\big) - \mu_2\big(h_2(\bar{x})\big) - \cdots \quad (1)$$

where, $\mathcal{L}$ is Lagrange function, $f(\bar{x})$ is objective function, $\bar{x}$ is dependent or state variable vector, $g_1(\bar{x})$, $g_2(\bar{x})$,… are the equality constraints, $h_1(\bar{x})$, $h_2(\bar{x})$,… are the inequality constraints and $\lambda_1, \lambda_2, \dots, \mu_1, \mu_2, \dots$ are Lagrange multipliers.
In the proposed method of OPF solution, minimization of total active power loss[P_{loss}] in the transmission lines is considered as objective function with two equality constraints as real and reactive power balance equations and inequality constraint as voltage magnitude error. Bus voltage magnitudes [$|V|$] and bus voltage angles [δ] in all the buses except slack bus are considered as dependent variables or state variables.
The formation of objective function, equality constraints, inequality constraints and Lagrange function are derived in terms of state variables and are explained below.

A. *Objective function*

$$\min f(\bar{x}) = min\ (P_{loss}) \quad (2)$$

To solve the OPF problem, the objective function must be expressed in terms of dependent variables.
Consider a transmission line between i^{th} bus and j^{th} bus as shown in Fig.1. The transmission line is represented by a Pi network with transfer admittance y_{ij} and shunt admittances y_{ij0} and y_{ji0}. The transmission line parameters form Y_{BUS} matrix for an interconnected system. The Y_{BUS} matrix elements are further used to form active power flows and reactive power flows through transmission lines.

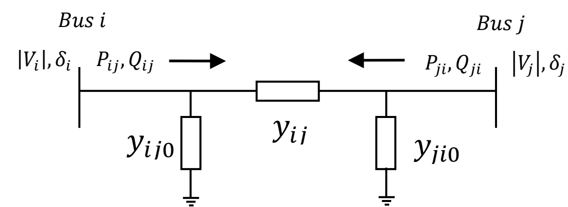

Fig.1. Single line representation of transmission line between i^{th} and j^{th} bus

The active power flow[P_{ij}] through transmission line between i^{th} and j^{th} bus is obtained by separating real part of apparent power flow equation as given below.

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$$P_{ij} = -|V_i|^2.\left|Y_{ij}\right|.cos\theta_{ij} + |V_i|.\left|V_j\right|.\left|Y_{ij}\right|.cos\left(\theta_{ij} - \delta_i + \delta_j\right) \quad (3)$$

where, $\left|Y_{ij}\right|$ is magnitude of short circuit transfer admittance of transmission line between i^{th} and j^{th} bus, θ_{ij} is angle of admittance, $|V_i|$ and $\left|V_j\right|$ are magnitudes of bus voltages and δ_i and δ_j are bus voltage angles at bus i and j respectively. The active power loss in line $i - j$ is the algebraic sum of P_{ij} and P_{ji}. For a power system with number of buses as $'N_b'$, the total active power loss can be written as:

$$P_{loss} = \sum_{i=1,i\neq j}^{Nb} \left(\sum_{j=1,j\neq i}^{Nb} P_{ij} \right) \quad (4)$$

By substituting equation (3) in equation (4), P_{loss} can be obtained in terms of state variables.
On the similar basis, the reactive power flow $[Q_{ij}]$ through transmission line between i^{th} bus and j^{th} bus is obtained by separating imaginary parts of apparent power flow equation as given below.

$$Q_{ij} = |V_i|^2.\left|Y_{ij}\right|.sin\theta_{ij} - |V_i|.\left|V_j\right|.\left|Y_{ij}\right|sin\left(\theta_{ij} - \delta_i + \delta_j\right) \quad (5)$$

The equation (5) is further used to calculate total reactive power loss in the transmission line, Q_{loss} as:

$$Q_{loss} = \sum_{i=1,i\neq j}^{Nb} \left(\sum_{j=1,j\neq i}^{Nb} Q_{ij} \right) \quad (6)$$

By substituting equation (5) in (6), Q_{loss} can be obtained in terms of state variables. This equation is used for forming equality constraints.

B. *Equality constraints*

Two equality constraints are considered in the proposed method. It is clear that under balanced condition, the total power generation should be equal to the addition of power demand and transmission losses. Hence, to meet this condition, both active and reactive power balance equations are considered as equality constraints.
Active power balance equation $[g_1(\bar{x})]$ is given by:

$$g_1(\bar{x}) = \sum_{i=1}^{Ng} P_{Gi} - \sum_{i=1}^{Nl} P_{Di} - P_{loss} \quad (7)$$

Reactive power balance equation $[g_2(\bar{x})]$is given by:

$$g_2(\bar{x}) = \sum_{i=1}^{Ng} Q_{Gi} - \sum_{i=1}^{Nl} Q_{Di} - Q_{loss} \quad (8)$$

where, P_{Gi} and Q_{Gi} are active and reactive power generation at i^{th} bus, P_{Di} and Q_{Di} are active and reactive power demands at i^{th} bus, N_g are number of generator buses and N_l are number of load buses. By substituting equations (4) and (6) in (7) and (8) respectively $g_1(\bar{x})$ and $g_2(\bar{x})$ can be obtained in state variables.

