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Generation of Fractal Flower by Two Chaotic Attractors

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Abstract— This work finds a link between the two important mathematical topics: fractal and chaos theory. From this link we generate a fractal model of flower. Fractal processes and chaotic systems were widely studied in many areas of research. The numerical simulations are presented to illustrate the effectiveness of the theory using C++.

Index Terms—Fractal theory, multiplication process, separation process, duplication process, chaos theory.

I. INTRODUCTION

In the literature, we found many scientists who proposed dynamical systems based on chaotic attractors and explored them in many applications. This paper presents a new fractal behavior or fractal flower chaotic system based on Julia's fractal process, chaotic attractors, and recurrent equation system.

II. MATHEMATICAL FORMULATION

In this section, we introduce the mathematical formulation and method to obtain the novel system using the combination between two chaotic attractors, fractal process of Julia and the recurrent equation system.

This approach generates a form of fractal flower.

The chaotic attractor is equivalent to the system described as follows:

Let S denote the 2-sphere in R^3 ,

$S = \{(x, y, z) \in R^3 , \quad |x|^2 + |y|^2 + |z|^2 = 1\}$.

$$\begin{cases} x = \cos(\psi)\cos(\varphi) \\ y = \cos(\psi)\sin(\varphi) \\ z = \sin(\psi) \end{cases} \quad (1)$$

With $\psi = \theta_1 t$; $\varphi = \theta_2 t$;

We elaborate from the previous equation the following recurrent system:

$$\begin{cases} x_{n+1} = \mu x_n - \nu y_n + \beta \frac{z_n}{\sqrt{x_n^2+y_n^2}}(\gamma x_n - \lambda y_n) \\ y_{n+1} = \mu x_n - \nu y_n + \beta \frac{z_n}{\sqrt{x_n^2+y_n^2}}(\lambda x_n + \gamma y_n) \\ z_{n+1} = 2\alpha z_n - z_{n-1} \end{cases} \quad (2)$$

with $\alpha^2 + \beta^2 = 1$, $\gamma^2 + \lambda^2 = 1$; $\mu = \alpha\gamma$ and $\nu = \alpha\lambda$.

We modify the condition of $\alpha^2 + \beta^2 = 1$ by $\alpha^2 + \beta^2 \simeq 1$.

We combine the system 2 with the Lorenz system. The latter has become one of the paradigms in the research of chaos, and it is described by y_1, y_2 and y_3 which are the system states whose constant parameters are σ, ρ and β.

$$M1 \begin{cases} \dot{y}_1 = \sigma(y_2 - y_1) \\ \dot{y}_2 = \rho y_1 - y_2 - y_1 y_3 \\ \dot{y}_3 = (y_1 y_2 - \beta y_3) \end{cases} \quad (3)$$

Then we treat the last value of Lorenz with the following system:

$$M2 \begin{cases} \dot{z}_1 = \sigma(\dot{y}_2 - \dot{y}_1) \\ \dot{z}_2 = \rho \dot{y}_1 - \dot{y}_2 - \dot{y}_1 \dot{y}_3 \\ \dot{z}3 = (\dot{y}_1 \dot{y}_2 - \beta \dot{y}_3) \end{cases} \quad (4)$$

After that, we apply the Julia process on cascades on the difference states of M1 and M2.

$$(x_g, y_g) = PoP(\dot{y}_1 - \dot{z}_1, \dot{y}_2 - \dot{z}_2)$$

The simulation result shows a new fractal flower see figure 1.

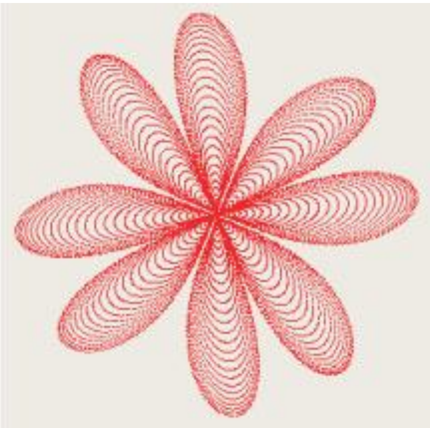

Fig 1: Fractal flower

A. Simulation of multiplication process

Figure 2 shows some results of applying the multiplication process on the fractal flower.

The number of leaves is doubled, for each following fractal flower.

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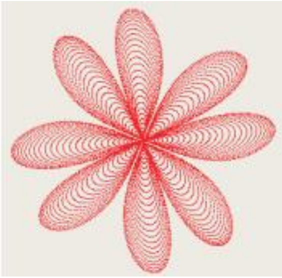
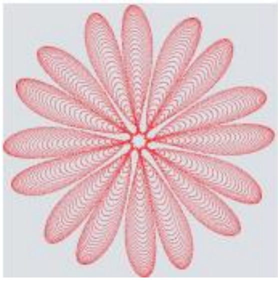


(a)



(b)

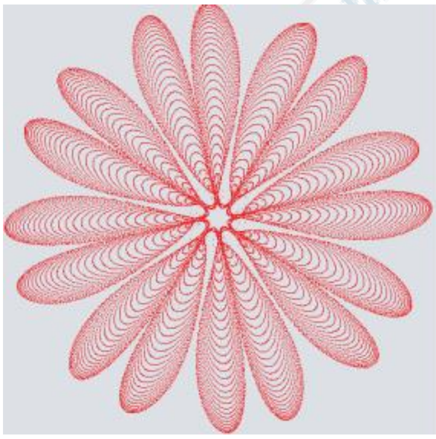
(c)

Fig 2: Multiplication process applied on fractal flower.

The same methodologies of the multiplication processes are applied in a chaotic attractor to generate more scrolls see paper [5].

B. Simulation of separation process

Figure 3 shows the result of applying the separation process on fractal flower. Figure 3(b) shows two similar flowers. Each one has different scales of leaves, and the number of leaves is divided by two.



(a) Fractal flower with 16 leaves

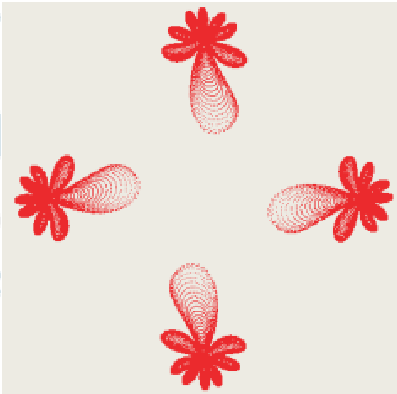
(b) Two fractal flowers with 8 leaves

Fig 3: Separation process applied on fractal flower.

C. Simulation of duplication process

Figure 4 shows some results of applying the duplication process on the fractal flower.

(a) Two fractal flowers



(b) Four fractal flowers

Fig 4: Duplication process

III. CONCLUSION

In this paper, we present the methodology of generation a fractal form of flower. Some numerical simulation results are provided to show the effectiveness of the theory proposed in this work.

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