C. *Inequality constraints*

With change in load, the bus voltage magnitude changes and at a particular load, it may exceed the specified limit or there is a possibility of loss of voltage stability. This can be prevented by adding inequality constraints so that the bus voltage magnitude can be forced within the specified limits. In other words, the load bus will be a voltage controlled bus. For example, if i^{th}bus voltage magnitude, $|V_i|$ is violated the specified voltage, $\left|V_{spec}\right|$ then, voltage error is present which should be below 5%.

$$voltage\ error = \left|\left(\left|V_{spec}\right| - |V_i|\right)\right| \leq 0.05 \quad (9)$$

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Thus, inequality constraint is formed as:

$$h(\bar{x}) = 0.05 - \left|\left(\left|V_{spec}\right| - \left|V_i\right|\right)\right| \quad (10)$$

D. Inequality constraint

Lagrange Function with objective function and constraints is written as:

$$\mathcal{L} = P_{loss} - \lambda_1\left(\sum_{i=1}^{Ng} P_{Gi} - \sum_{i=1}^{Nl} P_{Di} - P_{loss}\right) - \lambda_2\left(\sum_{i=1}^{Ng} Q_{Gi} - \sum_{i=1}^{Nl} Q_{Di} - Q_{loss}\right) - \mu\left(0.05 - \left|\left(\left|V_{spec}\right| - \left|V_i\right|\right)\right|\right) \quad (11)$$

The unknowns can be obtained by equating partial derivatives with respect to $|V|$ and δ in all buses except slack bus as well as with respect to λ_1, λ_2 and μ to zero and solving them simultaneously by Kuhn-Tucker conditions [18] as given below:

$$\frac{\partial\mathcal{L}}{\partial|V_2|} = 0, \frac{\partial\mathcal{L}}{\partial|V_3|} = 0, \dots, \frac{\partial\mathcal{L}}{\partial|V_{Nb}|} = 0 \quad (12)$$

$$\frac{\partial\mathcal{L}}{\partial|\delta_2|} = 0, \frac{\partial\mathcal{L}}{\partial|\delta_3|} = 0, \dots, \frac{\partial\mathcal{L}}{\partial|\delta_{Nb}|} = 0 \quad (13)$$

$$\frac{\partial\mathcal{L}}{\partial\lambda_1} = 0, \quad \frac{\partial\mathcal{L}}{\partial\lambda_2} = 0, \quad \frac{\partial\mathcal{L}}{\partial\bar{\mu}} = 0 \quad (14)$$

$$\bar{\mu}.\left(h(\bar{x})\right) = 0 \quad (15)$$

By solving $(12), (13), (14)$ and (15), value of unknown $|V|$ and δ in all buses can be found.

III. SIMULATION RESULTS

To validate the algorithm, following cases are considered for clarity. CASE (1) and (2) are tested in a three bus system.

CASE (1): Optimal load flow by considering only equality constraints.

CASE (2): Optimal load flow by considering only equality and inequality constraints.

CASE (3): Optimal load flow for IEEE 5 bus system.

The single line diagram of a three bus system considered for analysis is shown in Fig.2. Bus no. 1 is taken as slack bus and 2 and 3 are load buses. The system bus data and line data is given in TABLE I and II respectively.

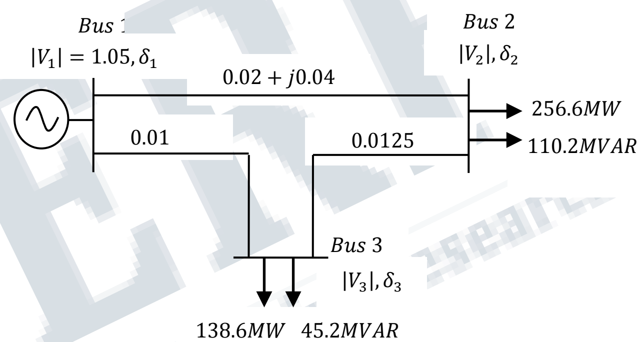

Fig.2. Three bus system under consideration. [19]

SYSTEM LINE DATA [19]

Sending end bus	Receiving end bus	Resistance $(p.u.)$	Reactance $(p.u.)$
1	2	0.02	0.04
2	3	0.0125	0.025
3	1	0.01	0.03

SYSTEM BUS DATA [19]

Bus	Generation		Load		$\|V\|$ $(p.u.)$	δ (deg)
	P_G (MW)	Q_G $(MVAR)$	P_D (MW)	Q_D $(MVAR)$		
1	0	0	0	0	1.05	0
2	0	0	256.6	110.2	1	0
3	0	0	138.6	45.2	1	0

CASE (1): Optimal load flow solution by considering only equality constraint

A MATLAB program is developed to implement OPF solution and results are shown in the table below.

The classical Newton-Raphson load flow (NRLF) is also carried out and the state variables obtained by both methods are compared in TABLE III. Also, a MATLAB simulation model for the test system is developed and load flow solution is obtained. It is observed that the NRLF program results are exactly matching with the simulation results. Further line-flows obtained using both the methods are shown in TABLE IV and total power loss and generation powers obtained are shown TABLE V.

STATE VARIABLES FROM NRLF AND OPF- CASE(1)

Sr. No.	Variables	NRLF Method	OPF Method
1	$\|V_2\|\ (p.u)$	0.9818	0.98887
2	$\delta_2\ (deg.)$	-3.5035	-3.04962
3	$\|V_3\|\ (p.u.)$	1.0012	0.99778
4	$\delta_3\ (deg.)$	-2.8624	-3.19275

LINE FLOWS FROM NRLF AND OPF SOLUTION- CASE(1)

Sr. No.	Power Flows in lines	NRLF Method	OPF Method
1	$P_{12}+jQ_{12}$	$199.5+j84$	$175.71+j75.89$
2	$P_{21}+jQ_{21}$	$-191-j67$	$-169-j62.58$
3	$P_{23}+jQ_{23}$	$-65.6-j43.2$	$-6.23-j32$
4	$P_{32}+jQ_{32}$	$66.4+j44.8$	$6.24+j32.29$
5	$P_{13}+jQ_{13}$	$210+j105$	$231.2+j111.019$
6	$P_{31}+jQ_{31}$	$-205-j90$	$-225.26-j93.1$

P_{LOSS} AND P_G FROM NRLF AND OPF -CASE(1)

Sr. No.	Variables	NRLF Method	OPF Method
1	$P_{loss}\ (MW)$	14.3	12.792
2	$Q_{loss}(MVAR)$	33.6	31.568
3	$P_G(MW)$	409.500	408.02
4	$Q_G(MVAR)$	189.000	187.04

From TABLE III, IV and V, it is observed that for reactive power demand of $155.4MVAR$, the state variables that is bus voltage magnitudes and angles obtained by OPF solution are such that, the active power loss is minimized from $14.3MW$ to $12.792MW$ satisfying the equality constraints. With the OPF solution values, the active and reactive power flows through the lines gets changed which reduces the total active power loss, P_{loss}. This further reduces the active power generation, P_G and reactive power generation, Q_G by the slack bus.

However, if the reactive power demand, Q_D is increased from $155.4MVAR$ to $380.0MVAR$ keeping active power demand constant, it is observed that, the voltage at bus 2 is reduced from $0.9818p.u.$ to $0.9224p.u.$ which is below specified voltage limit. The line flows are such that there is increase in active power loss and reactive power loss. This further burdens the slack bus generator to generate more active and reactive power to supply the increased demand as well as additional losses. OPF solution with only equality constraint is obtained and compared with NRLF in TABLE VI.

NRLF AND OPF SOLUTION WITH $Q_D = 380.0$MVAR CASE(1)

Sr. No.	Variables	NRLF Method	OPF solution
1	$\|V_2\|\ (p.u.)$	0.9224	0.93958
2	$\delta_2\ (deg.)$	-2.2739	-2.4689
3	$\|V_3\|\ (p.u.)$	0.9687	0.95926
4	$\delta_3\ (deg.)$	-2.3841	-2.30306
5	$P_{loss}\ (MW)$	29.6168	24.514
6	$Q_{loss}(MVAR)$	67.6109	58.896
7	$P_G(MW)$	424.818	419.74
8	$Q_G(MVAR)$	447.611	438.95

By comparing the results obtained by NRLF and proposed OPF solution, it is observed from TABLE VI that the total active power loss is reduced from 29.6168MW to 24.514MW but the voltage at bus 2 is still below specified voltage limit in both methods. Therefore, to limit the bus voltage within the specified limit, inequality constraints are required to be added which is explained in CASE (2).

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CASE (2): Optimal load flow by considering only equality and inequality constraints

Considering 5% voltage variation, voltage is allowed to vary from $0.95 p.u.$ to $1.05\ p.u.$ As voltage at bus 2 is violated, inequality constraint is put to limit voltage at bus 2 only. The OPF solution by adding inequality constraints is given in TABLE VII.

It is observed from TABLE VI and VII that, with inequality constraints, the voltage at bus 2 is forced to remain within the specified limit and increased from $0.93958\ p.u.$ to $0.9528\ p.u.$ Also, the active power loss gets reduced from $24.514 MW$ to $22.74 MW$.

OPF SOLUTION FOR CASE(2)

Sr. No.	Variables	OPF solution
1	$\|V_2\|\ (p.u.)$	0.9528
2	$\delta_2\ (deg.)$	-2.2288
3	$\|V_3\|\ (p.u.)$	0.9511
4	$\delta_3\ (deg.)$	-2.53247
5	$P_{loss}\ (MW)$	22.74
6	$Q_{loss}(MVAR)$	57.21
7	$P_G(MW)$	417.83
8	$Q_G(MVAR)$	437.06

CASE (3): Optimal load flow for IEEE 5 bus system.

The proposed OPF solution method is implemented on IEEE-5 bus system as shown in Fig.3. The IEEE-5 bus system data is given in TABLE VIII and IX.

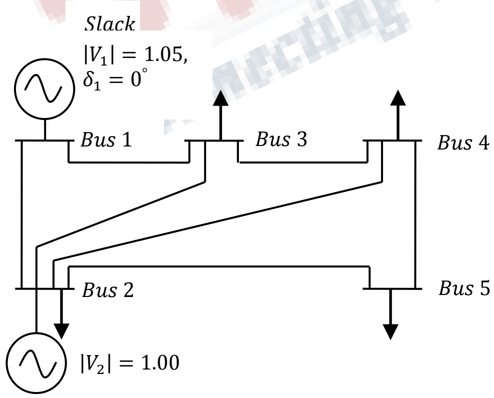

Fig.3. IEEE-5 bus system [20]

The OPF is implemented on IEEE 5 bus system in which bus 1 is slack bus, bus 2 is a PV bus and bus 3, 4 and 5 are load buses. The voltage at this bus is kept constant by generating required reactive power by generator connected to it.

SYSTEM LINE DATA FOR IEEE 5 BUS SYSTEM-CASE(3)[20]

Sending end bus	Receiving end bus	Resistance $(p.u.)$	Reactance $(p.u.)$	B/2 $(p.u.)$
1	2	0.02	0.06	0.03
1	3	0.08	0.24	0.025
2	3	0.06	0.18	0.02
2	4	0.06	0.18	0.02
2	5	0.04	0.12	0.015
3	4	0.01	0.03	0.01
4	5	0.08	0.24	0.025

it. OPF is carried out by keeping voltage of PV bus, V_2 at a fixed value of $1.00 p.u.$ to meet the objective of minimization of active power loss. The OPF solution and NRLF solution are compared in TABLE X.

SYSTEM BUS DATA FOR IEEE 5 BUS SYSTEM-CASE(3)[20]

Bus	Generation		Load		$\|V\|$	δ
	P_G (MW)	Q_G $(MVAR)$	P_D (MW)	Q_D $(MVAR)$	$(p.u.)$	(deg)
1	0	0	0	0	1.06	0
2	40	30	20	10	1.00	0
3	0	0	45	15	1.00	0
4	0	0	40	5	1.00	0
5	0	0	60	10	1.00	0

NRLF AND OPF SOLUTION FOR IEEE 5 BUS SYSTEM-CASE(3)

Sr. No.	Variables	NRLF Method	OPF Solution with $\|V_2\|$ fixed
1	$\|V_2\|\ (p.u)$	1.00	1.00
2	$\delta_2\ (deg.)$	-2.06	-2.5902
3	$\|V_3\|\ (p.u.)$	0.9872	0.99828
4	$\delta_3\ (deg.)$	-4.64	-2.3284

5	$\|V_4\|\ (p.u)$	0.9841	0.996
6	$\delta_4\ (deg.)$	-4.96	-2.3301
7	$\|V_5\|\ (p.u)$	0.9717	0.98995
8	$\delta_5\ (deg.)$	-5.76	-2.3346
9	$P_{loss}\ (MW)$	6.1232	3.6276

It is observed that OPF solution with fixed voltage at PV bus gives voltage magnitudes and angles such that the active power loss is reduced from 6.1232MW to3.6276MW. Thus, by proposed method, it is possible to reduce transmission loss using optimal power flow solution while maintaining voltage within the limit.

CONCLUSION

The proposed optimal power flow problem is solved by considering active power loss minimization in transmission line as objective taking power balance equations as equality constraints and voltage limits as inequality constraint. The system unknowns are directly calculated without carrying out conventional load flow. This makes the algorithm simple and easy to implement. Complete analysis is carried out for 3 bus system and results are compared with NRLF solution and observed that the active power transmission loss is reduced by 23.21%. The algorithm is also tested for IEEE 5 bus system. It is observed that, the OPF solution maintains specified voltage at PV bus meeting the objective of active power loss minimization while maintaining equality constraints. Further, the method can be implemented on the systems with nonlinear loads.

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